Fault detection for position estimation

I. Punčochář and M. Šimandl

Department of Cybernetics
and
Research Centre Data–Algorithms–Decision Making
Faculty of Applied Sciences
University of West Bohemia
Pilsen, Czech Republic

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Outline

Introduction

Relationship to traffic problem

- Trend – replace fixed-cycle controllers by more advanced controllers
- Aims of traffic control – maximize intersection throughput, minimize waiting times, balance load in microregions, ...
- Prerequisites for controller design – model and data

Source of data

- Fixed detectors
  - Fixed inductive loop detectors
  - Video cameras and radars
- Floating detectors
  - Fleet of vehicles (taxis, buses, etc.) equipped with receivers for Global Navigation Satellite System (GNSS)

Introduction – cont'd

Main conditions for a correct function of receivers for GNSS

- Clear sky view
- Synchronized atomic clocks
- Accurate information about satellite trajectories

Definition of faults

All factors that deteriorate precision of position estimate beyond acceptable limits.

Goals of presentation

- Provide overview of suitable fault detection (FD) methods
- Present two fault detection methods in more detail

Overview of fault detection methods for GNSS

Classification based on available data

- Position estimates
  - A dynamical model of the vehicle is required
  - FD method checks consistency between the dynamical model and position estimates
  - The quality of detection is mainly determined by the quality of the dynamical model
- Raw data (satellite positions, pseudoranges)
  - There are more advanced FD methods
  - Both a dynamical model of the vehicle and a static model of measurements can be used
  - Just the static model of measurements is utilized

Overview of fault detection methods for GNSS – cont'd

The static nonlinear model of measurements

$$\rho_k^i = h\left(\mathbf{x}_k, \mathbf{x}_k^i\right) + c\Delta t_k + f_k^i + v_k^i, \quad {}_{i=1,\ldots,n(k)}^{k=0,1,\ldots} \tag{1}$$

ρ_k^i – known pseudorange between receiver and i-th satelite
$h\left(\mathbf{x}_k, \mathbf{x}_k^i\right)$ – Euclidian distance $\left|\mathbf{x}_k - \mathbf{x}_k^i\right|$
$\mathbf{x}_k^i$ – known position of i-th satellite
$\mathbf{x}_k$ – unknown position of receiver
c – the speed of light
Δt_k – unknown difference between receiver's and satellites' clocks
f_k^i – non-zero value represents fault in i-th measurement
v_k^i – noise with pdf $\mathcal{N}\left\{v_k^i : 0, \sigma^2\right\}$

Overview of fault detection methods for GNSS – cont'd

Position estimation

- Pseudoranges ρ_k^i and satellite positions $\mathbf{x}_k^i, i = 1, \ldots, n(k)$

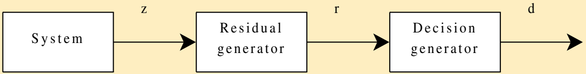
  $\Downarrow$

  Estimate of position $\hat{\mathbf{x}}_k$ and clock bias $\Delta\hat{t}_k$
- Position estimation requires at least four measurements (i.e. $n(k) \geq 4$)
- Analytical computation – uses just four measurements, worse quality, no problems with initial condition and convergence
- Numerical computation (Gauss-Newton algorithm) – uses all available measurements, possible problems with initial condition and convergence

Overview of fault detection methods for GNSS – cont'd

Fault detection

- Fault detection requires at least five measurements (i.e. $n(k) \geq 5$)
- Standard fault detection scheme



- Cluster analysis – idea is to use analytically computed position estimates (based of different four-element subsets) and test whether they create just one cluster
- Parity relation – idea is to use numerically computed position estimate and check the mutual consistency of all measurements

Model specification

Linearized model of measurements at position estimate

$$\mathbf{z}_k = \mathbf{H}_k \bar{\mathbf{x}}_k + \mathbf{f}_k + \mathbf{v}_k \quad (2)$$

$\mathbf{z}_k$ – vector of transformed measurements
$\mathbf{H}_k$ – Jacobian matrix
$\bar{\mathbf{x}}_k = [\mathbf{x}_k,\ c\Delta t_k]^T$
$\mathbf{f}_k$ – vector of faults
$\mathbf{v}_k$ – noise, pdf $\mathcal{N}\left\{\mathbf{v}_k : \mathbf{0}, \sigma^2\mathbf{I}\right\}$

$$\mathbf{H}_k = \begin{bmatrix} \left.\frac{\partial h(\mathbf{x}_k,\mathbf{x}_k^1)}{\partial \mathbf{x}_k}\right|_{\mathbf{x}_k=\hat{\mathbf{x}}_k} & 1 \\ \left.\frac{\partial h(\mathbf{x}_k,\mathbf{x}_k^2)}{\partial \mathbf{x}_k}\right|_{\mathbf{x}_k=\hat{\mathbf{x}}_k} & 1 \\ \vdots & \vdots \\ \left.\frac{\partial h(\mathbf{x}_k,\mathbf{x}_k^{n(k)})}{\partial \mathbf{x}_k}\right|_{\mathbf{x}_k=\hat{\mathbf{x}}_k} & 1 \end{bmatrix}$$

Residual generator

Residual generator based on parity relation

If $\mathbf{H}_k$ has full column rank than there is a $(n(k)-4)\times n(k)$ full row rank matrix $\mathbf{G}_k$ such that $\mathbf{G}_k\mathbf{H}_k=\mathbf{0}$.

$$\mathbf{r}_k=\mathbf{G}_k\mathbf{z}_k=\underbrace{\mathbf{G}_k\mathbf{f}_k+\mathbf{G}_k\mathbf{v}_k}_{\text{internal form}} \tag{3}$$

$\mathbf{r}_k$ – vector of residual signals

Statistical property of $\mathbf{r}_k$ based on $\mathbf{f}_k$

$\mathbf{f}_k=\mathbf{0}\Rightarrow\mathcal{N}\left\{\mathbf{r}_k:\mathbf{0},\mathbf{\Sigma}_k\right\}$ $\mathbf{f}_k\neq\mathbf{0}\Rightarrow\mathcal{N}\left\{\mathbf{r}_k:\mathbf{G}_k\mathbf{f}_k,\mathbf{\Sigma}_k\right\}$

The covariance matrix $\mathbf{\Sigma}_k=\sigma^2\mathbf{G}_k\mathbf{G}_k^T$ is positive definite, and it is possible to choose $\mathbf{G}_k$ such that $\mathbf{\Sigma}_k=\mathbf{I}$.

Decision generators

Decision generator based on the χ^2 test

Statistic

$$t_k = \mathbf{r}_k^T \mathbf{r}_k \tag{4}$$

Its properties

$\mathbf{f}_k = \mathbf{0} \Rightarrow \chi^2 \{t_k, n(k) - 4\}$

$\mathbf{f}_k \neq \mathbf{0} \Rightarrow \chi^2 \{t_k, n(k) - 4, \lambda_k\}$
$\lambda_k = \mathbf{f}_k^T \mathbf{G}_k^T \mathbf{G}_k \mathbf{f}_k$

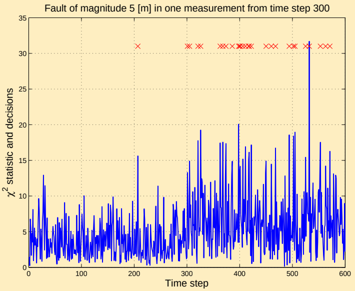
Decision rule

- If $t_k \leq T_{1-\alpha}$ then $d_k = 0$
- If $t_k > T_{1-\alpha}$ then $d_k = 1$

Threshold $T_{1-\alpha}$ is $(1-\alpha)$-quantile of central χ^2 distribution with $n(k)-4$ degrees of freedom, and the significance level α is the probability of type I error.

Decision generators – cont'd

Typical behavior of the χ^2 test statistic for different magnitudes of a fault



Decision generators – cont'd

Decision generator based on the cumulative sum (CUSUM) test

Statistic

$$t_k = \max\left(t_{k-1} + \underbrace{\ln \frac{\mathcal{N}\left\{\mathbf{r}_k : \mathbf{G}_k \bar{\mathbf{f}}_k, \mathbf{I}\right\}}{\mathcal{N}\left\{\mathbf{r}_k : \mathbf{0}, \mathbf{I}\right\}}}_{\Delta t_k}, 0 \right), \quad \begin{matrix} t_{-1}=0 \\ \bar{\mathbf{f}}_k - \text{expected fault} \end{matrix} \qquad (5)$$

Its properties

$\mathbf{f}_k = \mathbf{0} \Rightarrow \mathrm{E}\left\{\Delta t_k\right\} < 0$ $\qquad$ $\mathbf{f}_k = \bar{\mathbf{f}}_k \Rightarrow \mathrm{E}\left\{\Delta t_k\right\} > 0$

Decision rule

- If $t_k \leq T_{1-\alpha}$ then $d_k = 0$
- If $t_k > T_{1-\alpha}$ then $d_k = 1$

The threshold can be chosen as $T_{1-\alpha} = -\ln \alpha$.

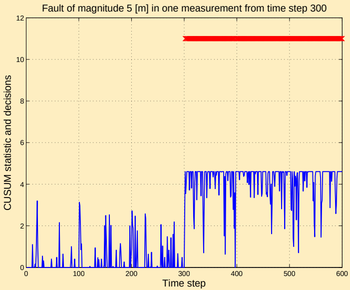
Decision generators – cont'd

Decision generator based on the CUSUM test - modifications

- The actual fault $\mathbf{f}_k$ can differ from the expected fault $\bar{\mathbf{f}}_k$
  - Weighted CUSUM test
  - Generalized likelihood ratio test
  - Usage of $2n(k)$ parallel CUSUM tests with $\bar{\mathbf{f}}_k \in \{\pm \bar{f} \mathbf{e}_i\},\ i = 1, \ldots, n(k),\ \bar{f}$ – expected magnitude, $\mathbf{e}_i$ – standard basis vectors
- The uninterrupted function of the CUSUM test has to be provided
  - Whenever a change is detected a new CUSUM test is stated and started
  - The statistic of the CUSUM test t_k is bounded from above by the threshold $T_{1-\alpha}$

Decision generators – cont'd

Typical behavior of the CUSUM test statistic for different magnitudes of a fault



Decision generators – cont'd

Comparison of the χ^2 test and CUSUM test

- The χ^2 test
  - It is not optimal for mean change detection
  - Implementation is simple
  - Computational demands are quite small
- The CUSUM test
  - It is optimal for mean change detection provided that all assumptions are satisfied
  - There are implementation issues
  - Computational demands are slightly higher

Conclusion

Concluding remarks

- Fault detection methods make it possible to verify correctness of position estimates before they are further utilized in traffic control and transportation.
- Two presented fault detection methods do not need any dynamical model of a vehicle and thus model identification is avoided.
- The presented fault detection methods can by used also in conjunction with a dynamical model of a vehicle.