Neural Network Based
Bicriterial Dual Control with Multiple Linearization

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Contents

1. Introduction
2. Problem statement
3. Controller design
4. Example
5. Conclusion

Introduction

- adaptive control of nonlinear stochastic systems
- nonlinear functions of the system are considered to be unknown functional adaptive control,

characteristic of functional adaptive control

- nonlinear system is modeled by a universal approximator
- simultaneously optimizing control performance and reducing uncertainty
- avoid the time consuming process of off-line identification of the system

Introduction - motivation, goal

⇒ Summary of functional adaptive control

- functional adaptive control defined in 2001 by Fabri & Kadirkamanathan,
- utilizing of bicriterial dual control methodology brings an excellent control quality (*Šimandl at al, 2005, IFAC World Congress*),
- functional adaptive control extended for MIMO systems (*Král and Šimandl, 2008, IFAC World Congress*)
- functional adaptive control extended for slowly time variant systems (*Král and Šimandl, 2009, SYSID*)

Goal

To design an alternative bicriterial dual controller by fully utilizing the character of the Gaussian Sum estimator and so to improve quality of control.

Problem statement

Nonlinear stochastic discrete-time system

$$y_k = f(\boldsymbol{x}_{k-1}) + g(\boldsymbol{x}_{k-1})u_{k-1} + e_k,$$

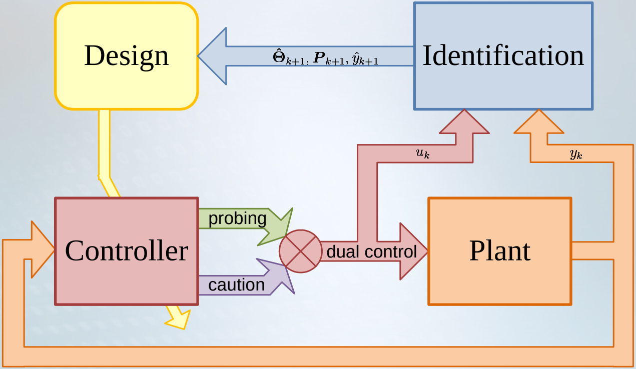
- $f(\boldsymbol{x}_{k-1})$ and $g(\boldsymbol{x}_{k-1})$ are unknown nonlinear functions
- $\boldsymbol{x}_{k-1} \triangleq [y_{k-p}, \ldots, y_{k-1}, u_{k-1-s}, \ldots, u_{k-2}]$ is known measurable state
- y_k is output
- u_k is input
- e_k is additive white noise, pdf $\mathcal{N}\left\{0, \sigma^2\right\}$

Bicriterial dual controller

$$u_k = h\left(y^r_{k+1}, \boldsymbol{I}_k\right)$$

- output y_k should follow reference signal y^r_k
- $\boldsymbol{I}_k$ contains information received up to time k

Dual control - principle



Bicriterial dual controller – basic idea

The bicriterial dual controller design is based on two separate criteria. Each of those criteria introduces one of opposing aspects between control and estimation: **caution** and **probing**.

The caution control component

$$J_k^c = E\left\{(y_{k+1} - y_{k+1}^r)^2 + q u_k^2 | \boldsymbol{I}_k\right\}$$
$$u_k^c = \underset{\mathrm{u_k}}{\operatorname{argmin}}\, J_k^c$$

The probing control component

$$J_k^a = -E\left\{(y_{k+1} - \hat{y}_{k+1})^2 | \boldsymbol{I}_k\right\}$$
$$\Omega_k = [u_k^c - \delta_k, u_k^c + \delta_k]$$
$$\delta_k = \eta \operatorname{tr}(\boldsymbol{P}_{k+1})$$

The final control

$$u_k = \underset{\mathrm{u_k \in \Omega_k}}{\operatorname{argmin}}\, J_k^a$$

Bicriterial dual controller – cont'd

Bicriterial dual controller

- control law can be obtained analytically ➠ low computational demands
- $u_k = h(\eta, y^r_{k+1}, \hat{\boldsymbol{\Theta}}_{k+1}, \boldsymbol{P}_{k+1})$: η - *designer parameter*

  : y^r_{k+1} - *known variable*

  : $\hat{\boldsymbol{\Theta}}_{k+1}, \boldsymbol{P}_{k+1}$ - *estimation*

Neural network - model choice

Model of the system

- The unknown nonlinear functions $f(\boldsymbol{x}_{k-1})$ and $g(\boldsymbol{x}_{k-1})$ are approximated by Multi-Layer Perceptron (MPL) networks ➠ **model**
- Compromise between complexity and accuracy of the estimator and dual controller

$$\begin{aligned}\hat{y}_k &= \hat{f}(\boldsymbol{x}_{k-1}, \boldsymbol{w}^f, \boldsymbol{c}^f) + \hat{g}(\boldsymbol{x}_{k-1}, \boldsymbol{w}^g, \boldsymbol{c}^g)u_{k-1} \\ \hat{f} &= (\boldsymbol{c}^f)^T \boldsymbol{\phi}^f(\boldsymbol{x}_{k-1}, \boldsymbol{w}^f) \\ \hat{g} &= (\boldsymbol{c}^g)^T \boldsymbol{\phi}^g(\boldsymbol{x}_{k-1}, \boldsymbol{w}^g)\end{aligned}$$

$$\boldsymbol{\Theta} = \left[(\boldsymbol{c}^f)^T, (\boldsymbol{w}^f)^T, (\boldsymbol{c}^g)^T, (\boldsymbol{w}^g)^T\right]^T \Rightarrow \hat{\boldsymbol{\Theta}}_{k+1},\ \boldsymbol{P}_{k+1} = ?$$

Neural network - parameter estimation

Neural network model

- Neural network can be rewritten into state space estimation model

$$\begin{aligned}\boldsymbol{\Theta}_{k+1} &= \boldsymbol{\Theta}_k \\ \hat{y}_k &= \hat{f}(\boldsymbol{x}_{k-1}, \boldsymbol{w}^f, \boldsymbol{c}^f) + \hat{g}(\boldsymbol{x}_{k-1}, \boldsymbol{w}^g, \boldsymbol{c}^g)u_{k-1} + e_k\end{aligned}$$

- The measurement equation is nonlinear
- It is possible to use non-linear estimation methods - Gaussian Sum (GS) Filter
- Prior information about parameters given by pdf in the form of the Gaussian mixture as

$$p(\boldsymbol{\Theta}|\mathbf{I}^k) = \sum_{\ell=1}^{N_0} \alpha_{k+1,\ell}\mathcal{N}\left\{\boldsymbol{\Theta} : \hat{\boldsymbol{\Theta}}_{k+1,\ell}, \mathbf{P}_{k+1,\ell}\right\},$$

where $\alpha_i > 0$, $\Sigma_{i=1}^{\ell}\alpha_i = 1$

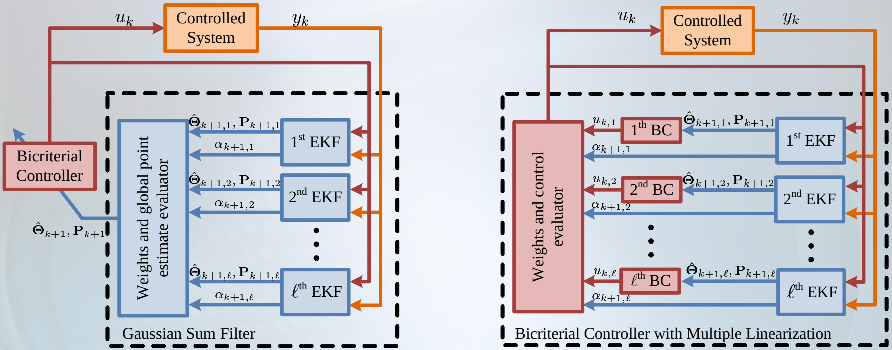
Gaussian sum method

Main characteristic

- a bank of N parallel running extended Kalman filters (EKF)
- comes out from the EKF ➠ easy implementation
  ➠ feasible computational demands
- respects features of disturbance
- high quality estimation
- provides probability density function of the parameter estimates

$$p(\boldsymbol{\Theta}_{k+1}|\boldsymbol{I}^k) \Rightarrow \hat{\boldsymbol{\Theta}}_{k+1},\ \boldsymbol{P}_{k+1}$$

Control design - two alternatives



$$u_k^{GS} = u_k^c + \delta_k \text{sign}(\varpi_k)$$

$$u_k^{ML} = \sum_{i=1}^{\ell} \alpha_i u_{k,i}$$
$$u_{ki} = u_{k,i}^c + \delta_k \sum_{i=1}^{\ell} \alpha_i \text{sign}(\varpi_{k,i})$$

$$u_k^{GS} \neq u_k^{ML}$$

Example

Synthetic benchmark system

$$y_k = \frac{1.5 y_{k-1} y_{k-2}}{1 + y_{k-1}^2 + y_{k-2}^2} + 0.2 \sin(y_{k-1} + y_{k-2}) + (\sin(y_{k-1} y_{k-2}) - 1.3) u_{k-1} + e_k,$$

two controllers were compared

- Cautious (CA)
- Bicriterial dual (BD)

two types of 'design' were used:

- Gaussian Sum filter estimator (GS)
- Multiple Linearization technique (ML)

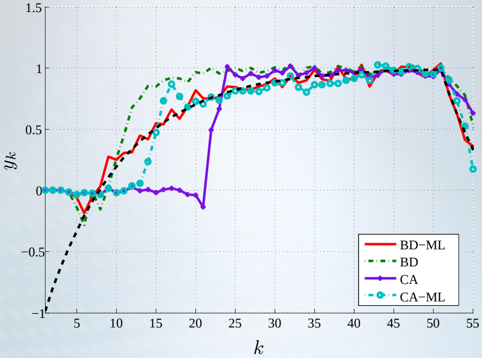
Example - The results

The quality of control is measured by the mean of sums of square errors between reference value $y^r_{k+1,j}$ and system output $y_{k+1,j}$ over 5.000 trials: $\hat{J} = \frac{1}{5000}\sum_{j=1}^{5000}\sum_{k=1}^{250}(y_{k+1,j} - y^r_{k+1,j})^2 + qu^2_{k,j}$

controller	$\hat{J}$	$\text{var}\{\hat{J}\}$
BD-ML	5.55	$3.10 \cdot 10^{-4}$
BD	14.9	$2.76 \cdot 10^{-2}$
CA-ML	7.34	$1.21 \cdot 10^{-3}$
CA	15.72	$2.14 \cdot 10^{-2}$

Example - The results (cont'd)

Typical output of the system



Conclusion

☞

- ✯ The bicriterial dual controller with multiple linearization for the discrete-time stochastic system was presented.
- ✯ The model of the system is given by the multilayer perceptron network where unknown parameters are estimated on-line.
- ✯ Proposed controller consists of a set of local bicriterial controllers connected with corresponding local estimators.
- ✯ Final dual controller exploits of the whole information provided by the Gaussian Sum filter and not just a point estimate.
- ✯ It achieves better control quality in comparison with controller that uses the global point estimate only.