Multiple-model Filtering with Multiple Constraints

Jindřich Duník, Miroslav Šimandl, Ondřej Straka

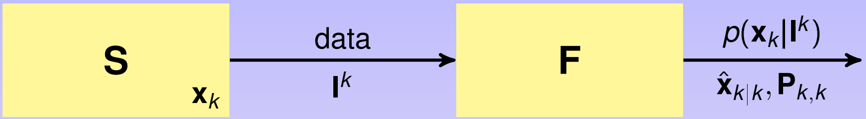
Research Center Data, Algorithms, and Decision Making,
Department of Cybernetics,
University of West Bohemia,
Pilsen, Czech Republic

ACC 2010, June/July

ZÁPADOČESKÁ UNIVERZITA V PLZNI

Outline

Unconstrained MM Filtering



S: system

- behavior approx. by a set of M models Σ_i
  $\mathbf{x}_{k+1} = \mathbf{f}_k^i(\mathbf{x}_k, \mathbf{u}_k) + \mathbf{g}_k^i(\mathbf{w}_k)$, $p_i(\mathbf{x}_0)$, $p(\mathbf{w}_k)$
- measured output
  $\mathbf{z}_k = \mathbf{h}_k(\mathbf{x}_k) + \mathbf{v}_k$, $p(\mathbf{v}_k)$

F: filter

Aim of filtering

$p(\mathbf{x}_k|\mathbf{I}^k)$ with state $\mathbf{x}_k$ and measured data $\mathbf{I}^k = [\mathbf{z}^k, \mathbf{u}^{k-1}]$,

$\mathbf{z}^k \triangleq [\mathbf{z}_0^{\mathrm{T}}, \mathbf{z}_1^{\mathrm{T}}, \ldots, \mathbf{z}_k^{\mathrm{T}}]^{\mathrm{T}}$ and $\mathbf{u}^{k-1} \triangleq [\mathbf{u}_0^{\mathrm{T}}, \mathbf{u}_1^{\mathrm{T}}, \ldots, \mathbf{u}_{k-1}^{\mathrm{T}}]^{\mathrm{T}}$

Unconstrained nonlinear filtering methods

Linear systems

Kalman filter (KF)

Nonlinear systems

Local methods (validity within a small region)

- extended KF, unscented KF, etc.

Global methods (validity within almost whole state space)

- Analytical approach (Gaussian sum method)
- Monte Carlo approach (particle filters)
- Numerical approach (point-mass method)

Equality constraints

Reasons for considering constraints

- physical realizability
- technological limitations of the state variable
- kinematic constraints
- geometric considerations

Constraint specification

- $\mathbf{c}(\mathbf{x}_k) = 0$ - nonlinear
- $\mathbf{C}\mathbf{x}_k = 0$ - linear

Approaches for constraining the state estimate

description-modifying approach

- *reparametrization methods:* integrate the constraint into the system description by reparametrizing the system
- *pseudo-observation methods:* transform the constraint into a deterministic measurement equation

optimization approach

take into account the constraint during the estimation process and provide directly a constrained estimate, (single-step optimization)

Approaches for constraining the state estimate (cont.)

estimate-constraining approach

- *truncation methods:* trim the conditional pdf of the state with respect to constraints
- *projection methods:* propose a projection operator that transforms the estimate onto the constraint surface, (two-step optimization)

linear estimation	L/NL constraints	✓
nonlinear estimation	L constraints	✓
nonlinear estimation	L/NL constraints	missing

Goal of the paper

- to consider multiple equality constraints (generalization of the single constraint case)
- to design a global nonlinear filter for the multiple constraints case

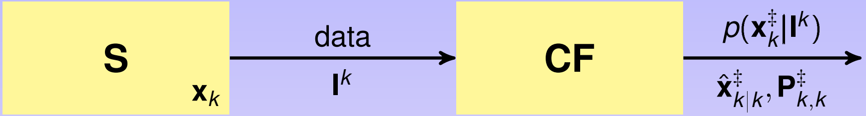
Choice for the filtering stage:

- **Gaussian sum filter** as a global analytical filter

Choice for constraining stage:

- **Two-step projection method** proposed in *S.J. Julier and J.J. LaViola. On Kalman Filtering with Nonlinear Equality Constraints. IEEE Transactions on Signal Processing, 55(6), 2007.*

MM MCon problem specification



S: system

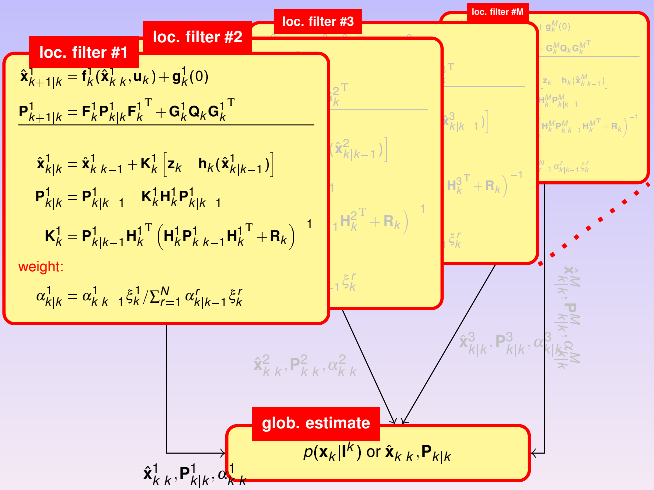
- behavior approx. by a set of M models Σ_i
  $\mathbf{x}_{k+1} = \mathbf{f}_k^i(\mathbf{x}_k, \mathbf{u}_k) + \mathbf{g}_k^i(\mathbf{w}_k)$, $p_i(\mathbf{x}_0)$, $p(\mathbf{w}_k)$
- measured output
  $\mathbf{z}_k = \mathbf{h}_k(\mathbf{x}_k) + \mathbf{v}_k$, $p(\mathbf{v}_k)$
- a set of N constraints: $\{\mathbf{c}_j(\mathbf{x}_k) = 0\}_{j=1}^N$

CF: constrained filter

Find

$$p(\mathbf{x}_k|\mathbf{I}^k)$$

filtering stage – GSF solution



constraining stage – two-step projection method

Projection methods

based on projections π_j s.t. $\mathbf{c}_j\left(\pi_j(\mathbf{x}_k)\right) = 0$ for any $\mathbf{x}_k \in \mathbb{R}^{n_x}$

Two-step projection

Step 1: projecting the unconstrained estimate given by $p(\mathbf{x}_k|\mathbf{I}^k)$ using π_j to $p^{\dagger}_{\cdot,j}(\mathbf{x}^{j\dagger}_k|\mathbf{I}^k)$

Step 2: projecting a point estimate $\hat{\mathbf{x}}^{\dagger}_{k|k}$ from $p^{\dagger}_{\cdot,j}(\mathbf{x}^{\dagger}_k|\mathbf{I}^k)$ (which for nonlinear constraints usually does not fulfill the constraint) onto the constraint surface to obtain $\hat{\mathbf{x}}^{\ddagger}_{k|k}$ s.t. $\mathbf{c}_j(\hat{\mathbf{x}}^{\ddagger}_{k|k}) = 0$

Projecting a Gaussian sum pdf – I

- Start with unconstrained RV $\mathbf{x}_k$ given by

$$p(\mathbf{x}_k|\mathbf{I}^k) = \sum_{i=1}^{M} \alpha^i_{k|k} \mathcal{N}\{\mathbf{x}_k; \hat{\mathbf{x}}^i_{k|k}, \mathbf{P}^i_{k|k}\},$$

- and project it using π_j to the constrained RV $\mathbf{x}_k^{j\dagger}$

$$p^\dagger_{\cdot,j}(\mathbf{x}_k^{j\dagger}|\mathbf{I}^k) = p\left(\pi_j^{-1}(\mathbf{x}_k^{j\dagger})\middle|\mathbf{I}^k\right)/|\Pi_j|$$

(Π_j is Jacobian of π_j) which leads to

$$p^\dagger_{\cdot,j}(\mathbf{x}_k^{j\dagger}|\mathbf{I}^k) = \sum_{i=1}^{M} \alpha^i_{k|k} p^\dagger_{i,j}(\mathbf{x}_k^{j\dagger}|\mathbf{I}^k)$$

Projecting a Gaussian sum pdf – II

- keep reproducibility of the densities $p^{\dagger}_{i,j}(\mathbf{x}^{j\dagger}_k|\mathbf{I}^k)$:

$$p^{\dagger}_{\cdot,j}(\mathbf{x}^{j\dagger}_k|\mathbf{I}^k) = \sum_{i=1}^{M} \alpha^i_{k|k} \mathcal{N}\{\mathbf{x}^{j\dagger}_k; \hat{\mathbf{x}}^{i,j\dagger}_{k|k}, \mathbf{P}^{i,j\dagger}_{k|k}\},$$

Thus the first step consist in transforming

$$\begin{array}{ccc} \text{unconstrained} & & \text{constrained} \\ \{\alpha^i_{k|k}, \hat{\mathbf{x}}^i_{k|k}, \mathbf{P}^i_{k|k}\}_{i=1}^M & \rightarrow & \{\alpha^i_{k|k}, \hat{\mathbf{x}}^{i,j\dagger}_{k|k}, \mathbf{P}^{i,j\dagger}_{k|k}\}_{i=1}^M \end{array}$$

- choose a point estimate of $\mathbf{x}^{\dagger}_k$ and execute the second projection step

MM Mcon filtering - summary

Modes - obtained by combining models and constraints

$$\left.\begin{array}{l} M \text{ models } \{\Sigma_i\}_{i=1}^{M} \\ N \text{ constraints } \{\mathbf{c}_j\}_{j=1}^{N} \end{array}\right\} \quad M \cdot N \text{ modes } \{\langle \Sigma_i, \mathbf{c}_j \rangle\}_{i=1,j=1}^{M,N}$$

Correction of the weights

correcting the weight (not affected by the constraints yet) by replacing the unconstrained predictive mean and covariance matrix by their constrained counterparts $\hat{\mathbf{x}}_{k|k-1}^{i,j\ddagger}$ and $\mathbf{P}_{k|k-1}^{i,j\ddagger}$ in the weight calculation

MM MCon filtering

mode 1,1 | mode 1,2 | mode 1,3 | mode M,N

$$\hat{\mathbf{x}}^{1,1}_{k+1|k} = \mathbf{f}^{1,1}_k(\hat{\mathbf{x}}^{1,1}_{k|k}, \mathbf{u}_k) + \mathbf{g}^{1,1}_k(0)$$

$$\mathbf{P}^{1,1}_{k+1|k} = \mathbf{F}^{1,1}_k \mathbf{P}^{1,1}_{k|k} \mathbf{F}^{1,1\,T}_k + \mathbf{G}^{1,1}_k \mathbf{Q}_k \mathbf{G}^{1,1\,T}_k$$

$$\hat{\mathbf{x}}^{1,1}_{k|k} = \hat{\mathbf{x}}^{1,1}_{k|k-1} + \mathbf{K}^{1,1}_k \left[\mathbf{z}_k - \mathbf{h}_k(\hat{\mathbf{x}}^{1,1}_{k|k-1})\right]$$

$$\mathbf{P}^{1,1}_{k|k} = \mathbf{P}^{1,1}_{k|k-1} - \mathbf{K}^{1,1}_k \mathbf{H}^{1,1}_k \mathbf{P}^{1,1}_{k|k-1}$$

$$\mathbf{K}^{1,1}_k = \mathbf{P}^{1,1}_{k|k-1} \mathbf{H}^{1,1\,T}_k \left(\mathbf{H}^{1,1}_k \mathbf{P}^{1,1}_{k|k-1} \mathbf{H}^{1,1\,T}_k + \mathbf{R}_k\right)^{-1}$$

projecting:

$$\hat{\mathbf{x}}^{1}_{k|k}, \mathbf{P}^{1}_{k|k} \rightarrow, \hat{\mathbf{x}}^{1,1\ddagger}_{k|k}, \mathbf{P}^{1,1\ddagger}_{k|k}$$

weight: (corrected)

$$\alpha^{1,1\ddagger}_{k|k} = \alpha^{1,1\ddagger}_{k|k-1} \xi^{1,1\ddagger}_k / \sum_{r=1}^{N} \sum_{s=1}^{M} \alpha^{r,s\ddagger}_{k|k-1} \xi^{r,s\ddagger}_k$$

$\hat{\mathbf{x}}^{1,1\ddagger}_{k|k}, \mathbf{P}^{1,1\ddagger}_{k|k}, \alpha^{1,1\ddagger}_{k|k}$

$\hat{\mathbf{x}}^{1,2\ddagger}_{k|k}, \mathbf{P}^{1,2\ddagger}_{k|k}, \alpha^{1,2\ddagger}_{k|k}$

$\hat{\mathbf{x}}^{1,3\ddagger}_{k|k}, \mathbf{P}^{1,3\ddagger}_{k|k}, \alpha^{1,3\ddagger}_{k|k}$

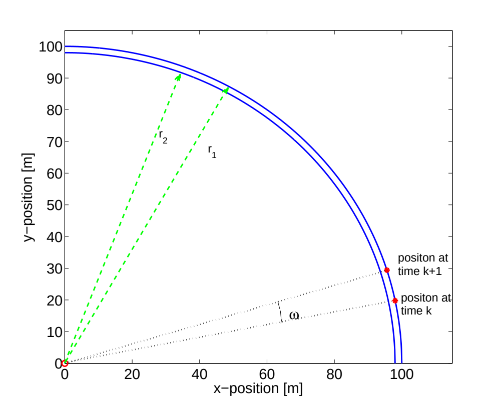
$\hat{\mathbf{x}}^{M,N\ddagger}_{k|k}, \mathbf{P}^{M,N\ddagger}_{k|k}, \alpha^{M,N\ddagger}_{k|k}$

glob. estimate

$p(\mathbf{x}^{\ddagger}_k|\mathbf{I}^k)$ or $\hat{\mathbf{x}}^{\ddagger}_{k|k}, \mathbf{P}^{\ddagger}_{k|k}$

Numerical example - I (outline)

Positioning a vehicle along two circular road segments



Numerical example - II (model)

- constant angular velocity $\omega = 5.7$ degrees per second
- dynamics of the vehicle is modeled for estimation purposes using

$$\mathbf{x}_{k+1} = \begin{bmatrix} 1 & T & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & T \\ 0 & 0 & 0 & 1 \end{bmatrix} \mathbf{x}_k + \begin{bmatrix} 0.5T^2 & 0 \\ T & 0 \\ 0 & 0.5T^2 \\ 0 & T \end{bmatrix} \tilde{\mathbf{w}}_k,$$

- $\mathbf{x}_k = [x_{1,k},\ x_{2,k},\ x_{3,k},\ x_{4,k}]^T = [x_k,\ \dot{x}_k,\ y_k,\ \dot{y}_k]^T$
- $M = 1$ models

Numerical example – III (measurement, constraints)

- measuring the position

$$\mathbf{z}_k = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \mathbf{x}_k + \mathbf{v}_k,$$

- $N = 2$ constraints

$$c_j(\mathbf{x}_k) = x_{1,k}^2 + x_{3,k}^2 - r_j^2 = 0, \quad j = 1, 2$$

  $(r_1 = 100,\ r_2 = 98)$

- corresponding projections

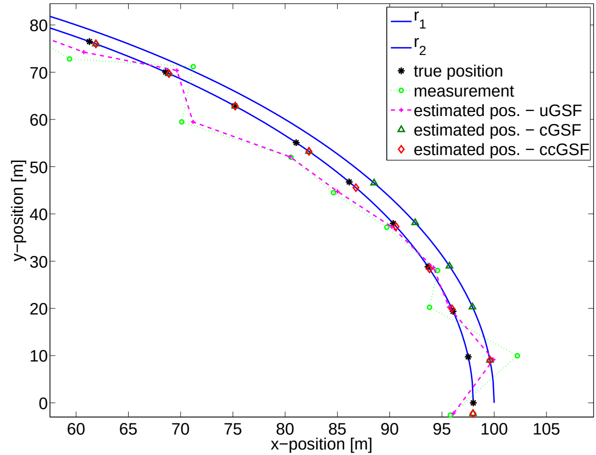
$$\pi_j(\mathbf{x}_k) = \left[\frac{x_{1,k} r_j}{\sqrt{x_{1,k}^2 + x_{3,k}^2}}, x_{2,k}, \frac{x_{3,k} r_j}{\sqrt{x_{1,k}^2 + x_{3,k}^2}} x_{4,k} \right]^{\mathrm{T}}$$

Numerical example – IV (filters)

- considered filters:
  - unconstrained GSF (uGSF)
  - constrained GSF with std. weights computation (cGSF)
  - constrained GSF with weights respecting constraints (ccGSF)
- criteria:
  - mean square-error (MSE)
  - correct road determination (CRD)

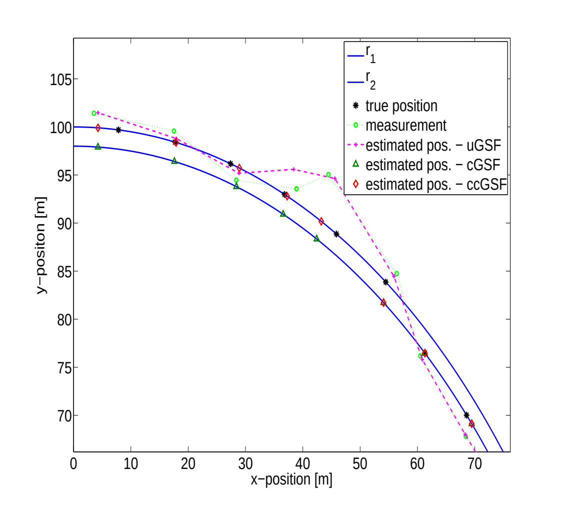
Numerical example – V (scenario 1 - no switching)

	MSE	CRD
uGSF	5.68	N/A
cGSF	3.98	40.03
ccGSF	2.76	58.06



Numerical example – VI (scenario 2 – switching)

	MSE	CRD
uGSF	5.96	N/A
cGSF	5.10	25.39
ccGSF	3.76	46.35



Concluding remarks

Conclusion

- A new multiple-model multiple-constraint filtering methods were proposed.
- It is a generalization of the two-step constraint application method, originally proposed in the Kalman filtering framework for a single constraint.
- The weight computation of the filter was analyzed and a correction to the computation was introduced.
- The performance of the proposed method is much better than that of the unconstrained Gaussian sum filter with only a slight increase of computational costs.