Rolling horizon for active fault detection

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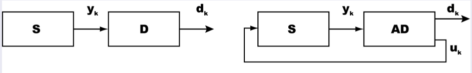
Outline

Introduction

Passive and active fault detection

- Passive detector – it provides only decision about faults [Willsky(1976), Basseville&Nikiforov(1993), etc.]
- Active detector – in addition to decision it generates input signal to improve fault detection [Zhang(1989), Kerestecioğlu(1993), Campbell&Nikoukhah(2004)]



a) Passive fault detection b) Active fault detection

S	system
D	detector
AD	active detector
$\mathbf{y}_k$	output
d_k	decision
u_k	input

Active detector design

Active detector design

- Generally, there are many methods to the detector design and it is not easy to sort them out
- There are only a few works on the active detector design, the availability of the future information is not considered and input signal design is based on an additional criterion
- Finally, there is not easy way to take into account real costs caused by the wrong decisions

Introduction – cont'd

Information processing strategies (IPS's)

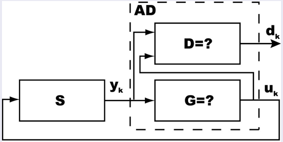
To sort out design methods the following strategies are considered in [Šimandl&Herejt(2003)]

- Open loop (OL) - uses only an a priori information and possible future information is not considered
- Open loop feedback (OLF) - uses an a priori information and available information up to current time, but the future information is not considered
- Closed loop (CL) - uses an a priori information, available information up to current time and it is considered that the future information will be available too

Introduction – cont'd

Goals of the article

- Present a compact formulation of the active detector design in the multiple models framework and provide corresponding solution using CL IPS



Structutre of the active detector

- Propose a feasible suboptimal solution based on rolling horizon technique and compared it with OLF IPS and Pseudo Random Binary Sequence (PRBS) as input signal

Problem formulation

System description for $k \in \mathcal{T} = \{0, 1, \ldots, F\}$

$$\begin{aligned} \mathbf{x}_{k+1} &= A(\theta_k)\mathbf{x}_k + B(\theta_k)u_k + G(\theta_k)\mathbf{w}_k \\ \mathbf{y}_k &= C(\theta_k)\mathbf{x}_k + H(\theta_k)\mathbf{v}_k \\ P_{i,j} &= P\left(\theta_{k+1} = j | \theta_k = i\right), \;\; i, j \in \mathcal{M} \end{aligned}$$

$\mathbf{x}_k$ unknown state; $\theta_k \in \mathcal{M} = \{1, 2, \ldots, N\}$ unknown index of the model; u_k input; $\mathbf{y}_k$ output; $\mathbf{w}_k$ and $\mathbf{v}_k$ mutually independent white Gaussian noises; $A(\theta_k)$, $B(\theta_k)$, $G(\theta_k)$, $C(\theta_k)$, $H(\theta_k)$ known matrices; $P_{i,j}$ known transition probabilities; $p(\mathbf{x}_0)$ known Gaussian pdf; $P(\theta_0)$ known probability distribution

Problem formulation – cont'd

Active detector description for $k \in \mathcal{T}$

- Detector $d_k = \sigma_k\left(\mathbf{I}_0^k\right)$
- Input signal generator $u_k = \gamma_k\left(\mathbf{I}_0^k\right)$

with notation $\mathbf{I}_0^k = \left[{\mathbf{y}_0^k}^T, {u_0^{k-1}}^T\right]^T$, $\mathbf{y}_0^k = \left[\mathbf{y}_0^T, \dots, \mathbf{y}_k^T\right]^T$

Criterion on finite detection horizon F

$$J\left(\sigma_0^F, \gamma_0^F\right) = \mathrm{E}\left\{\sum_{k=0}^{F} L_k\left(\theta_k, d_k\right)\right\} \rightarrow \min$$

So, the aim is to find functions σ_0^F and γ_0^F which minimize this criterion

Active detector design

Active detector based on CL IPS

- Backward recursive equation with initial condition $V_{F+1}^* = 0$

$$V_k^*(\mathbf{I}_0^k) = \min_{d_k \in \mathcal{M}} \mathrm{E}\left\{L\left(\theta_k, d_k\right) | \mathbf{I}_0^k, d_k\right\} + \min_{u_k \in \mathcal{U}_k} \mathrm{E}\left\{V_{k+1}^*\left(\mathbf{I}_0^{k+1}\right) | \mathbf{I}_0^k, u_k\right\}, \quad k = F, \ldots, 0$$

- Detector $d_k^* = \arg\min_{d_k \in \mathcal{M}} \mathrm{E}\left\{L(\theta_k, d_k) | \mathbf{I}_0^k, d_k\right\}$
- Input signal generator
  $u_k^* = \arg\min_{u_k \in \mathcal{U}_k} \mathrm{E}\left\{V_{k+1}^*\left(\mathbf{I}_0^{k+1}\right) | \mathbf{I}_0^k, u_k\right\}$

Active detector design – cont'd

Active detector based on OLF IPS

- Modified aim is to solve OL problem at time step k for future time steps

$$\bar{J} = \min_{d_k^F, u_k^F} \mathrm{E}\left\{ \sum_{i=k}^{F} L\left(\theta_i, d_i\right) | \mathbf{I}_0^k, d_k^F, u_k^F \right\}$$

- Detector $d_k^{OLF} = \arg\min_{d_k \in \mathcal{M}} \mathrm{E}\left\{ L\left(\theta_k, d_k\right) | \mathbf{I}_0^k, d_k \right\}$
- Input signal generator is not determined $\Rightarrow$ use of additional criterion or choose some probing signal (e.g. PRBS signal)

Active detector design – cont'd

Active detector based on rolling horizon technique

- Aim is to numerically solve optimization problem for truncated horizon l with criterion

$$\tilde{J}\left(\sigma_k^{\bar{l}}, \gamma_k^{\bar{l}}\right) = \mathrm{E}\left\{\sum_{i=k}^{\bar{l}} L\left(\theta_i, d_i\right)\right\}, \; \bar{l} = \min\{k+l-1, F\}$$

  and only $\sigma_k(\mathbf{I}_0^k)$, $\gamma_k(\mathbf{I}_0^k)$ are used
- Detector $d_k^{RH} = \arg\min_{d_k \in \mathcal{M}} \mathrm{E}\left\{L\left(\theta_k, d_k\right) | \mathbf{I}_0^k, d_k\right\}$
- Input signal generator
  $u_k^* = \arg\min_{u_k \in \mathcal{U}_k} \mathrm{E}\left\{\tilde{V}_{k+1}^*\left(\mathbf{I}_0^{k+1}\right) | \mathbf{I}_0^k, u_k\right\}$

State estimation in multiple models framework

Exact state estimation

- Given model sequence θ_0^k the state pdf $p\left(\mathbf{x}_k|\mathbf{I}_0^k,\theta_0^k\right)$ is computed using Kalman filter $\Rightarrow N^{k+1}$ filters are needed
- The state estimation is given as

$$p\left(\mathbf{x}_k|\mathbf{I}_0^k\right) = \sum_{\theta_0^k} p(\mathbf{x}_k|\mathbf{I}_0^k,\theta_0^k)P(\theta_0^k|\mathbf{I}_0^k)$$

$$P\left(\theta_k|\mathbf{I}_0^k\right) = \sum_{\theta_0^{k-1}} P(\theta_0^k|\mathbf{I}_0^k) = \sum_{\theta_0^{k-1}} \frac{p(\mathbf{y}_k|\mathbf{y}_0^{k-1},\mathbf{u}_0^{k-1},\theta_0^k)P(\theta_k|\theta_{k-1})P(\theta_0^{k-1}|\mathbf{I}_0^{k-1})}{c}$$

State estimation in multiple models framework – cont'd

Merging of the model sequences

- After each h time steps the model sequences which terminate in the same model are merged and resulting Gaussian mixture is replaced by single Gaussian distribution with the same first two moments

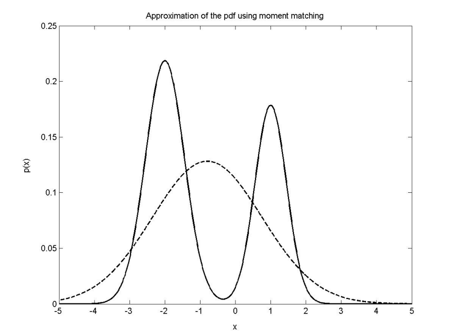
$$P(\theta_k|\mathbf{I}_0^k) = \sum_{\theta_{k-h}^{k-1}} P(\theta_{k-h}^k|\mathbf{I}_0^k)$$

$$p(\mathbf{x}_k|\mathbf{I}_0^k,\theta_k) = \sum_{\theta_{k-h}^{k-1}} \alpha(\theta_{k-h}^k)p(\mathbf{x}_k|\mathbf{I}_0^k,\theta_{k-h}^k) \approx p_A(\mathbf{x}_k|\mathbf{I}_0^k,\theta_k)$$

$$\alpha(\theta_{k-h}^k) = P(\theta_{k-h}^k|\mathbf{I}_0^k)/P(\theta_k|\mathbf{I}_0^k)$$

State estimation in multiple models framework – cont'd

Replacement of exact pdf by single Gaussian distribution



True pdf (-), approximative pdf (--)

Numerical examples

System description

$$\theta_k = 1: \quad x_{k+1} = 0.3x_k + u_k + \sqrt{0.25}w_k$$
$$y_k = -2x_k + \sqrt{0.25}v_k$$
$$\theta_k = 2: \quad x_{k+1} = 0.5x_k + 1.5u_k + \sqrt{0.25}w_k$$
$$y_k = 1.5x_k + \sqrt{0.25}v_k$$

$$P_{i,j} = \begin{bmatrix} 0.2 & 0.8 \\ 0.8 & 0.2 \end{bmatrix}$$
$$P(\theta_0 = 1) = P(\theta_0 = 2) = 0.5$$
$$w_k \sim \mathcal{N}\{0, 1\}$$

$$\theta_k = d_k \Rightarrow L(\theta_k, d_k) = 0$$
$$\theta_k \neq d_k \Rightarrow L(\theta_k, d_k) = 1$$
$$x_0 \sim \mathcal{N}\{0, 0.1\}$$
$$v_k \sim \mathcal{N}\{0, 1\}$$

Numerical examples – cont'd

Monte Carlo simulation results

- Parameters of the simulation $F = 50$, $l = 2$, $h = 1$, $\mathcal{U}_k = \{-0.5, 0.5\}$, $k \in \mathcal{T}$

	OLF+PRBS	RH
$\mathrm{E}\{\hat{J}\}$	6.6142	2.7548
$\mathrm{VAR}\{\hat{J}\}$	0.0083	0.0028

- Improvement $(6.6142 - 2.7548)/0.066142 = 58.35\%$

Numerical examples – cont'd

Monte Carlo simulation results

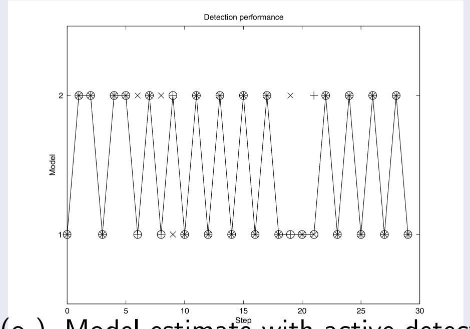
- Parameters of the simulation $F = 50,\ l = 2,\ h = 1,$ $\mathcal{U}_k = \{-\bar{u}, \bar{u}\},\ k \in \mathcal{T}$

$\bar{u}$	0.1	0.3	0.5	1	3
$\mathrm{E}\{\hat{J}^{OLF}\}$	18.5533	11.0350	6.6142	2.53	0.605
$\mathrm{VAR}\{\hat{J}^{OLF}\}$	0.0751	0.0977	0.0083	0.068	0.0018
$\mathrm{E}\{\hat{J}^{RH}\}$	17.0333	7.2633	2.7548	0.545	0.455
$\mathrm{VAR}\{\hat{J}^{RH}\}$	0.2687	0.1928	0.0028	0.0044	0.0024
D	1.52	3.7717	3.8594	1.985	0.15

$$D = \mathrm{E}\{\hat{J}^{OLF}\} - \mathrm{E}\{\hat{J}^{RH}\}$$

Numerical examples – cont'd

Example of detection run



True model (o-), Model estimate with active detector based on OLF strategy (x), Model estimate with active detector based on RH scheme (+)

Numerical examples – cont'd

System description

$$\theta_k = 1: \quad x_{k+1} = 0.98x_k + 2.6u_k + \sqrt{0.02}w_k$$
$$y_k = 4.5x_k + \sqrt{0.3}v_k$$
$$\theta_k = 2: \quad x_{k+1} = 0.99x_k + 2.78u_k + \sqrt{0.02}w_k$$
$$y_k = 4.4x_k + \sqrt{0.3}v_k$$

$$P_{i,j} = \begin{bmatrix} 0.99 & 0.01 \\ 0.01 & 0.99 \end{bmatrix}$$

$$\theta_k = d_k \Rightarrow L(\theta_k, d_k) = 0$$
$$\theta_k \neq d_k \Rightarrow L(\theta_k, d_k) = 1$$

$$P(\theta_0 = 1) = P(\theta_0 = 2) = 0.5$$
$$x_0 \sim \mathcal{N}\{0, 0.04\}$$
$$w_k \sim \mathcal{N}\{0, 1\}$$
$$v_k \sim \mathcal{N}\{0, 1\}$$

Numerical examples – cont'd

Monte Carlo simulation results

- Parameters of the simulation $F = 50,\ l = 2,\ h = 1,$ $\mathcal{U}_k = \{-0.5, 0.5\},\ k \in \mathcal{T}$

	OLF+PRBS	RH
$\mathrm{E}\{\hat{J}\}$	13.5520	4.4470
$\mathrm{VAR}\{\hat{J}\}$	0.8917	0.1110

- Improvement $(13.5520 - 4.4470)/0.13552 = 67.19\%$

Conclusion remarks

Conclusion remarks

- The compact formulation of the fault detection problem was shown
- The stress was laid on the active fault detection problem and the suboptimal feasible solution was proposed
- Utilization of the closed loop information processing strategy leads to the lower value of the criterion
- Extensive MC simulations confirm that rolling horizon technique can provides better results in comparison with open loop feedback information processing strategy
- It was shown that possible improvement is highly dependent on considered input signal

Some comparison

Some comparison

- The main difficulty of some comparison consist in different design aims
- It is possible to make a very rough comparison with approach presented in [Zhang(1989)], where the detector is based on SPRT test and the input signal is designed to minimize ASN (Average Sampling Number)

	RH	Zhang
ASN	6.0486	13.45

- Unfortunately, the detector based on criterion minimization does not guarantee any limits on false alarm probability and missed alarm probability