Prediction Error Dual Controller

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Dual control (Feldbaum 1960)

- Arises in control problem with insufficient knowledge of parameters
- Two conflicting goals – meet control objective and improve estimation
- Optimal dual control problem – mostly cannot be solved analytically

Suboptimal solutions (Tse *et al.* 1973, Wittenmark *et al.* 1975, Millito *et al.* 1982,. . .)

- Augmenting the cautious control law (Bicriterial controller,. . .)
- Modification of criterion (e.g. IDC, ASOD,. . .)
- Criterion approximation (e.g. WDC, Utility cost,. . .)

Requirements of feasible solution

- computationally moderate (only one step ahead horizon)
- clear interpretation
- guarantees sufficient control quality

Properties of feasible solutions

Bicriterial Control
(Filatov *et al.*,1997; Šimandl and Flídr,2001;Flídr and Šimandl,2005)

- ✔ two objectives ⇒ two criteria
- ✔ probing is binary signal with preset amplitude specified by designer
- ✘ How to choose probing amplitude?

Innovations Dual Control (ICD)
(Millito *et al.* 1982)

- ✔ criteria enhanced with weighted innovation sequence
- ✔ respects uncertainty but it is less cautious
- ✔ reasonable weight lies within the set $< 0, 1 >$
- ✘ The controller has only overall information about quality of estimation

Innovations Dual Controller

- ✓ The controlled system considered SISO ARMAX
- ✓ Parameter estimation by Recursive least square method

The criterion

$$J_k^c(u_k) = E\left\{(y_{k+1} - \bar{y}_{k+1})^2 - \lambda_{k+1}\nu_{k+1}^2 | \mathfrak{I}_k\right\}$$

$$\mathfrak{I}_k = (u_0, \ldots, u_{k-1}, y_0, \ldots, y_k)$$

- ➢ the parameter $\lambda \geq 0$ specifies the degree of compromise between control and estimation objectives
- ➢ the innovations sequence ν_{k+1} provides overall information about estimation quality
- ➢ the *certainty equivalent* and *cautious* controllers are special cases of IDC

Goal - Generalization of the basic IDC

- ➢ Generalization to the class of MIMO state space systems with random variables described by an arbitrary probability density functions (pdf's)
- ➢ Design of criteria that would rate the state augmented with uncertain parameters instead of only the system output
- ➢ Analysis of the designed criteria and the control law

Consider the MIMO stochastic system

$$\begin{aligned} \boldsymbol{s}_{k+1} &= \boldsymbol{A}(\boldsymbol{\theta}_k)\boldsymbol{s}_k + \boldsymbol{B}(\boldsymbol{\theta}_k)\boldsymbol{u}_k + \boldsymbol{w}_k, \\ \boldsymbol{\theta}_{k+1} &= \boldsymbol{\Phi}_k\boldsymbol{\theta}_k + \boldsymbol{\epsilon}_k, \qquad k = 0, \ldots, N-1 \\ \boldsymbol{y}_k &= \boldsymbol{C}\boldsymbol{s}_k + \boldsymbol{v}_k, \end{aligned}$$

$\boldsymbol{s}_k \in \mathbb{R}^n$	...	non-measurable state
$\boldsymbol{\theta}_k \in \mathbb{R}^p$	...	unknown parameters
$\boldsymbol{u}_k \in \mathbb{R}^r$	...	control
$\boldsymbol{y}_k \in \mathbb{R}^m$	...	measurement

- ✓ The elements of matrices $\boldsymbol{A}(\boldsymbol{\theta}_k)$ and $\boldsymbol{B}(\boldsymbol{\theta}_k)$ are known linear functions of the unknown parameters $\boldsymbol{\theta}_k$
- ✓ The random variables $\boldsymbol{s}_0$, $\boldsymbol{\theta}_0$, $\boldsymbol{w}_k$, $\boldsymbol{\epsilon}_k$ and $\boldsymbol{v}_k$ are described by known pdf's

Prediction error dual controller (PEDC)

The control objective criterion

$$J_k = E\left\{(\boldsymbol{x}_{k+1}-\bar{\boldsymbol{x}}_{k+1})^T\boldsymbol{V}_{k+1}(\boldsymbol{x}_{k+1}-\bar{\boldsymbol{x}}_{k+1})-\boldsymbol{\nu}_{k+1}^T\boldsymbol{\Lambda}_{k+1}\boldsymbol{\nu}_{k+1}+\boldsymbol{u}_k^T\boldsymbol{W}_k\boldsymbol{u}_k\middle|\mathfrak{I}_k\right\}$$

$$\boldsymbol{x}_k = \begin{pmatrix} \boldsymbol{s}_k \\ \boldsymbol{\theta}_k \end{pmatrix}, \qquad \boldsymbol{\nu}_{k+1} = \boldsymbol{x}_{k+1} - \hat{\boldsymbol{x}}'_{k+1}(\hat{\boldsymbol{s}}_k, \hat{\boldsymbol{\theta}}_k).$$

The prediction of the augmented state $\hat{\boldsymbol{x}}'_{k+1}$ is defined as

$$\hat{\boldsymbol{x}}'_{k+1} = \begin{pmatrix} \boldsymbol{A}(\hat{\boldsymbol{\theta}}_k) & \boldsymbol{\mathcal{O}} \\ \boldsymbol{\mathcal{O}} & \boldsymbol{\Phi}_k \end{pmatrix}\begin{pmatrix} \hat{\boldsymbol{s}}_k \\ \hat{\boldsymbol{\theta}}_k \end{pmatrix} + \begin{pmatrix} \boldsymbol{B}(\hat{\boldsymbol{\theta}}_k) \\ \boldsymbol{\mathcal{O}} \end{pmatrix}\boldsymbol{u}_k + \begin{pmatrix} \hat{\boldsymbol{w}}_k \\ \boldsymbol{\mathcal{O}} \end{pmatrix},$$

where

$$\hat{\boldsymbol{s}}_k \triangleq E\left\{\boldsymbol{s}_k\middle|\mathfrak{I}_k\right\}, \qquad \hat{\boldsymbol{\theta}}_k \triangleq E\left\{\boldsymbol{\theta}_k\middle|\mathfrak{I}_k\right\}, \qquad \hat{\boldsymbol{w}}_k \triangleq E\left\{\boldsymbol{w}_k\middle|\mathfrak{I}_k\right\}.$$

Analysis of the control law

Structure of the control law for considered MIMO system

Denote $\boldsymbol{\alpha}_k$, $\boldsymbol{\beta}_k$ and $\boldsymbol{\gamma}_k$ as the first, the second and the third moment of the augmented state $\boldsymbol{x}_k \triangleq (\boldsymbol{s}_k, \boldsymbol{\theta}_k)^T$ given by the pdf $p(\boldsymbol{x}_k|\boldsymbol{y}_0^k)$, respectively. After the control law derivation the dependency of the control law can be written as

$$\boldsymbol{u}_k = f_k(\boldsymbol{\alpha}_k, \boldsymbol{\beta}_k, \boldsymbol{\gamma}_k)$$

In order to generate the control $\boldsymbol{u}_k$, it is necessary to know the filtering pdf $p(\boldsymbol{x}_k|\boldsymbol{y}_0^k)$. Because the system is nonlinear system from the estimation point of view a suitable nonlinear filtering method has to be employed.

Analysis of the control law - special cases

Control law for special choices of $\boldsymbol{\Lambda}_{k+1}$

- $\boldsymbol{\Lambda}_{k+1} = \boldsymbol{V}_{k+1} \quad \Rightarrow \quad \boldsymbol{u}_k = \boldsymbol{u}_k^{\mathcal{CE}}$
- $\boldsymbol{\Lambda}_{k+1} = \mathcal{O} \quad \Rightarrow \quad \boldsymbol{u}_k = \boldsymbol{u}_k^{\mathcal{C}}$

Relation to CE and Cautious controllers

- $\boldsymbol{u}_k = \boldsymbol{M}_k^{\mathcal{CE}} \boldsymbol{u}_k^{\mathcal{CE}} + \boldsymbol{N}_k^{\mathcal{CE}}$
  - $\boldsymbol{M}_k^{\mathcal{CE}} \leq \boldsymbol{I}$

    $\Rightarrow$ PEDC is cautious compared to *CE controller*
- $\boldsymbol{u}_k = \boldsymbol{M}_k^{\mathcal{C}} \boldsymbol{u}_k^{\mathcal{C}} + \boldsymbol{N}_k^{\mathcal{C}}$
  - In case that $\boldsymbol{\Lambda}_{k+1}^{s,s} \leq \boldsymbol{V}_{k+1}^{s,s}$ then $\boldsymbol{M}_k^{\mathcal{C}} \geq \boldsymbol{I}$

    $\Rightarrow$ PEDC is less cautious than *Cautious controller*

Suitable choise: $\mathcal{O} \leq \boldsymbol{\Lambda}_{k+1}^{s,s} \leq \boldsymbol{V}_{k+1}^{s,s}$

Relation of PEDC to IDC

PEDC comprises the IDC as a special case

- ✓ The system is supposed as SISO.
- ✓ The criterion matrices are then chosen as follows

$$\boldsymbol{V}_{k+1}^{s,s} = \boldsymbol{C}^T\boldsymbol{C},$$
$$\boldsymbol{\Lambda}_{k+1}^{s,s} = \boldsymbol{C}^T\boldsymbol{C}\lambda_{k+1}.$$

- ✓ The matrices $\boldsymbol{V}_{k+1}^{s,\theta}$, $\boldsymbol{V}_{k+1}^{\theta,s}$, $\boldsymbol{V}_{k+1}^{\theta,\theta}$, $\boldsymbol{\Lambda}_{k+1}^{s,\theta}$, $\boldsymbol{\Lambda}_{k+1}^{\theta,s}$ and $\boldsymbol{\Lambda}_{k+1}^{\theta,\theta}$ are zero matrices.

$$\boldsymbol{V}_{k+1} = \begin{pmatrix} \boldsymbol{V}_{k+1}^{s,s} & \boldsymbol{V}_{k+1}^{s,\theta} \\ \boldsymbol{V}_{k+1}^{\theta,s} & \boldsymbol{V}_{k+1}^{\theta,\theta} \end{pmatrix}, \qquad \boldsymbol{\Lambda}_{k+1} = \begin{pmatrix} \boldsymbol{\Lambda}_{k+1}^{s,s} & \boldsymbol{\Lambda}_{k+1}^{s,\theta} \\ \boldsymbol{\Lambda}_{k+1}^{\theta,s} & \boldsymbol{\Lambda}_{k+1}^{\theta,\theta} \end{pmatrix}$$

Analysis of the criterion

Decomposition of the criterion

$$J_k = J_k^{\mathcal{CE}} + J_K^{\mathcal{C}} + J_k^{\mathcal{P}}$$

$\Rightarrow$ it comprises both aspects of the dual control

✓ Certainty equivalent part

$$J_k^{\mathcal{CE}} = \left(\hat{\boldsymbol{x}}'_{k+1} - \bar{\boldsymbol{x}}_{k+1}\right)^T \boldsymbol{V}_{k+1} \left(\hat{\boldsymbol{x}}'_{k+1} - \bar{\boldsymbol{x}}_{k+1}\right) + \boldsymbol{u}_k^T \boldsymbol{W}_k \boldsymbol{u}_k$$

✓ Cautious part

$$J_k^{\mathcal{C}} = E\left\{\left(\boldsymbol{x}_{k+1} - \hat{\boldsymbol{x}}'_{k+1}\right)^T \boldsymbol{V}_{k+1} \left(\boldsymbol{x}_{k+1} - \hat{\boldsymbol{x}}'_{k+1}\right) \middle| \mathfrak{I}_k \right\}$$

✓ Probing part

$$J_k^{\mathcal{P}} = -E\left\{\left(\boldsymbol{x}_{k+1} - \hat{\boldsymbol{x}}'_{k+1}\right)^T \boldsymbol{\Lambda}_{k+1} \left(\boldsymbol{x}_{k+1} - \hat{\boldsymbol{x}}'_{k+1}\right) \middle| \mathfrak{I}_k \right\}$$

Numerical example

Considered system

$$\mathbf{s}_{k+1} = \begin{pmatrix} 0 & 1 \\ \theta_1 & \theta_2 \end{pmatrix} \mathbf{s}_k + \begin{pmatrix} \theta_{3k} \\ \theta_{4k} \end{pmatrix} u_k + \mathbf{w}_k$$
$$\boldsymbol{\theta}_{k+1} = \boldsymbol{\theta}_k$$
$$y_k = (1,\ 1)\mathbf{s}_k + v_k$$

- ✓ Prior pdf of the state and the parameters
  - ➢ $p(\mathbf{s}_0) = \mathcal{N}\left((0,\ 0)^T,\ 5 \cdot \mathbf{I}\right)$
  - ➢ $p(\boldsymbol{\theta}_0) = \mathcal{N}\left(\hat{\boldsymbol{\theta}}_0, \mathrm{diag}(0.3, 0.3, 1.2, 1.2)\right),$
    $\hat{\boldsymbol{\theta}}_0 = (-2.0427,\ 0.3427,\ 0,\ 1)^T$
- ✓ Noise pdf's
  - ➢ $p(\mathbf{w}_k) = \mathcal{N}\left((0,\ 0)^T, 10^{-4}\mathbf{I}\right)$
  - ➢ $p(v_k) = \mathcal{N}\left(0,\ 10^{-3}\right)$

Criteria parameters

- Matrix $\boldsymbol{V}_{k+1}$ such as to enable comparison with IDC and Bicriterial controller

$$\boldsymbol{V}_{k+1} = \begin{pmatrix} \boldsymbol{V}_{k+1}^{s,s} & \boldsymbol{V}_{k+1}^{s,\theta} \\ \boldsymbol{V}_{k+1}^{\theta,s} & \boldsymbol{V}_{k+1}^{\theta,\theta} \end{pmatrix} = \begin{pmatrix} \boldsymbol{C}^T\boldsymbol{C} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} \end{pmatrix}$$

- Matrix $\boldsymbol{\Lambda}_{k+1}$ designed to support reduction of uncertainty of parameters θ_{3k} and θ_{4k}

$$\boldsymbol{\Lambda}_{k+1} = \begin{pmatrix} \boldsymbol{\Lambda}_{k+1}^{s,s} & \boldsymbol{\Lambda}_{k+1}^{s,\theta} \\ \boldsymbol{\Lambda}_{k+1}^{\theta,s} & \boldsymbol{\Lambda}_{k+1}^{\theta,\theta} \end{pmatrix} = \begin{pmatrix} 0.64 \cdot \boldsymbol{C}^T\boldsymbol{C} & \boldsymbol{\Lambda}_{k+1}^{\theta,s} \\ \boldsymbol{\Lambda}_{k+1}^{s,\theta} & \boldsymbol{0} \end{pmatrix},$$

$$\boldsymbol{\Lambda}_{k+1}^{s,\theta} = \boldsymbol{\Lambda}^{s,\theta}{}_{k+1}^{T} = \begin{pmatrix} 0 & 0 & 0.1 & 0.1 \\ 0 & 0 & 1 & 0.1 \end{pmatrix},$$

- $W_k = 0.001$

Comparison to other controllers

The following index is chosen as a measure of the control performance

$$\mathcal{M} = \sqrt{\frac{1}{N}\sum_{k=1}^{N}(s_k - \bar{s}_k)^2},$$

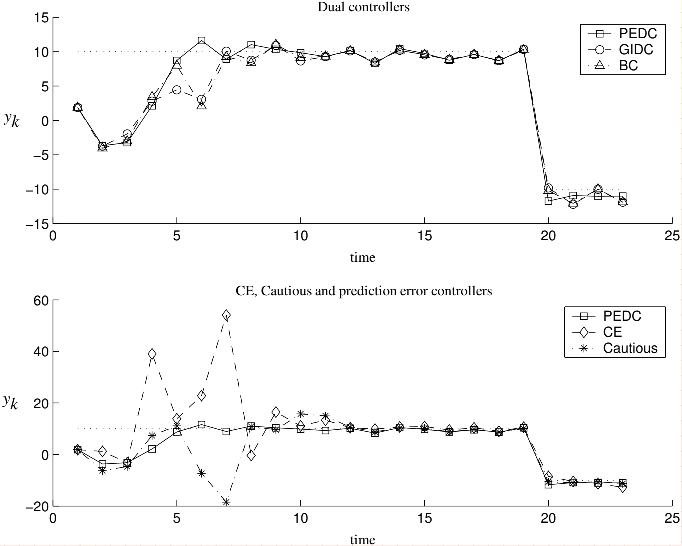
- ✓ y_k represents the measurement
- ✓ $\bar{y}_k$ represents the reference value
- ✓ N determines length of one simulation run

The expected value $\hat{\mathcal{M}} = E\{\mathcal{M}\}$ is estimated using 1000 MC simulations.

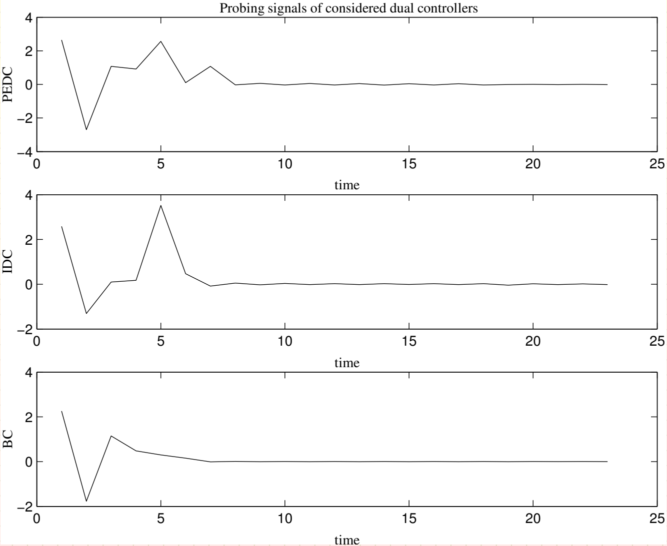
Control quality comparison using the index $\hat{\mathcal{M}}$

	$\hat{\mathcal{M}}$
Certainty equivalent controller	5.475879
Cautious controller	3.472613
Bicriterial controller	2.988189
Innovations dual controller	2.636660
Prediction error dual controller	2.580099

Comparison to other controllers



Comparison to other controllers



Concluding remarks

- Generalization of the IDC was presented
- Some aspects of the control law and criterion were discussed
- The PEDC controller can directly influence reduction of uncertainty of the parameter estimates
- The PEDC provides better performance compared to IDC

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Prediction error dual controller

Control law

$$
\begin{aligned}
\boldsymbol{u}_k = -\Big[& \boldsymbol{W}_k + \boldsymbol{B}^T(\hat{\boldsymbol{\theta}}_k)\boldsymbol{V}^{s,s}_{k+1}\boldsymbol{B}(\hat{\boldsymbol{\theta}}_k) \\
& + E\left\{ [\boldsymbol{B}(\boldsymbol{\theta}_k) - \boldsymbol{B}(\hat{\boldsymbol{\theta}}_k)]^T \boldsymbol{L}^{s,s}_{k+1} [\boldsymbol{B}(\boldsymbol{\theta}_k) - \boldsymbol{B}\hat{\boldsymbol{\theta}}_k)] \middle| \mathfrak{I}_k \right\} \Big]^{-1} \\
& \times \Big[E\left\{ [\boldsymbol{B}(\boldsymbol{\theta}_k) - \boldsymbol{B}(\hat{\boldsymbol{\theta}}_k)]^T \boldsymbol{L}^{s,\theta}_{k+1} [\boldsymbol{\Phi}_k \boldsymbol{\theta}_k - \boldsymbol{\Phi}_k \hat{\boldsymbol{\theta}}_k] \middle| \mathfrak{I}_k \right\} \\
& + \boldsymbol{B}^T(\hat{\boldsymbol{\theta}}_k)\boldsymbol{V}^{s,\theta}_{k+1}\boldsymbol{\Phi}_k \hat{\boldsymbol{\theta}}_k \\
& + E\left\{ [\boldsymbol{B}(\boldsymbol{\theta}_k) - \boldsymbol{B}(\hat{\boldsymbol{\theta}}_k)]^T \boldsymbol{L}^{s,s}_{k+1} [\boldsymbol{A}(\boldsymbol{\theta}_k)\boldsymbol{s}_k - \boldsymbol{A}(\hat{\boldsymbol{\theta}}_k)\hat{\boldsymbol{s}}_k] \middle| \mathfrak{I}_k \right\} \\
& + \boldsymbol{B}^T(\hat{\boldsymbol{\theta}}_k)\boldsymbol{V}^{s,s}_{k+1} \left(\boldsymbol{A}(\hat{\boldsymbol{\theta}}_k)\hat{\boldsymbol{s}}_k + \hat{\boldsymbol{w}}_k - \bar{\boldsymbol{x}}_{k+1} \right) \Big],
\end{aligned}
$$

Relation to *certainty equivalent* control

$$\boldsymbol{u}_k = \boldsymbol{M}_k^{\mathcal{CE}} \boldsymbol{u}_k^{\mathcal{CE}} + \boldsymbol{N}_k^{\mathcal{CE}},$$

$$\begin{aligned}\boldsymbol{M}_k^{\mathcal{CE}} = &\Big[\boldsymbol{W}_k + \boldsymbol{B}^T(\hat{\boldsymbol{\theta}}_k)\boldsymbol{V}_{k+1}^{s,s}\boldsymbol{B}(\hat{\boldsymbol{\theta}}_k) + \Big\{[\boldsymbol{B}(\boldsymbol{\theta}_k) - \boldsymbol{B}(\hat{\boldsymbol{\theta}}_k)]^T \boldsymbol{L}_{k+1}^{s,s}[\boldsymbol{B}(\boldsymbol{\theta}_k) - \boldsymbol{B}\hat{\boldsymbol{\theta}}_k)] \Big| \mathfrak{I}_k\Big\}\Big]^{-1} \times \\ &\times \Big[\boldsymbol{W}_k + \boldsymbol{B}^T(\hat{\boldsymbol{\theta}}_k)\boldsymbol{V}_{k+1}^{s,s}\boldsymbol{B}(\hat{\boldsymbol{\theta}}_k)\Big]\end{aligned}$$

$$\begin{aligned}\boldsymbol{N}_k^{\mathcal{CE}} = -&\Big[\boldsymbol{W}_k + \boldsymbol{B}^T(\hat{\boldsymbol{\theta}}_k)\boldsymbol{V}_{k+1}^{s,s}\boldsymbol{B}(\hat{\boldsymbol{\theta}}_k) + \Big\{[\boldsymbol{B}(\boldsymbol{\theta}_k) - \boldsymbol{B}(\hat{\boldsymbol{\theta}}_k)]^T \boldsymbol{L}_{k+1}^{s,s}[\boldsymbol{B}(\boldsymbol{\theta}_k) - \boldsymbol{B}\hat{\boldsymbol{\theta}}_k)] \Big| \mathfrak{I}_k\Big\}\Big]^{-1} \times \\ &\times \Big[E\Big\{[\boldsymbol{B}(\boldsymbol{\theta}_k) - \boldsymbol{B}(\hat{\boldsymbol{\theta}}_k)]^T \boldsymbol{L}_{k+1}^{s,\theta}[\boldsymbol{\Phi}_k\boldsymbol{\theta}_k - \boldsymbol{\Phi}_k\hat{\boldsymbol{\theta}}_k] \Big| \mathfrak{I}_k\Big\} + \\ &+ E\Big\{[\boldsymbol{B}(\boldsymbol{\theta}_k) - \boldsymbol{B}(\hat{\boldsymbol{\theta}}_k)]^T \boldsymbol{L}_{k+1}^{s,s}[\boldsymbol{A}(\boldsymbol{\theta}_k)\boldsymbol{s}_k - \boldsymbol{A}(\hat{\boldsymbol{\theta}}_k)\hat{\boldsymbol{s}}_k] \Big| \mathfrak{I}_k\Big\}\Big]\end{aligned}$$

Relation to *cautious* control

$$\boldsymbol{u}_k = \boldsymbol{M}_k^{\mathfrak{C}} \boldsymbol{u}_k^{\mathfrak{C}} + \boldsymbol{N}_k^{\mathfrak{C}},$$

$$\boldsymbol{M}_k^{\mathfrak{C}} = \Big[\boldsymbol{W}_k + \boldsymbol{B}^T(\hat{\boldsymbol{\theta}}_k)\boldsymbol{V}_{k+1}^{s,s}\boldsymbol{B}(\hat{\boldsymbol{\theta}}_k) + \Big\{[\boldsymbol{B}(\boldsymbol{\theta}_k) - \boldsymbol{B}(\hat{\boldsymbol{\theta}}_k)]^T \boldsymbol{L}_{k+1}^{s,s}[\boldsymbol{B}(\boldsymbol{\theta}_k) - \boldsymbol{B}\hat{\boldsymbol{\theta}}_k)] \Big| \mathfrak{I}_k\Big\}\Big]^{-1} \times$$
$$\times \Big[\boldsymbol{W}_k + \boldsymbol{B}^T(\hat{\boldsymbol{\theta}}_k)\boldsymbol{V}_{k+1}^{s,s}\boldsymbol{B}(\hat{\boldsymbol{\theta}}_k) + \Big\{[\boldsymbol{B}(\boldsymbol{\theta}_k) - \boldsymbol{B}(\hat{\boldsymbol{\theta}}_k)]^T \boldsymbol{V}_{k+1}^{s,s}[\boldsymbol{B}(\boldsymbol{\theta}_k) - \boldsymbol{B}(\hat{\boldsymbol{\theta}}_k)] \Big| \mathfrak{I}_k\Big\}\Big]$$

$$\boldsymbol{N}_k^{\mathfrak{C}} = \Big[\boldsymbol{W}_k + \boldsymbol{B}^T(\hat{\boldsymbol{\theta}}_k)\boldsymbol{V}_{k+1}^{s,s}\boldsymbol{B}(\hat{\boldsymbol{\theta}}_k) + \Big\{[\boldsymbol{B}(\boldsymbol{\theta}_k) - \boldsymbol{B}(\hat{\boldsymbol{\theta}}_k)]^T \boldsymbol{L}_{k+1}^{s,s}[\boldsymbol{B}(\boldsymbol{\theta}_k) - \boldsymbol{B}\hat{\boldsymbol{\theta}}_k)] \Big| \mathfrak{I}_k\Big\}\Big]^{-1} \times$$
$$\times \Big[E\Big\{[\boldsymbol{B}(\boldsymbol{\theta}_k) - \boldsymbol{B}(\hat{\boldsymbol{\theta}}_k)]^T \boldsymbol{\Lambda}_{k+1}^{s,\theta}[\boldsymbol{\Phi}_k\boldsymbol{\theta}_k - \boldsymbol{\Phi}_k\hat{\boldsymbol{\theta}}_k] \Big| \mathfrak{I}_k\Big\}$$
$$+ E\Big\{[\boldsymbol{B}(\boldsymbol{\theta}_k) - \boldsymbol{B}(\hat{\boldsymbol{\theta}}_k)]^T \boldsymbol{\Lambda}_{k+1}^{s,s}[\boldsymbol{A}(\boldsymbol{\theta}_k)\boldsymbol{s}_k - \boldsymbol{A}(\hat{\boldsymbol{\theta}}_k)\hat{\boldsymbol{s}}_k] \Big| \mathfrak{I}_k\Big\}\Big]$$