A survey of research activities at FAV in filtering, detection and control

Research Centre Data - Algorithms - Decision Making

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Main topics of research at the Faculty of Applied Sciences

- Fault detection
- Nonlinear filtering
  - Global methods
    - Point mass approach
    - Particle filter approach
    - Gaussian sum approach (presented by Ondřrej Straka)
  - Local methods
    - Derivative free approach
  - Nonlinear filtering toolbox (presented by Miroslav Flídr)
  - Application of nonlinear filtering in traffic control
- Adaptive dual control (marginally in 2006)

Fault detection: Introduction

Motivation

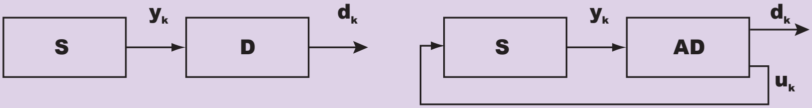
There are different fault detection approaches that have some common features:

- The fault detection problem is not usually formulated as a general optimization problem
- It is difficult to determine whether a detector, designed using standard approaches, utilizes all available information in the best way
- Active fault detection and control problem are not considered to be solved simultaneously

Fault detection: Introduction – cont'd

Passive and active fault detection

- **Passive fault detection** - the detector **D** passively uses measurements $\mathbf{y}_k$ to provide decisions $\mathbf{d}_k$ about faults in the system **S** (most publications on fault detection deal with this problem)
- **Active fault detection** - the active detector **AD** provides decisions and also input signal $\mathbf{u}_k$ that should improve fault detection and eventually control the system (there are only a few contributions and it is a developing area of fault detection)



a) Passive fault detection b) Active fault detection

Fault detection: Introduction – cont'd

Active fault detection and control – goals

- Propose a unified formulation of the active fault detection problem
- Specify three basic special cases
  - Optimal detector for given input signal generator
  - Optimal detector and optimal input signal generator
  - Optimal detector and optimal dual controller
- Find solutions of considered special cases using closed loop information processing strategy (IPS)

Fault detection: Problem formulation

Description of the observed system for $k \in \mathcal{T} = \{0, \ldots, F\}$

$$
\begin{aligned}
\text{System}: \mathbf{x}_{k+1} &= \mathbf{f}_k\left(\mathbf{x}_k, \mu_k\, \mathbf{u}_k, \mathbf{w}_k\right) \\
\mu_{k+1} &= \mathbf{g}_k\left(\mu_k, \mathbf{e}_k\right) \\
\mathbf{y}_k &= \mathbf{h}_k\left(\mathbf{x}_k, \mu_k, \mathbf{v}_k\right)
\end{aligned}
$$

$\mathbf{f}_k$, $\mathbf{g}_k$ and $\mathbf{h}_k$ are known function; $\mathbf{x}_k \in \mathcal{R}^{n_x}$ is controllable part of the state; $\mu_k \in \mathcal{M} \subset \mathcal{R}^{n_\mu}$ is uncontrollable part of the state and represents faults; $\mathbf{u}_k \in \mathcal{U}_k \subset \mathcal{R}^{n_u}$ is input, $\mathbf{y}_k \in \mathcal{R}^{n_y}$ is output; $\{\mathbf{w}_k\}$, $\{\mathbf{e}_k\}$ and $\{\mathbf{v}_k\}$ are mutually independent random sequences

Fault detection: Problem formulation – cont'd

Description of the general active detector for $k \in \mathcal{T}$

$$\text{Active detector}: \begin{bmatrix} \mathbf{d}_k \\ \mathbf{u}_k \end{bmatrix} = \begin{bmatrix} \boldsymbol{\sigma}_k\left(\mathbf{I}_0^k\right) \\ \gamma_k\left(\mathbf{I}_0^k, \mathbf{d}_k\right) \end{bmatrix}$$

$\boldsymbol{\sigma}_k$ and γ_k are generally unknown functions; $\mathbf{d}_k \in \mathcal{M}$ is an estimate of the decision μ_k; all available information at time k is stored in $\mathbf{I}_0^k = [{\mathbf{y}_0^k}^T, {\mathbf{u}_0^{k-1}}^T, {\mathbf{d}_0^{k-1}}^T]^T$

Fault detection: Problem formulation – cont'd

Additive general criterion

$$J(\boldsymbol{\sigma}_0^F, \gamma_0^F) = \mathrm{E}\left\{\sum_{i=0}^{F} L_i(\mathbf{d}_i, \mu_i, \mathbf{x}_i, \mathbf{u}_i)\right\} \to \min$$

Information processing strategies

- Open loop (OL) - only a priori information is used
- Open loop feedback (OLF) - information received up to current time is used, but the future information is not considered
- Closed loop (CL) - information received up to current time is used and also the future information is considered ($J^{CL} \leq J^{OLF} \leq J^{OL}$)

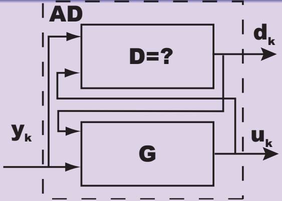
General solution - Bellman recursive equation

$$V_k^*(\mathbf{I}_0^k) = \min_{\substack{\mathbf{d}_k \in \mathcal{M} \\ \mathbf{u}_k \in \mathcal{U}_k}} \mathrm{E}\left\{L_k(\mathbf{d}_k, \mu_k, \mathbf{x}_k, \mathbf{u}_k) + V_{k+1}^*(\mathbf{I}_0^{k+1}) | \mathbf{I}_0^k, \mathbf{u}_k, \mathbf{d}_k\right\}$$

Fault detection: Specifications and solutions of the special cases

Case I: Optimal detector for given input signal generator

- The input signal generator is given by functions $\gamma_k(\mathbf{I}_0^k, \mathbf{d}_k)$ (e.g. existing controller) and the detector $\boldsymbol{\sigma}_k(\mathbf{I}_0^k)$ has to be found



- Criterion which has to be minimized

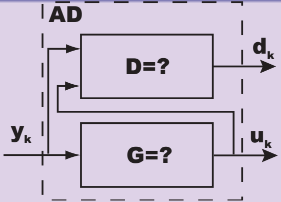
$$J_{ADGG}(\boldsymbol{\sigma}_0^F) = \mathrm{E}\left\{\sum_{i=0}^{F} L_i^d(\mathbf{d}_i, \mu_i)\right\}$$

- It follows from solution that optimal decision $\mathbf{d}_k^*$ is a trade-off between the evaluation of current decision and the evaluation of future decisions which are determined by this decision

Fault detection: Specifications and solutions of the special cases – cont'd

Case II: Optimal detector and optimal input signal generator

- Both the detector $\boldsymbol{\sigma}_k(\mathbf{I}_0^k)$ and input signal generator $\gamma_k(\mathbf{I}_0^k, \mathbf{d}_k)$ have to be found



- Criterion which has to be minimized

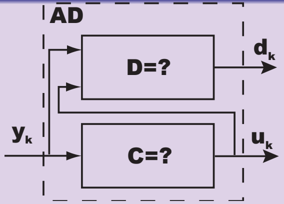
$$J_{ADG}(\boldsymbol{\sigma}_0^F, \gamma_0^F) = \mathrm{E}\left\{\sum_{i=0}^{F} L_i^d(\mathbf{d}_i, \mu_i)\right\}$$

- It follows from solution that optimal decision $\mathbf{d}_k^*$ and optimal input signal $\mathbf{u}_k^*$ are chosen independently, so $\gamma_k(\mathbf{y}_0^k, \mathbf{u}_0^{k-1}, \mathbf{d}_0^k) = \gamma_k(\mathbf{y}_0^k, \mathbf{u}_0^{k-1})$

Fault detection: Specifications and solutions of the special cases – cont'd

Case III: Optimal detector and optimal dual controller

- Both the detector $\boldsymbol{\sigma}_k(\mathbf{I}_0^k)$ and the dual controller $\gamma_k(\mathbf{I}_0^k, \mathbf{d}_k)$ have to be found



- Criterion which has to be minimized

$$J_{ADC}(\boldsymbol{\sigma}_0^F, \gamma_0^F) = \mathrm{E}\left\{\sum_{i=0}^{F} L_i^d(\mathbf{d}_i, \mu_i) + \alpha_i L_i^c(\mathbf{x}_i, \mathbf{u}_i)\right\}$$

- It follows from solution that optimal decision $\mathbf{d}_k^*$ and optimal input signal $\mathbf{u}_k^*$ are also independent, but optimal input signal is trade-off between control objective and the excitation of the observed system

Fault detection: Specifications and solutions of the special cases – cont'd

Comments on special cases

- The solution was obtained using closed loop information processing strategy and it has form of backward recursive equation in all three special cases
- The optimal decision $\mathbf{d}_k^*$ and the optimal input signal $\mathbf{u}_k^*$ are independent in the second and the third special case
- The optimal input signal is a compromise between two opposite goals in the first and the last special case

Fault detection: Multiple model framework an illustrative example

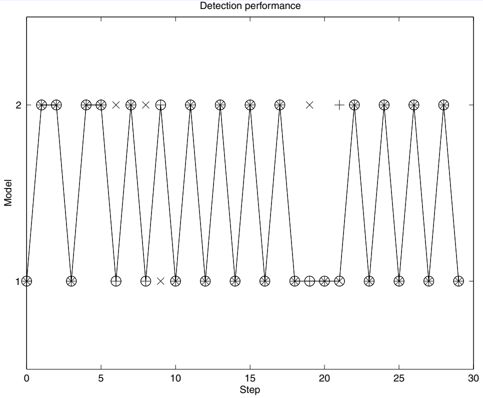
Specification of the multiple model framework

- The observed system is supposed to be described at each time step by one of the model from a priori given finite set of models
- The set $\mathcal{M} = \{1, \ldots, N\}$ is discrete and μ_k denotes scalar index to the model valid at time k
- The state equation $\mu_{k+1} = \mathbf{g}_k(\mu_k, \mathbf{e}_k)$ is replaced by transition probabilities $P_{i,j} = P(\mu_{k+1} = j | \mu_k = i)$, $i, j \in \mathcal{M}$

Numerical example

- System is described by two switching linear Gaussian scalar models
- Two different active detectors generating probing input signal (D1 - designed using OLFIPS, D2 - designed by CLIPS) are used to detect correct model at each time
- Quality of decisions is illustrated in a simulation run and compared through values of criterion

Fault detection: Results of a numerical example



○ true model

× decisions provided by the detector D1

+ decisions provided by the detector D2

Criterion values

J^*_{D1}	J^*_{D2}
6.6142	2.7548

Fault detection: Concluding remarks and publications

Concluding remarks

- The general formulation of the active fault detection and control problem was proposed
- Three interesting special cases were derived and solved using closed loop information processing strategy

Publications

- Šimandl, M. and I. Punčochář (2006). Closed loop information processing strategy for optimal fault detection and control. In: Preprints of the 14th IFAC Symposium on System Identification. Newcastle, Australia.
- Šimandl, M., I. Punčochář and J. Královec (2005). Rolling horizon for active fault detection. In: Proceedings of the 44th IEEE Conference on Decision and Control and European Control Conference. Seville, Spain.
- Šimandl, M., I. Punčochář and P. Herejt (2005). Optimal input and decision in multiple model fault detection. In: Preprints of the 16th IFAC World Congress. Prague, Czech Republic.

N.F.M.: Nonlinear stochastic system

$$x_{k+1} = f_k(x_k) + w_k \quad k = 0, 1, 2, \ldots$$

- x_k is nx dimensional state vector at time instant t_k,
- w_k is nx dimensional state noise at time t, where $t_k \leq t < t_{k+1}$,
- $f_k(\cdot)$ is known vector function of corresponding dimension
- random process $\{w_k\}$ is white noise with known pdf $p(w_k)$
- pdf of the initial state $p(x_0)$ is known

$$z_k = h_k(x_k) + v_k \quad k = 0, 1, 2, \ldots$$

- z_k is nz dimensional vector of measurements at time instant t_k
- v_k is nz dimensional measurement noise at time t_k
- random process $\{v_k\}$ is white noise with known pdf $p(v_k)$
- processes $\{w_k\}, \{v_k\}$ and random variable x_0 are mutually independent

N.F.M.: General solution

Recursive state estimation

- Bayesian relation $p(a,b) = p(a \mid b)p(b) = p(b \mid a)p(a)$
- Filtering $p(x_k \mid z^k)$, prediction $p(x_{k+l} \mid z^k)$, smoothing $p(x_k \mid z^{k+l})$, $l > 0$
- Bayesian recursive relations

$$p(x_k \mid z^k) = \frac{p(x_k \mid z^{k-1}).p(z_k \mid x_k)}{p(z_k \mid z^{k-1})}$$

$$p(x_k \mid z^{k-1}) = \int_{-\infty}^{\infty} p(x_{k-1} \mid z^{k-1})p(x_k \mid x_{k-1})dx_{k-1}$$

$$p(z_k \mid z^{k-1}) = \int_{-\infty}^{\infty} p(x_k \mid z^{k-1})p(z_k \mid x_k)dx_k$$

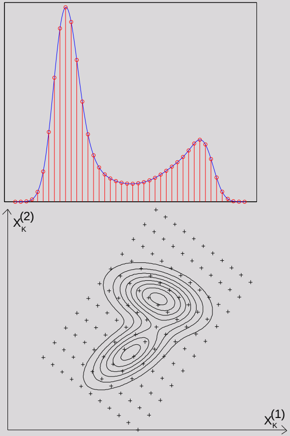
- Analytical solution especially for linear Gaussian systems

N.F.M.: Basic approximations of pdf

Point mass method

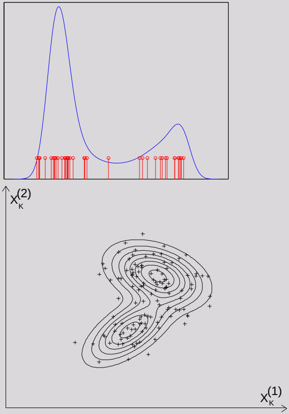
$$p(\mathbf{x}_k|\mathbf{z}^k)=\{P_i;\, P_i = p(\mathbf{x}_k \in \text{okoli}\,\xi_i|\mathbf{z}^k)\},$$

$\xi_i \in \Xi(N)$



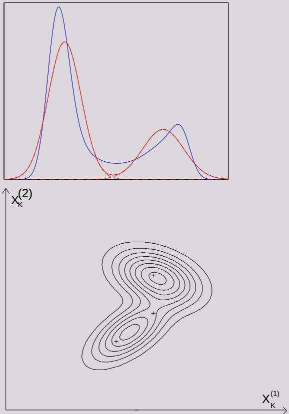
Particle filters

$$p(\mathbf{x}_k|\mathbf{z}^k) = \sum_{i=1}^{N} w^{(i)}\,\delta(\mathbf{x}_k - \mathbf{x}_k^{(i)})$$



Gaussian sum method

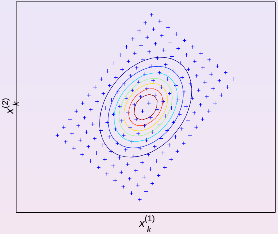
$$p(\mathbf{x}_k|\mathbf{z}^k) = \sum_{i=1}^{N} \alpha^{(i)}\mathcal{N}\{\mathbf{x}_k : \mu_k^i, \mathbf{P}_k^i\}$$



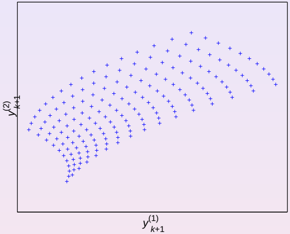
N.F.M.: Point mass method - basic algorithm

0. Initialization Define initial grid ξ_0 for prior pdf $p(\mathbf{x}_0|\mathbf{z}^{-1})$:
$$\xi_0 = \{\xi_0^{(i)}; \quad \text{i=1,2,\ldots,N}\}$$
1. Filtering: Filtering pdf $\hat{p}(\mathbf{x}_k|\mathbf{z}^k)$ for ξ_k is given by
$$\hat{p}(\xi_k^{(i)}|\mathbf{z}^k) = c_k^{-1}\hat{p}(\xi_k^{(i)}|\mathbf{z}^{k-1})\, p_{\mathbf{v}_k}(\mathbf{z}_k - \mathbf{h}_k(\xi_k^{(i)}))$$
$$c_k = \sum_{i=1}^N \Delta\xi_k^{(i)}\, \hat{p}(\xi_k^{(i)}|\mathbf{z}^{k-1})\, p_{\mathbf{v}_k}(\mathbf{z}_k - \mathbf{h}_k(\xi_k^{(i)}))$$
2. Grid transformation: Points $\xi_k^{(i)}$ of grid ξ_k are transformed to $\eta_{k+1}^{(i)} = \mathbf{f}_k(\xi_k^{(i)})$ and redefined to $\xi_{k+1}^{(j)}$.
3. Prediction: Predictive pdf $\hat{p}(\mathbf{x}_{k+1}|\mathbf{z}^k)$ for $\xi_{k+1}^{(j)}$ is given by discretization of the convolution integral
$$\hat{p}(\xi_{k+1}^{(j)}|\mathbf{z}^k) = \sum_{i=1}^N \Delta\xi_k^{(i)}\, \hat{p}(\xi_k^{(i)}|\mathbf{z}^k) \times\; p_{\mathbf{w}_k}(\xi_{k+1}^{(j)} - \eta_{k+1}^{(i)}).$$

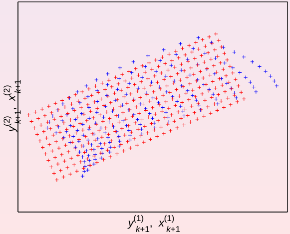
N.F.M.: Point mass method – Grid construction and transformation

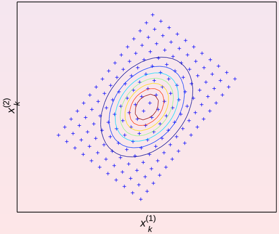


1. Filtering



2. Grid transformation





N.F.M.: Point mass method – Main problems and solution

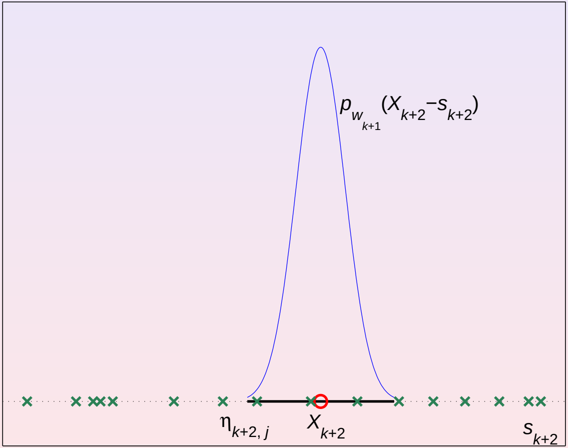
Main problems

- Grid design
- Number of grid points specification
- Numerical demands
- Not suitable for multimodal pdf

Improvements were achieved in the following issues

- Setting of grid point by anticipative grid design
- Boundary based grid placement
- Reduction of computational demands by thrifty convolution technique
- Multigrid representation of multimodal pdf's including multigrid management by splitting and merging

N.F.M.: Point mass method – Idea of the anticipative approach



N.F.M.: Point mass method – Example of multigrid representation (splitting, merging algorithm)

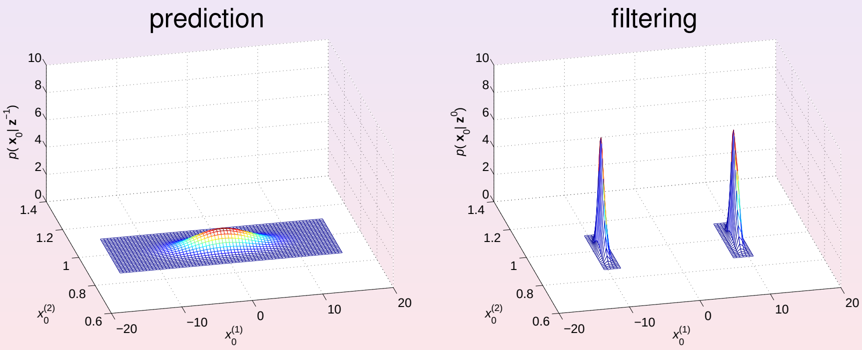
System specification

$$
\begin{aligned}
x^1_{k+1} &= x^1_k + w^1_k \\
x^2_{k+1} &= x^1_k x^2_k + w^2_k \\
z_k &= 0.2(x^2_k)^2 + v_k
\end{aligned}
$$

$$
\begin{aligned}
p(\mathbf{w}_k) &= \mathcal{N}\left\{\mathbf{w}_k; \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0.25 & 0 \\ 0 & 10^{-4} \end{bmatrix}\right\} \\
p(v_k) &= \mathcal{N}\{v_k; 0, 1\} \\
p(\mathbf{x}_0) &= \mathcal{N}\left\{\mathbf{x}_0; \begin{bmatrix} 0 \\ 0.5 \end{bmatrix}, \begin{bmatrix} 16 & 0 \\ 0 & 0.02 \end{bmatrix}\right\}
\end{aligned}
$$

N.F.M.: Point mass method – Example of multigrid representation (splitting, merging algorithm)

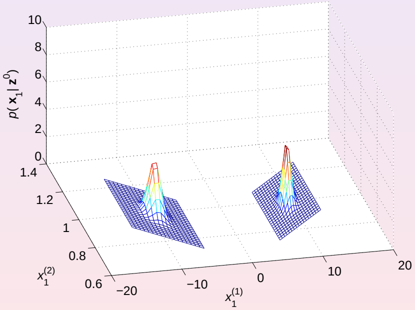
$\mathbf{x}_0$



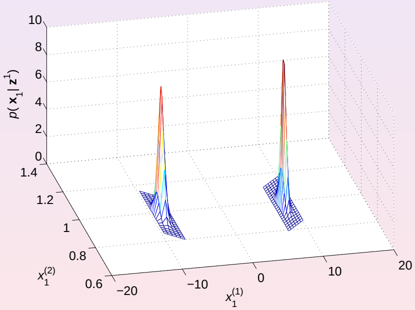
N.F.M.: Point mass method – Example of multigrid representation (splitting, merging algorithm)

$\mathbf{x}_1$

prediction

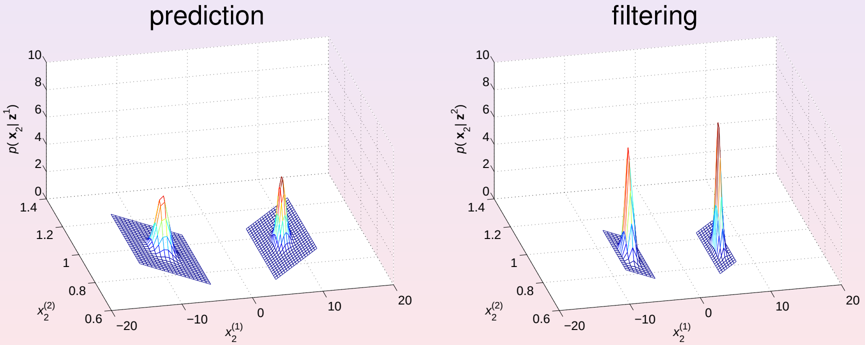


filtering



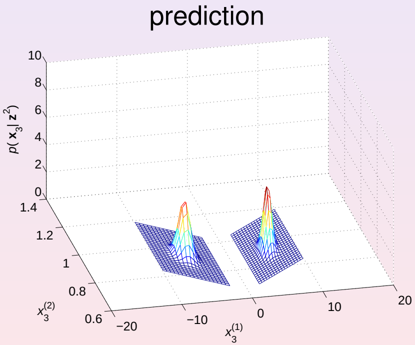
N.F.M.: Point mass method – Example of multigrid representation (splitting, merging algorithm)

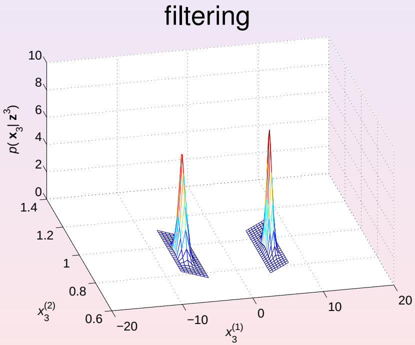
$\mathbf{x}_2$



N.F.M.: Point mass method – Example of multigrid representation (splitting, merging algorithm)

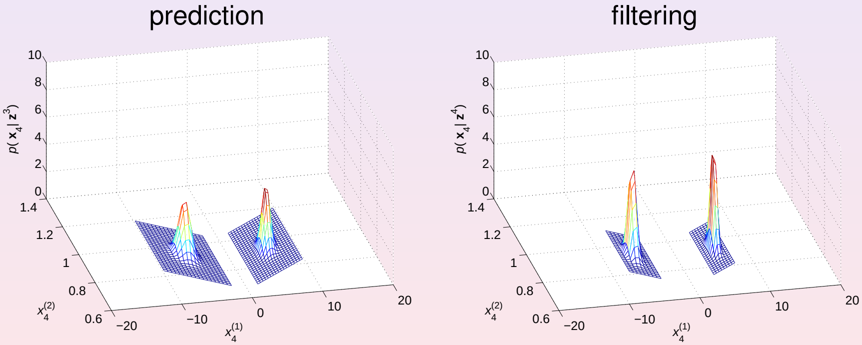
$\mathbf{x}_3$





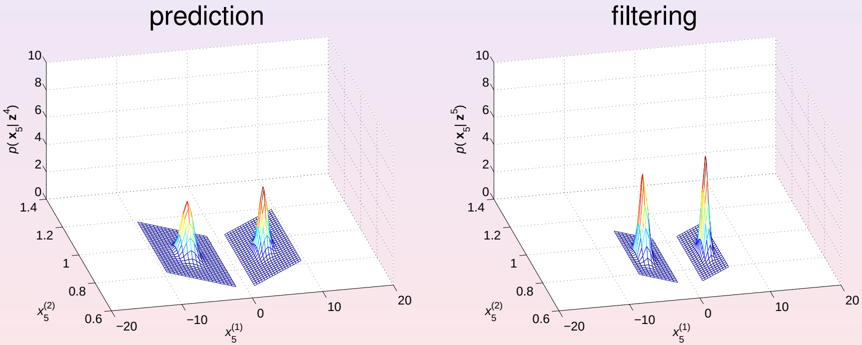
N.F.M.: Point mass method – Example of multigrid representation (splitting, merging algorithm)

$\mathbf{x}_4$

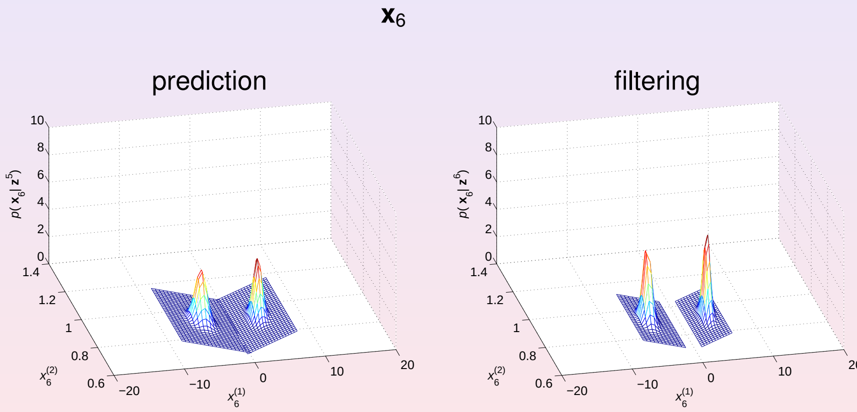


N.F.M.: Point mass method – Example of multigrid representation (splitting, merging algorithm)

$\mathbf{x}_5$

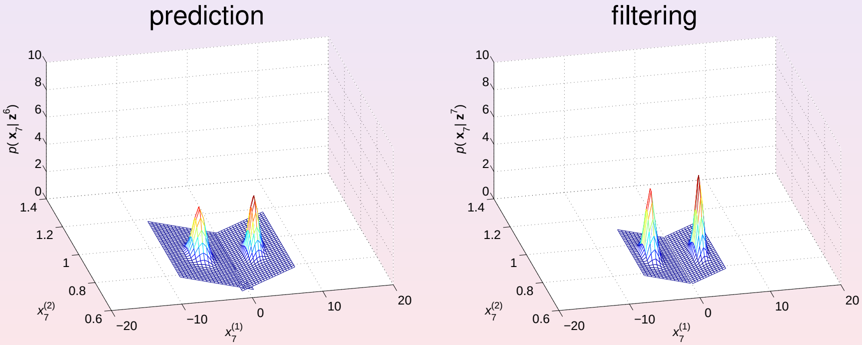


N.F.M.: Point mass method – Example of multigrid representation (splitting, merging algorithm)



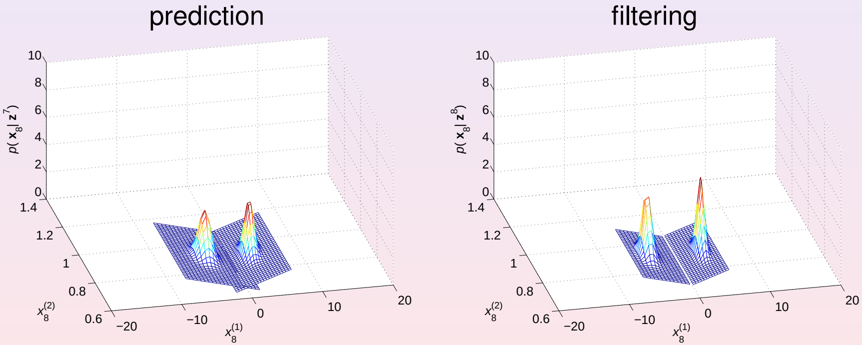
N.F.M.: Point mass method – Example of multigrid representation (splitting, merging algorithm)

$\mathbf{x}_7$

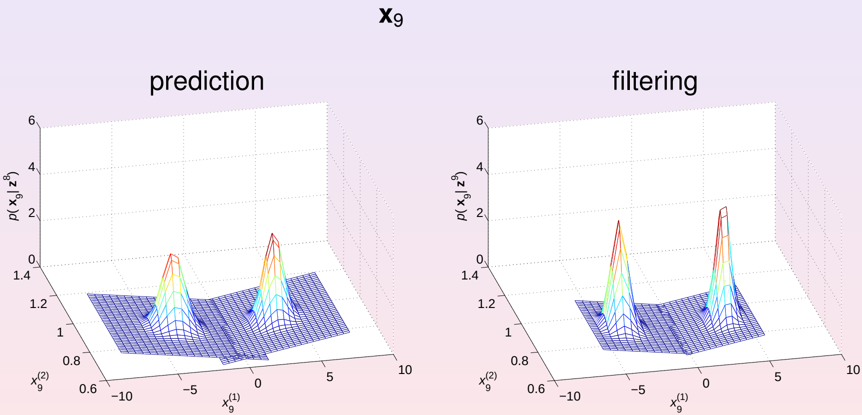


N.F.M.: Point mass method – Example of multigrid representation (splitting, merging algorithm)

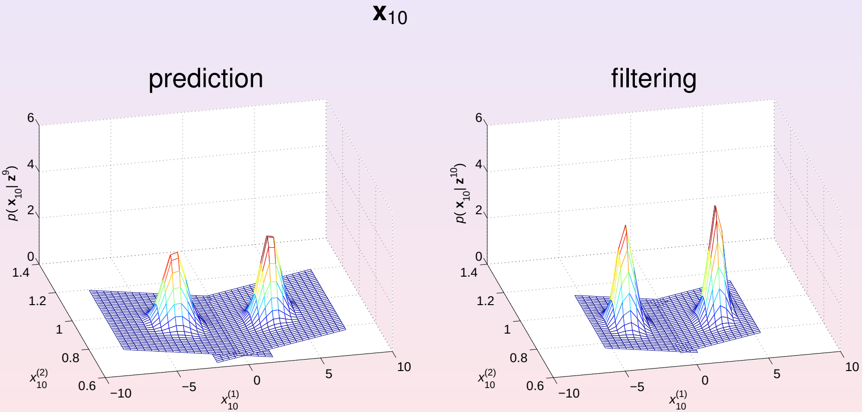
$\mathbf{x}_8$



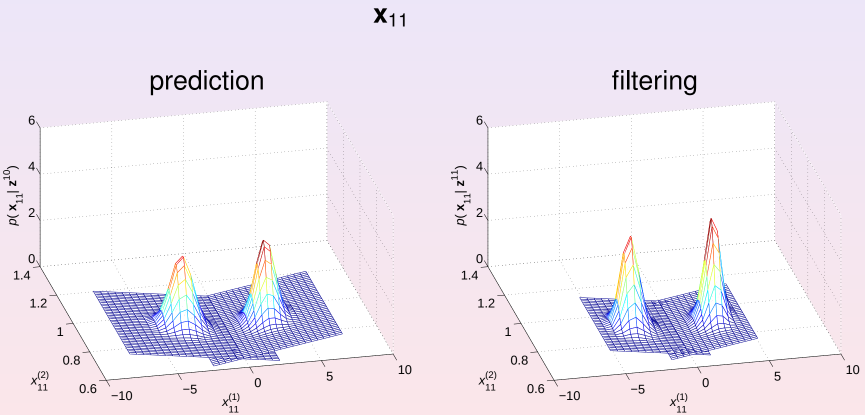
N.F.M.: Point mass method – Example of multigrid representation (splitting, merging algorithm)



N.F.M.: Point mass method – Example of multigrid representation (splitting, merging algorithm)

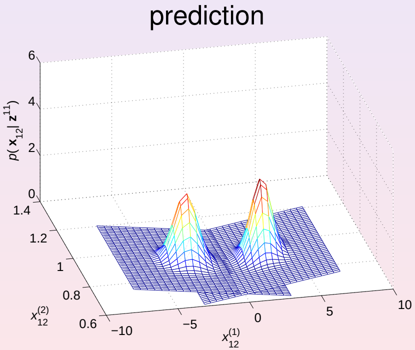


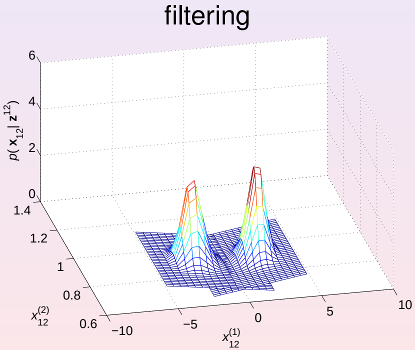
N.F.M.: Point mass method – Example of multigrid representation (splitting, merging algorithm)



N.F.M.: Point mass method – Example of multigrid representation (splitting, merging algorithm)

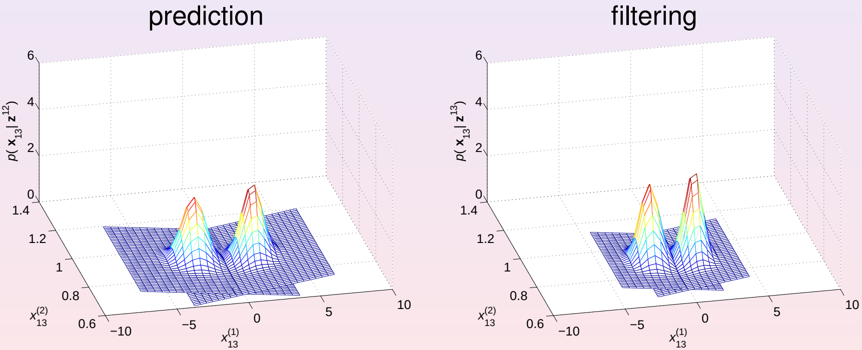
$\mathbf{x}_{12}$





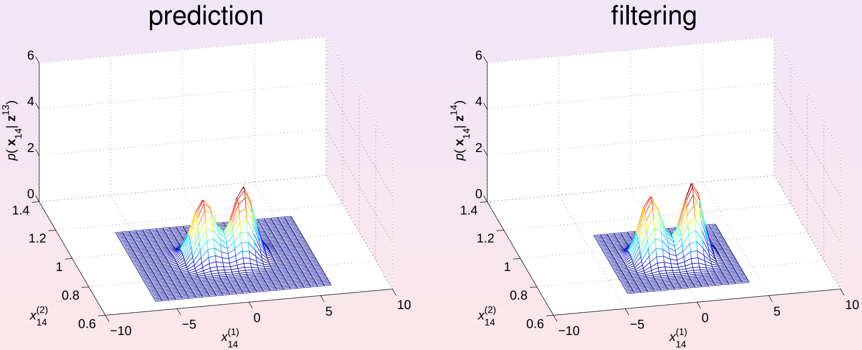
N.F.M.: Point mass method – Example of multigrid representation (splitting, merging algorithm)

$\mathbf{x}_{13}$



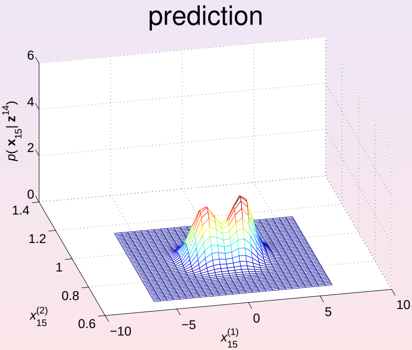
N.F.M.: Point mass method – Example of multigrid representation (splitting, merging algorithm)

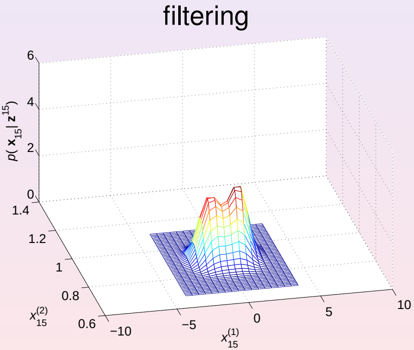
$\mathbf{x}_{14}$



N.F.M.: Point mass method – Example of multigrid representation (splitting, merging algorithm)

$\mathbf{x}_{15}$





N.F.M.: Point mass method – Main results

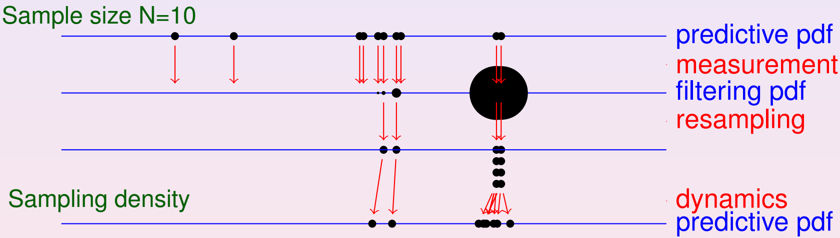
Advanced point mass method was designed. The new algorithms were built in a unified framework.

Main results published in

- Šimandl M., Královec J. , Soderstrom T. (2006): Advanced point mass method for nonlinear state estimation, Automatica 42, Issue 7, 1133-1145.
- Královec J. and M. Šimandl (2005). Numerical solution of filtering problem with multimodal densities, In: Preprints of the 16th IFAC World Congress, July 4-8, Prague, Czech Republic.

N.F.M.: Particle filter

- Solution of the Bayessian recursive relations by the Monte Carlo method
- Approximation of the filtering pdf by an empirical filtering pdf given by a set of samples and corresponding weights



Key aspects affecting estimate quality and computational demands

- Sample size
- Sampling density (also called importance function)

N.F.M.: Particle filter - recent proposed advances

- Functional sampling density based on utilization of additional information about a sample given in the measurement pdf and the transition pdf
- Adaptive sample size setting based on keeping constant efficient sample size (the proposed technique allows to keep estimate quality independent of sampling density

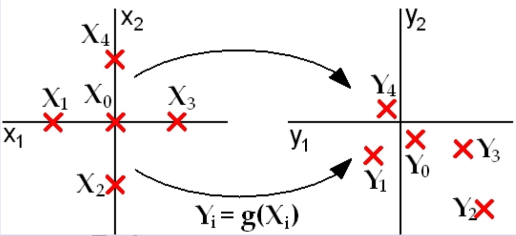
Publications

- Straka O. and M. Šimandl (2005). Using the Bhattcharyya distance in functional sampling density of particle filter, In: Preprints of the 16th IFAC World Congress, July 4-8, Prague, Czech Republic.
- Straka O., Šimandl M. (2006): Particle filter adaptation based on efficient sample size. In: Preprints of the 14th IFAC Symposium on System Identification, Newcastle, Australia, pp. 991-996.

N.F.M.: Derivative-free estimators – main idea

Unscented Kalman Estimators

Utilize a transformation of set of points $\{\mathbf{X_i}\}$, characterizing random variable $\mathbf{x}$ with mean $\hat{\mathbf{x}}$ and covariance matrix $\mathbf{P_x}$, through a nonlinear function.



Divided Difference Estimators

are based on the approximation of a nonlinear function with polynomial interpolation, e.g.

$$y = g(x) \approx g(\hat{x}) + \frac{g(\hat{x}+h)-g(\hat{x}-h)}{2h}(x-\hat{x}),$$

where h is scaling parameter.

N.F.M.: Derivative-free estimators – properties

Merits of derivative-free estimators

- Derivative-free filters provide at least the same estimation performance as the well-known Extended Kalman Filter.
- Moreover, design of the novel filters does not require computation of nonlinear function derivative.

Drawbacks of derivative-free estimators

- The impact of the scaling parameter in polynomial transformation or unscented transformation in the behaviour of estimators is unclear.
- The local derivative-free smoothers have not been derived.
- The utilization of novel local derivative-free filters in the Gaussian sum framework have not been properly discussed.

N.F.M.: Derivative-free estimators – results

- Thorough analysis of the divided difference and the unscented filters was given which allows to explain the impact scaling parameters in their estimation performance.
  Duník, J., Šimandl, M., Straka, O., Král, L. (2005). Performance analysis of derivative-free filters. In: *Proceedings of the 44th IEEE Conference on Decision and Control, and the European Control Conf.*, Seville, Spain.
- The divided difference and unscented smoothers, refilling class of derivative-free estimators, have been designed.
  Šimandl, M., Duník, J. (2006). Design of derivative-free smoothers and predictors. In: *Preprints of the 14th IFAC Symposium on System Identification*, Newcastle, Australia.
- The application of novel derivative-free local filters in the Gaussian sum framework has been analyzed.
  Šimandl, M., Duník, J. (2005). Sigma point Gaussian sum filters design using square root unscented filters. In: *Preprints of the 16th IFAC World Congress*, Prague, Czech Republic.

Application of local filtering techniques in traffic control

Traffic control problem

- The derivative-free local filters have been applied in the area of the traffic control for estimation of the directly immeasurable queue lengths in intersections.
- The stress has been laid especially on the problem of the on-line concurrent estimation of the queue lengths and unknown model parameters.
- With respect to ”standard” approach, off-line parameter estimation and queue length estimation with Kalman Filter, the improvement about 10% has been reached.

- Pecherková P., Nagy I., Duník J. (2005) : Odhad délky kolon. Research Report No.2149, UTIA AV ČR, Praha, 37 pp.
- Duník J., Pecherková P., Flídr M. (2006): State space model of traffic system and its estimation using derivative-Free methods. Internal publication of DAR - UTIA 2006/8, UTIA AV ČR, Praha, 25 pp.