Attenuation Imaging Using Ultrasound Transmission Tomography

Igor Peterlík, Radovan Jiřík, Jiří Jan, Dušan Hemzal, Nicole Ruiter, Rainer Stotzka

Brno University of Technology
Department of Biomedical Engineering
Brno, CZECH REPUBLIC

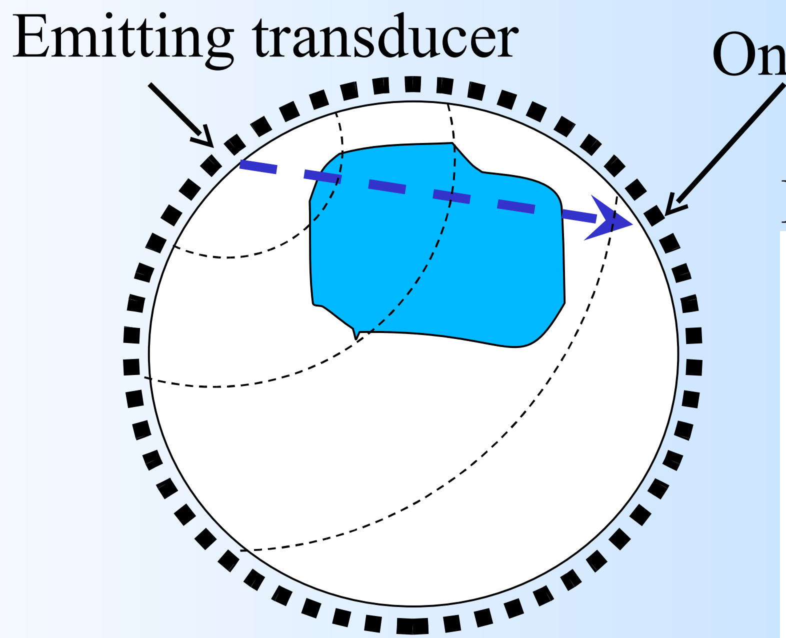
Forschungszentrum Karlsruhe
Institute for Data-Processing and Electronics
Eggenstein, GERMANY

1. Introduction

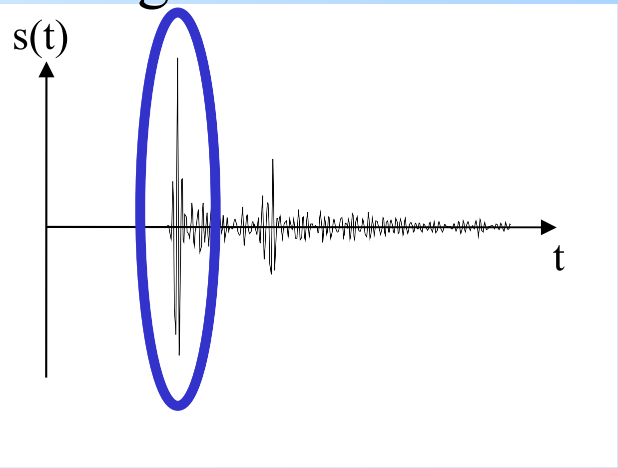
Ultrasound attenuation imaging:

– correction for ultrasound reflection tomography imaging
– stand-alone imaging modality – ultrasound attenuation coefficient closely related to the pathological tissue state

2. Transmission tomography imaging

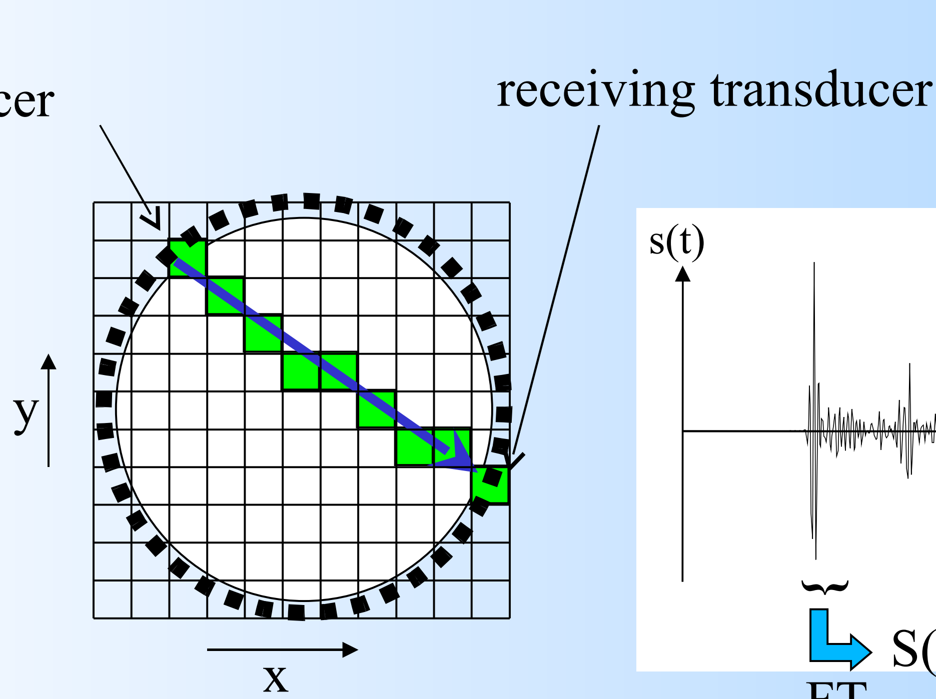


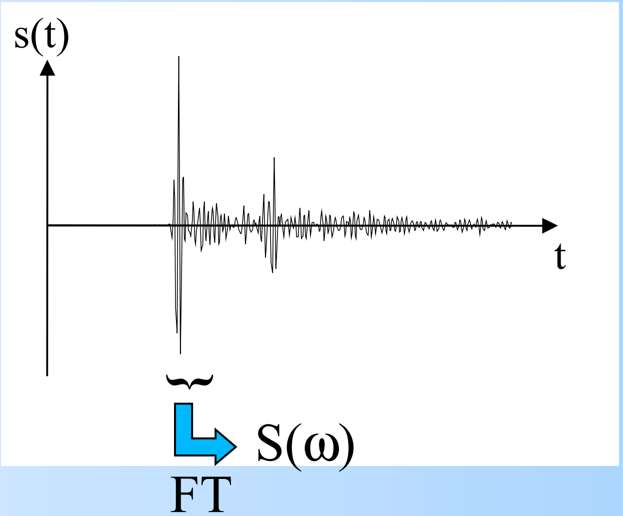
RF signal measured at transducer:



– each time one transducer in the emitter mode, all other transducers record the received RF signals
– all combinations of sending and receiving elements
– undirected wave

3. Attenuation imaging





measured — empty meassurement — attenuation

$$|S(\omega)| = |S_0(\omega)| e^{-\bar{\beta} d \left|\frac{\omega}{2\pi}\right|}$$

$\bar{\beta}$ – mean attenuation coef. along the path

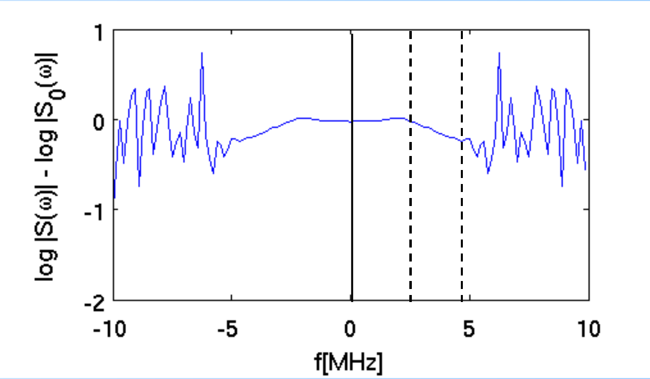
d – length of the path

3. Attenuation imaging

Estimation of the mean attenuation coefficient:

$$|S(\omega)| = |S_0(\omega)| e^{-\bar{\beta} d \left|\frac{\omega}{2\pi}\right|}$$

$$\bar{\beta} = \frac{\log|S(\omega)| - \log|S_0(\omega)|}{-d\left|\frac{\omega}{2\pi}\right|}$$



Estimation of the local attenuation coefficients (ART):

$$\bar{\beta} d = \sum_{i \in l} \beta_i d_i$$

l — pixels along the path

β_i — local att. coef. within i-th pixel along the path

d_i — length of the pixel along the path

Path 1: $\sum_{i \in l_1} \beta_i d_i = \bar{\beta}_1 d_1$

Path 2: $\sum_{i \in l_2} \beta_i d_i = \bar{\beta}_2 d_2$ ⇨ **Rf = p**

…

Path m: $\sum_{i \in l_m} \beta_i d_i = \bar{\beta}_m d_m$

3. Attenuation imaging

Problems (errors in estimates of mean att. coef.)	**Possible solutions**
diffraction	synthetic aperture focusing, subset of correct equations, *regularization of the ART*
pulse detection (uncertainty in the pulse position)	constrained maximum search, speed-of-sound and geometry correction, *regularization of the ART*
overlap of the transmission pulse and the scattered/ reflected wave signal	synthetic aperture focusing, *regularization of the ART*
noise in RF data	*regularization of the ART*

Regularization Technique for USCT

Igor Peterlík, Radovan Jiřík

Technical University Brno, Brno, Czech Republic

3rd International Workshop on Data – Algorithms – Decision Making 2007

Non-Regularized Solution

- the system $Rf = p$, where R and p are input data, f is the image being reconstructed
- M equations with N variables $\rightarrow$ overdetermined $M \times N$ system $\rightarrow$ no exact solution
- the solution computed as minimization:

$$\hat{f} = argmin_f(J_1(f))$$

where $J(f)$ is a functional being minimized:

$$J_1(f) = \|p - Rf\|^2$$

- naïve approach: solution of *normal equations*—$N \times N$ square system $R^T Rf = R^T p$

Edge-Preserving Regularization

- inspired by the techniques from image deconvolution:
  - homogeneous regions
  - edges (step changes of the att. coefficient)
- minimization of augmented functional [Char99]:

$$f = argmin_f(J_1(f) + \lambda^2 J_2(f))$$

- J_2 is regularizing term

$$J_2(f) = \sum_k \varphi[(D_x f)_k] + \sum_k \varphi[(D_y f)_k]$$

where

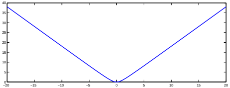
$$(D_x f)_{ij} = (f_{i,j+1} - f_{i,j})/\delta \quad (D_y f)_{ij} = (f_{i+1,j} - f_{i,j})/\delta$$

- parameters: two scalars (λ, δ) and the *potential function* φ

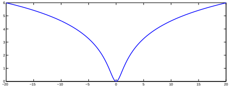
Potential Functions and Parameters

- the potential function assigns cost to every value of the image gradient
- threshold for the edge penalization (smoothing)

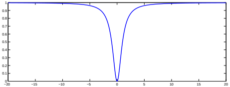
- $\varphi_{HS}(t) = 2\sqrt{1 + t^2} - 2$



- $\varphi_{HL}(t) = log(1 + t^2)$



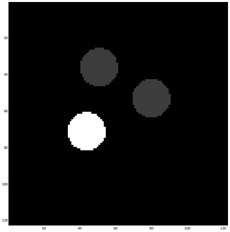
- $\varphi_{GM}(t) = \frac{t^2}{1+t^2}$



- parameter δ sets a threshold for edges, compromise edges preservation vs. noise suppression
- parameter λ is weight of the regularization

Experimental Data

- simulated RF data (based on Huygens principle)
- reconstruction of image 50×50 pixels, (1576 variables)
- overdetermined system of 7228 equations
- right-hand side vector affected by additive Gaussian noise
- the parameters chosen experimentally together with φ_{GM} potential function

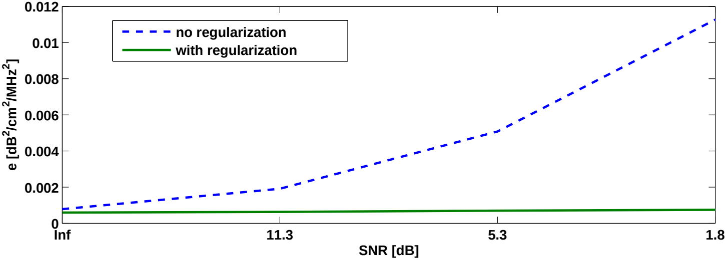


Experimental Results I.

- error measured by square image difference

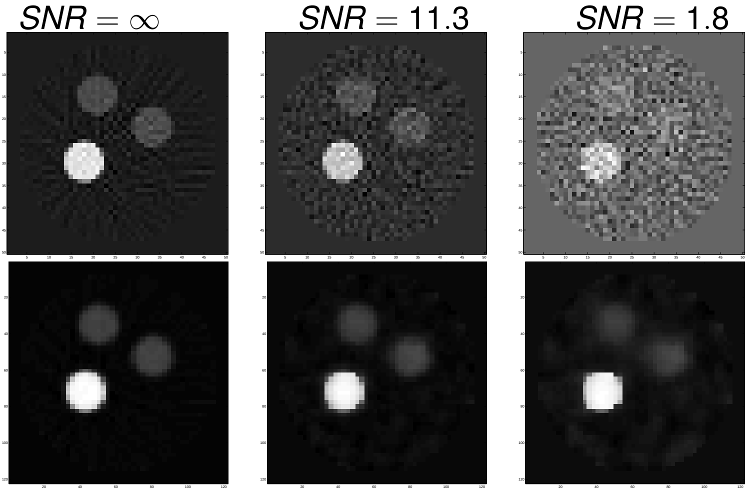
$$e = \frac{1}{MN} \sum_{m=1}^{M} \sum_{n=1}^{N} \left[f_{ref}(m,n) - \hat{f}(m,n) \right]^2$$

- effect of the regularization for various SNR



Experimental Results II.

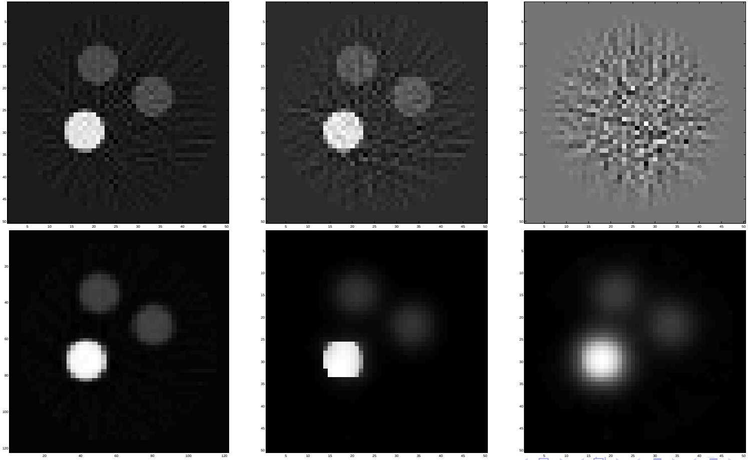
- effect of the regularization for various SNR (cont'd)



Experimental Results III.

- some equations left out (e.g. due to out-of-range right-hand value)

7228 equations | 2409 equations | 1807 equations



Conclusion and Future Work

- one specific topic of the project aimed at attenuation image reconstruction in ultrasound transmition tomography
- importance of the regularization to cope with
  - inaccurate estimates of the mean attenuation coefficients
  - low degree of overdetermination
- future work and goals
  - non-negativity constraints
  - testing on real 2D and 3D data
  - larger local neighborhood