Unified solution of optimal active fault detection and optimal control

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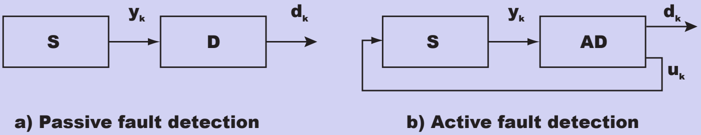
Outline

Introduction

Passive and active fault detection problem

- Passive fault detection - detector passively uses available information to decide on faults
- Active fault detection - active detector provides decision and input signal that should improve fault detection



Introduction – cont'd

Active fault detection problem

- Deterministic [Campbell&Nikoukhah(2004)] and stochastic [Zhang(1989), Kerestecioglu(1993)] models of the observed system are used
- A general formulation of the active fault detection problem in stochastic framework is missing and the relation between active fault detection and the optimal control is not considered
- Known approaches use information is such way that the consequences of the current decision in future steps are not considered and the future losses are not taken into account

Introduction – cont'd

Information processing strategies

- Open loop (OL) - only a priori information is used
- Open loop feedback (OLF) - all available information up to current time step is used, but the future information is not considered
- Closed loop (CL) - all available information up to current time step is used and the availability of the future information is considered as well; so the future losses are taken into account and this strategy provides the lowest value of a criterion (i.e. $J^{CL} \leq J^{OLF} \leq J^{OL}$)

Introduction – cont'd

Goals

- Propose a unified formulation of active fault detection problem
- Solve given problem using CL information processing strategy
- Focus on design of active detector containing dual controller
- Discuss two interesting special cases of active fault detection

Problem formulation

Description of the observed system for $k \in \mathcal{T} = \{0, \ldots, F\}$

$$
\begin{aligned}
\mathbf{x}_{k+1} &= \mathbf{f}_k\left(\mathbf{x}_k, \boldsymbol{\mu}_k\, \mathbf{u}_k, \mathbf{w}_k\right) \\
\boldsymbol{\mu}_{k+1} &= \mathbf{g}_k\left(\boldsymbol{\mu}_k, \mathbf{e}_k\right) \\
\mathbf{y}_k &= \mathbf{h}_k\left(\mathbf{x}_k, \boldsymbol{\mu}_k, \mathbf{v}_k\right)
\end{aligned}
$$

$\mathbf{f}_k$, $\mathbf{g}_k$ and $\mathbf{h}_k$ are known function; $\mathbf{x}_k \in \mathcal{R}^{n_x}$ is controllable part of the state; $\boldsymbol{\mu}_k \in \mathcal{M} \subset \mathcal{R}^{n_\mu}$ is uncontrollable part of the state and represents faults; $\mathbf{u}_k \in \mathcal{U}_k \subset \mathcal{R}^{n_u}$ is input, $\mathbf{y}_k \in \mathcal{R}^{n_y}$ is output; $\{\mathbf{w}_k\}$, $\{\mathbf{e}_k\}$ and $\{\mathbf{v}_k\}$ are white and mutually independent random sequences

Problem formulation – cont'd

Description of general active detector $k \in \mathcal{T}$

$$\begin{bmatrix} \mathbf{d}_k \\ \mathbf{u}_k \end{bmatrix} = \boldsymbol{\rho}_k\left(\mathbf{I}_0^k\right) = \begin{bmatrix} \boldsymbol{\sigma}_k\left(\mathbf{I}_0^k\right) \\ \gamma_k\left(\mathbf{I}_0^k, \mathbf{d}_k\right) \end{bmatrix}$$

Functions $\boldsymbol{\rho}_k$ are unknown and should be designed. Both decision $\mathbf{d}_k$ and input $\mathbf{u}_k$ are generated by the active detector. Information vector $\mathbf{I}_0^k = \left[\mathbf{y}_0^{k\,T}, \mathbf{u}_0^{k-1\,T}, \mathbf{d}_0^{k-1\,T}\right]^T$ contains all information received up to time step k.

Problem formulation – cont'd

Design criterion for active detector containing dual controller

$$J\left(\rho_0^F\right) = \mathrm{E}\left\{\sum_{i=0}^{F} L_k^{\mathrm{d}}(\mathbf{d}_k, \boldsymbol{\mu}_k) + \alpha_k L_k^{\mathrm{c}}(\mathbf{x}_k, \mathbf{u}_k)\right\}$$

The cost function L_k^{d} penalizes a difference between $\mathbf{d}_k$ and $\boldsymbol{\mu}_k$. The cost function L_k^{c} penalizes $\mathbf{x}_k$ and $\mathbf{u}_k$. The coefficient $\alpha_k \geq 0$ sets a desired compromise between active detection and dual control objectives.

Unified solution

Solution of dynamic optimization problem

- Minimization of the criterion $J(\rho_0^F)$ is solved using the backward recursive equation (BRE)

$$V_k^*\left(\mathbf{I}_0^k\right) = \min_{\substack{\mathbf{d}_k \in \mathcal{M} \\ \mathbf{u}_k \in \mathcal{U}_k}} \mathrm{E}\Big\{ L_k^{\mathrm{d}}(\mathbf{d}_k, \boldsymbol{\mu}_k) + \alpha_k L_k^{\mathrm{c}}(\mathbf{x}_k, \mathbf{u}_k) + V_{k+1}^*\left(\mathbf{I}_0^{k+1}\right) |\mathbf{I}_0^k, \mathbf{u}_k, \mathbf{d}_k \Big\}$$

  with initial condition $V_{F+1}^* = 0$ and minimal value of the criterion J is $J^* = \mathrm{E}\left\{V_0(\mathbf{I}_0)\right\}$
- The filtering pdf $p\left(\mathbf{x}_k, \boldsymbol{\mu}_k | \mathbf{y}_0^k, \mathbf{u}_0^k, \mathbf{d}_0^k\right)$ and the predictive pdf $p\left(\mathbf{y}_{k+1} | \mathbf{y}_0^k, \mathbf{u}_0^k, \mathbf{d}_0^k\right)$ are required to solve BRE

Unified solution – cont'd

Filtering and predictive pdf's

- It can be shown the following identities hold

$$p\left(\mathbf{x}_k, \boldsymbol{\mu}_k | \mathbf{y}_0^k, \mathbf{u}_0^k, \mathbf{d}_0^k\right) = p\left(\mathbf{x}_k, \boldsymbol{\mu}_k | \mathbf{y}_0^k, \mathbf{u}_0^{k-1}\right)$$

$$p\left(\mathbf{y}_{k+1} | \mathbf{y}_0^k, \mathbf{u}_0^k, \mathbf{d}_0^k\right) = p\left(\mathbf{y}_{k+1} | \mathbf{y}_0^k, \mathbf{u}_0^k\right)$$

- Note: Both identities were mathematically derived contrary to [Peterka, Automatica, January 1981] where the only first identity was simply stated as natural condition of control.

Unified solution – cont'd

Active detector containing dual controller design

- Using previous identities the original BRE can be rewritten to the following form

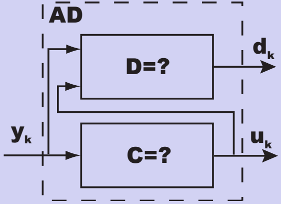
$$V_k^*\left(\mathbf{y}_0^k,\mathbf{u}_0^{k-1}\right)=\min_{\mathbf{d}_k\in\mathcal{M}}\mathrm{E}\left\{L_k^{\mathrm{d}}(\mathbf{d}_k,\boldsymbol{\mu}_k)|\mathbf{y}_0^k,\mathbf{u}_0^{k-1},\mathbf{d}_k\right\}+$$
$$\min_{\mathbf{u}_k\in\mathcal{U}_k}\mathrm{E}\left\{\alpha_k L_k^{\mathrm{c}}(\mathbf{x}_k,\mathbf{u}_k)+V_{k+1}^*\left(\mathbf{y}_0^{k+1},\mathbf{u}_0^k\right)|\mathbf{y}_0^k,\mathbf{u}_0^k\right\}$$

- Main new result

$$\mathbf{d}_k^*=\arg\min_{\mathbf{d}_k\in\mathcal{M}}\mathrm{E}\left\{L_k^{\mathrm{d}}(\mathbf{d}_k,\boldsymbol{\mu}_k)|\mathbf{y}_0^k,\mathbf{u}_0^{k-1},\mathbf{d}_k\right\}$$
$$\mathbf{u}_k^*=\arg\min_{\mathbf{u}_k\in\mathcal{U}_k}\mathrm{E}\left\{\alpha_k L_k^{\mathrm{c}}(\mathbf{x}_k,\mathbf{u}_k)+V_{k+1}^*\left(\mathbf{y}_0^{k+1},\mathbf{u}_0^k\right)|\mathbf{y}_0^k,\mathbf{u}_0^k\right\}$$

Unified solution – cont'd

Active detector containing dual controller design – remarks



- The optimal decision $\mathbf{d}_k^*$ and the optimal input $\mathbf{u}_k^*$ are chosen independently, but dual controller respect future decisions and inputs
- The block generating optimal decision $\mathbf{d}_k^*$ passively utilizes given information
- The optimal input $\mathbf{u}_k^*$ is a compromise between exciting and controlling

Special cases

Two special cases

- *Active detector*
  an active detector has to be designed in such a way that input signal improves fault detection only
- *Active detector containing given input signal generator*
  - a block generating input signal is given in advance
  - a block generating decision has to be designed

The first special case

Active detector design

- Control objective is not considered (i.e. $\alpha_k = 0, \forall k \in \mathcal{T}$) and then the BRE is given as

$$V_k^*(\mathbf{y}_0^k, \mathbf{u}_0^{k-1}) = \min_{\mathbf{d}_k \in \mathcal{M}} \mathrm{E}\left\{ L_k^d(\mathbf{d}_k, \boldsymbol{\mu}_k) | \mathbf{y}_0^k, \mathbf{u}_0^{k-1}, \mathbf{d}_k \right\} + \min_{\mathbf{u}_k \in \mathcal{U}_k} \mathrm{E}\left\{ V_{k+1}^*(\mathbf{y}_0^{k+1}, \mathbf{u}_0^k) | \mathbf{y}_0^k, \mathbf{u}_0^k \right\}$$

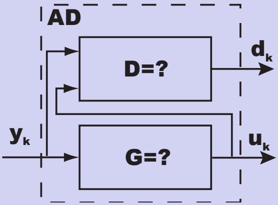
- Two minimization problems are solved simultaneously

$$\mathbf{d}_k^* = \arg \min_{\mathbf{d}_k \in \mathcal{M}} \mathrm{E}\left\{ L_k^{\mathrm{d}}(\mathbf{d}_k, \boldsymbol{\mu}_k) | \mathbf{y}_0^k, \mathbf{u}_0^{k-1}, \mathbf{d}_k \right\}$$

$$\mathbf{u}_k^* = \arg \min_{\mathbf{u}_k \in \mathcal{U}_k} \mathrm{E}\left\{ V_{k+1}^*\left(\mathbf{y}_0^{k+1}, \mathbf{u}_0^k\right) | \mathbf{y}_0^k, \mathbf{u}_0^k \right\}$$

The first special case – cont'd

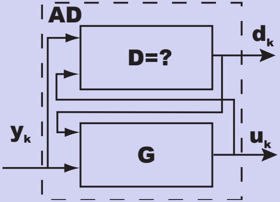
Active detector – remarks



- The optimal decision $\mathbf{d}_k^*$ and the optimal input $\mathbf{u}_k^*$ are independent
- The optimal input $\mathbf{u}_k^*$ only excites observed system to improve future decisions
- Therefore, the design of generator $\mathbf{G}$ respects future decisions

The second special case

Active detector containing given input signal generator



- Similarly to the first special case the control objective is not considered
- The input signal generator **G**, depending on decisions, is given by the known function $\gamma_k\left(\mathbf{I}_0^k, \mathbf{d}_k\right)$
- Note: If $\gamma_k\left(\mathbf{y}_0^k, \mathbf{u}_0^{k-1}\right)$ then a passive detection problem is solved

The second special case – cont'd

Active detector containing given input signal generator design

- For this special case, the BRE has the following form

$$V_k^*(\mathbf{I}_0^k) = \min_{\mathbf{d}_k \in \mathcal{M}} \left[\mathrm{E}\left\{ L_k^d(\mathbf{d}_k, \boldsymbol{\mu}_k) + V_{k+1}^*(\mathbf{I}_0^{k+1}) | \mathbf{I}_0^k, \mathbf{d}_k \right\} \right]_{\mathbf{u}_k = \gamma_k(\mathbf{I}_0^k, \mathbf{d}_k)}$$

- Only one minimization problem is solved

$$\mathbf{d}_k^* = \arg \min_{\mathbf{d}_k \in \mathcal{M}} \left[\mathrm{E}\left\{ L_k^d(\mathbf{d}_k, \boldsymbol{\mu}_k) + V_{k+1}^*(\mathbf{I}_0^{k+1}) | \mathbf{I}_0^k, \mathbf{d}_k \right\} \right]_{\mathbf{u}_k = \gamma_k(\mathbf{I}_0^k, \mathbf{d}_k)}$$

$$\mathbf{u}_k^* = \gamma_k\left(\mathbf{I}_0^k, \mathbf{d}_k^*\right)$$

Numerical example

Description of the observed system for $k \in \mathcal{T} = \{0, 1\}$

$$\mu_k = 1: \; x_{k+1} = 0.99x_k + u_k + \sqrt{0.25}w_k$$
$$y_k = 2x_k + \sqrt{0.25}v_k$$
$$\mu_k = 2: \; x_{k+1} = 1.01x_k + 0.99u_k + \sqrt{0.25}w_k$$
$$y_k = 2x_k + \sqrt{0.25}v_k$$
$$\mu_k = 3: \; x_{k+1} = 0.5x_k + 1.5u_k + \sqrt{0.25}w_k$$
$$y_k = 1.5x_k + \sqrt{0.25}v_k$$

$$P_{i,j} = \begin{cases} 0.9 \textit{ iff } i = j, & p(w_k) = p(v_k) = \mathcal{N}\{0, 1\}, p(x_0) = \mathcal{N}\{1, 0.1\} \\ 0.05 \textit{ iff } i \neq j, & P(\mu_0 = 1) = 0.4, P(\mu_0 = 2) = P(\mu_0 = 3) = 0.3 \end{cases}$$

Numerical example – cont'd

Cost functions for $k \in \mathcal{T}$

$$d_k = \mu_k \Rightarrow L_k^d(d_k, \mu_k) = 0 \qquad L_k^c(x_k, u_k) = x_k^2 + u_k^2$$

$$d_k \neq \mu_k \Rightarrow L_k^d(d_k, \mu_k) = 1 \qquad \alpha = 0.01$$

Input signal generator for $k \in \mathcal{T}$

- Set of input signal $\mathcal{U}_k = \{-1, 1\}$
- Description of the given generator $u_k = \gamma_k(d_k)$ for the second special case

$$d_k = 1 \vee d_k = 3 \Rightarrow u_k = -1,$$
$$d_k = 2 \Rightarrow u_k = 1$$

Numerical example – cont'd

Comparison of active detectors

- Active detector containing given input generator (ADGG)
- Active detector (ADG)
- Active detector containing dual controller (ADC)

- These approaches are compared by means of average number of wrong decisions (NWD)

Monte Carlo simulation results

	ADGG	ADG	ADC
NWD	1.2138	1.0931	1.1008

Conclusion

Summary

- A problem of simultaneous active fault detection and dual control was formulated and solved as an optimization problem
- It is shown that the optimal active detector containing dual controller consists of an optimal passive detector and an optimal dual controller which excites and controls the system
- Two interesting special cases were also discussed
  - Optimal active detector without fixed input signal generator
  - Optimal active detector with fixed input signal generator