Bicriterial dual control with multiple linearization

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Dual control (Feldbaum 1960)

- Arises in control problem with insufficient knowledge of parameters
- Two conflicting goals – meet control objective and improve estimation
- Optimal dual control problem – mostly cannot be solved analytically

Suboptimal solutions (Tse *et al.* 1973, Wittenmark *et al.* 1975, Millito *et al.* 1982,. . .)

- Augmenting the cautious control law (Bicriterial controller,. . .)
- Modification of criterion (e.g. IDC, ASOD,. . .)
- Criterion approximation (e.g. WDC, Utility cost,. . .)

Feasible solution - Bicriterial approach (Filatov *et al.* 1996)

- clear interpretation (two objectives ⇒ two criteria)
- computationally moderate (only one step ahead horizon)
- enhances the *cautious control* with a suitable *probing* signal

Bicriterial approach

Filatov, N. M., U. Keuchel and H. Unbehauen (1996). Dual control for an unstable mechanical plant. *IEEE Control Systems Magazine* **16**(4), 31–37.

- ✓ The controlled system considered SISO ARMAX
- ✓ Parameter estimation by Recursive leat square method

The criteria

- ➢ Control objective criterion - leading to cautious control

$$J_k^c(u_k) = E\left\{(\bar{y}_{k+1} - y_{k+1})^2 \middle| \Im_k\right\}, \Im_k = (u_0, \ldots, u_{k-1}, y_0, \ldots, y_k)$$

- ➢ Estimation objective criterion

$$J_k^e(u_k) = -E\left\{\left(y_{k+1} - \hat{y}_{k+1}\right)^2 \middle| \Im_k\right\}$$

$$u_k^* = \underset{u_k \in \Omega_k}{\operatorname{argmin}} J_k^e(u_k), \qquad \Omega_k = [u_k^c - \delta_k, u_k^c + \delta_k]$$

$$\delta_k = f(\boldsymbol{P}_k) = \eta \cdot \operatorname{tr} \boldsymbol{P}_k$$

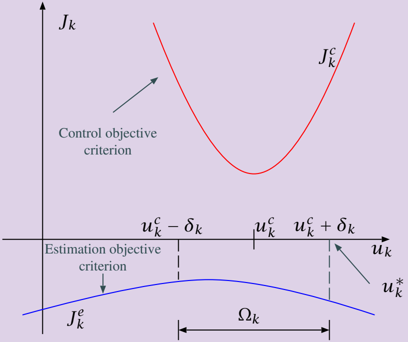
Bicriterial approach

Bicriterial control law

$$u_k^* = u_k^c + \delta_k \operatorname{sign}(\omega_k)$$

$$\omega_k = J_k^e\left(u_k^c + \delta_k\right) - J_k^e\left(u_k^c - \delta_k\right)$$

Maximization of criterion $J_k^e(u_k)$ on domain Ω_k



Goal - Generalization of the basic Bicriterial controller

- Generalization to the class of MIMO state space systems with random variables described by an arbitrary probability density functions (pdf's)
- Appropriate design of the criteria
- Usage of the Gaussian sum method for estimation and employment of its multiple linearization to the dual controller
- Analysis of the Bicriterial dual controller with multiple linearization

Consider the MIMO stochastic system

$$\begin{aligned} \boldsymbol{s}_{k+1} &= \boldsymbol{A}(\boldsymbol{\theta}_k)\boldsymbol{s}_k + \boldsymbol{B}(\boldsymbol{\theta}_k)\boldsymbol{u}_k + \boldsymbol{w}_k, \\ \boldsymbol{\theta}_{k+1} &= \boldsymbol{\Phi}_k\boldsymbol{\theta}_k + \boldsymbol{\epsilon}_k, \qquad\qquad k = 0, \ldots, N-1 \\ \boldsymbol{y}_k &= \boldsymbol{C}\boldsymbol{s}_k + \boldsymbol{v}_k, \end{aligned}$$

$\boldsymbol{s}_k \in \mathbb{R}^n$	...	non-measurable state
$\boldsymbol{\theta}_k \in \mathbb{R}^p$	...	unknown parameters
$\boldsymbol{u}_k \in \mathbb{R}^r$	...	control
$\boldsymbol{y}_k \in \mathbb{R}^m$	...	measurement

- ✓ The elements of matrices $\boldsymbol{A}(\boldsymbol{\theta}_k)$ and $\boldsymbol{B}(\boldsymbol{\theta}_k)$ are known linear function of the unknown parameters $\boldsymbol{\theta}_k$
- ✓ The random variables $\boldsymbol{s}_0$, $\boldsymbol{\theta}_0$, $\boldsymbol{w}_k$, $\boldsymbol{\epsilon}_k$ and $\boldsymbol{v}_k$ are described by known pdf's

The criteria

The control objective criterion

$$J_k^c\left(\boldsymbol{u}_k\right) = E\left\{\left(\boldsymbol{y}_{k+1} - \bar{\boldsymbol{y}}_{k+1}\right)^T \boldsymbol{V}_{k+1}\left(\boldsymbol{y}_{k+1} - \bar{\boldsymbol{y}}_{k+1}\right) + \boldsymbol{u}_k^T \boldsymbol{W}_k \boldsymbol{u}_k \middle| \Im_k\right\}$$

$$\boldsymbol{u}_k^c = \underset{\boldsymbol{u}_k}{\operatorname{argmin}}\, J_k^c\left(\boldsymbol{u}_k\right)$$

The estimation objective criterion

$$J_k^e\left(\boldsymbol{u}_k\right) = E\left\{\left(\boldsymbol{y}_{k+1} - \hat{\boldsymbol{y}}_{k+1}\right)^T \mathcal{V}_{k+1}\left(\boldsymbol{y}_{k+1} - \hat{\boldsymbol{y}}_{k+1}\right) \middle| \Im_k\right\}$$

$$\boldsymbol{u}_k^* = \underset{\boldsymbol{u}_k \in \Omega_k}{\operatorname{argmax}}\, J_k^e\left(\boldsymbol{u}_k\right)$$

$$\Omega_k = \left[\boldsymbol{u}_k^c - \delta_k, \boldsymbol{u}_k^c + \delta_k\right]$$

$$\delta_k = f\left(\boldsymbol{P}_k\right) = \eta \cdot \operatorname{tr} \boldsymbol{P}_k$$

Bicriterial control law for considered MIMO system

- Structure of the control law

$$\boldsymbol{u}_k^* = \boldsymbol{u}_k^c + \boldsymbol{\delta}_k \operatorname{sign}(\boldsymbol{\omega}_k)$$
$$\boldsymbol{\omega}_k = J_k^e\left(\boldsymbol{u}_k^c + \boldsymbol{\delta}_k\right) - J_k^e\left(\boldsymbol{u}_k^c - \boldsymbol{\delta}_k\right)$$

- Denote $\boldsymbol{\alpha}_k$, $\boldsymbol{\beta}_k$ and $\boldsymbol{\gamma}_k$ as the first, the second and the third moment of the augmented state $\boldsymbol{x}_k \triangleq (\boldsymbol{s}_k, \boldsymbol{\theta}_k)^T$ given by the pdf $p(\boldsymbol{x}_k|\boldsymbol{y}_0^k)$, respectively. After the control law derivation the dependency of the cautious and the probing part can be written as

$$\boldsymbol{u}_k^c = f(\boldsymbol{\alpha}_k, \boldsymbol{\beta}_k)$$
$$\boldsymbol{\omega}_k = f(\boldsymbol{\beta}_k, \boldsymbol{\gamma}_k)$$

Estimation

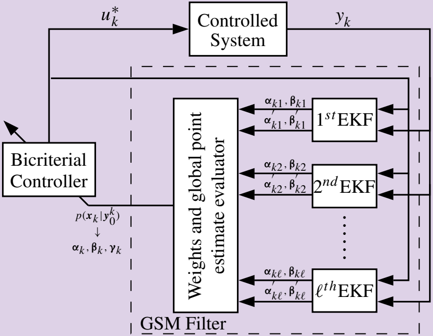
- To generate the $\boldsymbol{u}_k^*$, it is necessary to know the filtering pdf $p(\boldsymbol{x}_k|\boldsymbol{y}_0^k)$ (the system is nonlinear system from the estimation point of view)
- A suitable nonlinear filtering method has to be employed
- Non-Gaussian random variables $\Rightarrow$ usage of a global filtering method would be advantageous

Employment of global nonlinear filtering method

The Gaussian sum method was employed

$$p(\mathbf{x}_k|y_0^k) = \sum_{i=1}^{\ell} \alpha_i \mathcal{N}(\hat{\mathbf{x}}_{ki}, \operatorname{cov} \mathbf{x}_{ki})$$

Scheme of the Bicriterial controller



Two aspects of Bicriterial controller design

- ✓ The GSM filter is a bank of local Extended Kalman filters (EKF) which generates local point estimates (given by $\boldsymbol{\alpha}_{ki}$, $\boldsymbol{\beta}_{ki}$ and $\boldsymbol{\gamma}_{ki}$, $\forall i$)
- ✓ The controller makes use of the global point estimate given by moments $\boldsymbol{\alpha}_k$, $\boldsymbol{\beta}_k$ and $\boldsymbol{\gamma}_k$

⇒ Would it possible to take advantage of the local estimates?

Proposal – make use of local estimates in order to generate a probing signal that could support the estimation better

Bicriterial controller with multiple linearization

The i-th bicriterial controller coupled with the i-th EKF

$$\boldsymbol{u}_{ki}^{*} = \underset{\boldsymbol{u}_{ki} \in \Omega_{ki}}{\operatorname{argmax}} \, J_k^e\left(\boldsymbol{u}_{ki}\right), \qquad \Omega_{ki} = [\boldsymbol{u}_{ki}^c - \boldsymbol{\delta}_{ki}, \boldsymbol{u}_{ki}^c + \boldsymbol{\delta}_{ki}], \; \boldsymbol{\delta}_k = \eta \operatorname{tr} \boldsymbol{P}_{ki}$$

$$\boldsymbol{u}_{ki}^{*} = \boldsymbol{u}_{ki}^c + \boldsymbol{\delta}_{ki} \operatorname{sign}\left(\boldsymbol{\omega}_{ki}\right), \qquad \boldsymbol{\omega}_{ki} = J_k^e\left(\boldsymbol{u}_{ki}^c + \boldsymbol{\delta}_{ki}\right) - J_k^e\left(\boldsymbol{u}_{ki}^c - \boldsymbol{\delta}_{ki}\right)$$

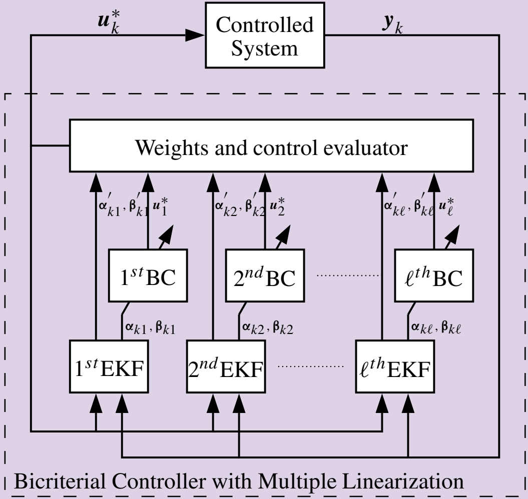
$$\boldsymbol{u}_k^{*} = \sum_{i=1}^{\ell} \alpha_i \boldsymbol{u}_{ki}^{*} = \boldsymbol{u}_k^c + \sum_{i=1}^{\ell} \alpha_i \boldsymbol{\delta}_{ki} \operatorname{sign}\left(\boldsymbol{\omega}_{ki}\right)$$

Does the controller induce different probing signal?

$$\boldsymbol{\delta}_k \operatorname{sign}\left(\boldsymbol{\omega}_k\right) \overset{?}{\neq} \sum_{i=1}^{\ell} \alpha_i \boldsymbol{\delta}_{ki} \operatorname{sign}\left(\boldsymbol{\omega}_{ki}\right).$$

Bicriterial controller with multiple linearization

Scheme of the Bicriterial controller with multiple linearization



Numerical example

Considered system

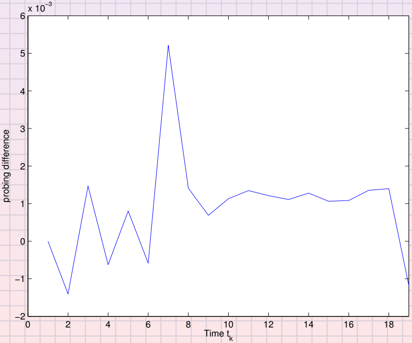
$$s_{k+1} = \begin{pmatrix} 0 & 1 \\ \theta_1 & \theta_2 \end{pmatrix} s_k + \begin{pmatrix} \theta_3 \\ \theta_{4k} \end{pmatrix} u_k + w_k$$
$$\theta_{4k+1} = \theta_{4k} + \epsilon_k$$
$$y_k = (1,\ 1) s_k + v_k$$

- ✓ Prior pdf of the state and the parameters
  - ➢ $p(s_0) = \mathcal{N}\left((0,\ 0)^T,\ 5 \cdot I\right)$
  - ➢ $p(\theta_0) = \mathcal{N}\left(\hat{\theta}_0, \text{diag}(0.8, 0.8, 1.3, 1.3)\right)$, $\hat{\theta}_0 = (-2.0427,\ 0.3427,\ 0,\ 1)^T$
- ✓ Noise pdf's
  - ➢ $p(w_k) = \mathcal{N}\left((0,\ 0)^T, 10^{-4} I\right)$
  - ➢ $p(v_k) = \mathcal{N}\left(0,\ 10^{-3}\right)$
  - ➢ $p(\epsilon_k) \sim \mathcal{U}(-0.1, 0.1)$ approximated by a Gaussian mixture with term number $\ell = 5$
- ✓ Probing parameter $\eta = .58$

Q: Does the inequality of probing signals hold?

⇒ The probing signals indeed differ!

$$\delta_k \operatorname{sign}(\omega_k) \neq \sum_{i=1}^{\ell} \alpha_i \delta_{ki} \operatorname{sign}(\omega_{ki}) \, .$$



Comparison to other controllers

The following index is chosen as a measure of the control performance

$$\mathcal{M} = \sqrt{\frac{1}{N}\sum_{k=1}^{N}(y_k - \bar{y}_k)^2},$$

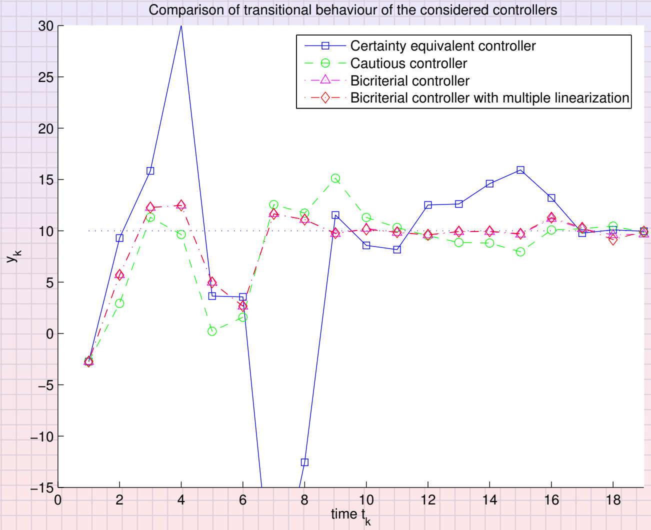
- ✓ y_k represents the measurement
- ✓ $\bar{y}_k$ represents the reference value
- ✓ N determines length of one simulation run

The expected value $\hat{\mathcal{M}} = E\{\mathcal{M}\}$ is estimated using 5000 Monte Carlo simulations.

Control quality comparison using the index $\hat{\mathcal{M}}$

	$\hat{\mathcal{M}}$
Certainty equivalent controller	9.9212
Cautious controller	4.9824
Bicriterial controller	4.8634
Bicriterial controller with multiple linearization	4.8578

Comparison to other controllers



Concluding remarks

- ➢ Generalization of the Bicriterial dual controller was presented
- ➢ Some aspects of Bicriterial controller were discused
- ➢ Bicriterial control scheme employing multiple linearization was suggested
- ➢ The cautious part of the multiple linearized control law is unchanged
- ➢ The two Bicriterial controller schemes induces different probing signal