Closed loop information processing strategy for optimal fault detection and control

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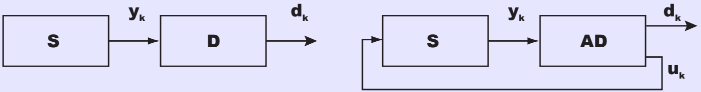
Outline

Introduction

Model-based approaches to the fault detection

- Uses a model of the observed system, a priori information and measurements to decide on faults
- Passive fault detection - detector passively uses available information to decide on faults
- Active fault detection - detector provides decision and input signal which should improve fault detection



a) Passive fault detection b) Active fault detection

Introduction – cont'd

Active fault detection problem

- Deterministic [Campbell&Nikoukhah(2004)] and stochastic [Zhang(1989), Kerestecioglu(1993)] models of the observed system are used
- A general formulation of the active fault detection problem in stochastic framework is missing and the relation between active fault detection and the optimal control is not considered
- Known approaches use information is such way that the consequences of the current decision in future steps are not considered and the future losses are not taken into account

Introduction – cont'd

Information processing strategies

- Open loop (OL) - only a priori information is used
- Open loop feedback (OLF) - all available information up to current time step is used, but the future information is not considered
- Closed loop (CL) - all available information up to current time step is used and the availability of the future information is considered as well; so the future losses are taken into account and this strategy provides the lowest value of a criterion (i.e. $J^{CL} \leq J^{OLF} \leq J^{OL}$)

Introduction – cont'd

Goals

- Propose a unified formulation of the active fault detection problem
- Specify three basic special cases
  - Optimal detector for given input signal generator
  - Optimal detector and optimal input signal generator
  - Optimal detector and optimal dual controller
- Find solutions of considered special cases using CL information processing strategy

Problem formulation

Description of the observed system for $k \in \mathcal{T} = \{0, \ldots, F\}$

$$
\begin{aligned}
\text{System}: \mathbf{x}_{k+1} &= \mathbf{f}_k\left(\mathbf{x}_k, \mu_k\, \mathbf{u}_k, \mathbf{w}_k\right) \\
\mu_{k+1} &= \mathbf{g}_k\left(\mu_k, \mathbf{e}_k\right) \\
\mathbf{y}_k &= \mathbf{h}_k\left(\mathbf{x}_k, \mu_k, \mathbf{v}_k\right)
\end{aligned}
$$

$\mathbf{f}_k$, $\mathbf{g}_k$ and $\mathbf{h}_k$ are known function; $\mathbf{x}_k \in \mathcal{R}^{n_x}$ is controllable part of the state; $\mu_k \in \mathcal{M} \subset \mathcal{R}^{n_\mu}$ is uncontrollable part of the state and represents faults; $\mathbf{u}_k \in \mathcal{U}_k \subset \mathcal{R}^{n_u}$ is input, $\mathbf{y}_k \in \mathcal{R}^{n_y}$ is output; $\{\mathbf{w}_k\}$, $\{\mathbf{e}_k\}$ and $\{\mathbf{v}_k\}$ are mutually independent random sequences

Problem formulation – cont'd

Description of the general active detector for $k \in \mathcal{T}$

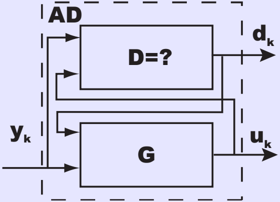
$$\text{Active detector}: \begin{bmatrix} d_k \\ \mathbf{u}_k \end{bmatrix} = \begin{bmatrix} \sigma_k\left(\mathbf{I}_0^k\right) \\ \gamma_k\left(\mathbf{I}_0^k, d_k\right) \end{bmatrix}$$

σ_k and γ_k are generally unknown functions; $d_k \in \mathcal{M}$ is an estimate of the μ_k (in special case called decision); all available information at time k is stored in $\mathbf{I}_0^k = [{\mathbf{y}_0^k}^T, {\mathbf{u}_0^{k-1}}^T, {d_0^{k-1}}^T]^T$

Specifications and solutions of the special cases

Case I: Optimal detector for given input signal generator

- The input signal generator is given by functions $\gamma_k(\mathbf{I}_0^k, d_k)$ (e.g. existing controller) and the detector $\sigma_k(\mathbf{I}_0^k)$ has to be found



- Criterion which has to be minimized

$$J_{ADGG}(\sigma_0^F) = \mathrm{E}\left\{\sum_{i=0}^{F} L_i^d(d_i, \mu_i)\right\}$$

Specifications and solutions of the special cases – cont'd

Case I: Optimal detector for given input signal generator

- Backward recursive equation with initial condition $V_{F+1}^* = 0$ and final value of the criterion $J_{ADGG}^{CL} = \mathrm{E}\{V_0^*(\mathbf{I}_0)\}$

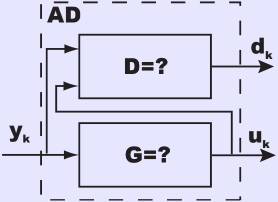
$$V_k^*(\mathbf{I}_0^k) = \min_{d_k \in \mathcal{M}} \mathrm{E}\left\{ L_k^d(d_k, \mu_k) + V_{k+1}^*(\mathbf{I}_0^{k+1}) | \mathbf{I}_0^k, d_k \right\}$$

- Optimal decision d_k^* is a trade-off between right decision at time step k and the excitation of the observed system through the law $\mathbf{u}_k = \gamma_k(\mathbf{I}_0^k, d_k)$

Specifications and solutions of the special cases – cont'd

Case II: Optimal detector and optimal input signal generator

- Both the detector $\sigma_k(\mathbf{I}_0^k)$ and input signal generator $\gamma_k(\mathbf{I}_0^k, d_k)$ have to be found



- Criterion which has to be minimized

$$J_{ADG}(\sigma_0^F, \gamma_0^F) = \mathrm{E}\left\{\sum_{i=0}^{F} L_i^d(d_i, \mu_i)\right\}$$

Specifications and solutions of the special cases – cont'd

Case II: Optimal detector and optimal input signal generator

- Backward recursive equation with the initial condition $V^*_{F+1} = 0$ and final value of the criterion $J^{CL}_{ADG} = \mathrm{E}\left\{V^*_0(\mathbf{y}_0)\right\}$

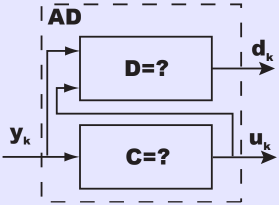
$$V^*_k(\mathbf{y}^k_0, \mathbf{u}^{k-1}_0) = \min_{d_k \in \mathcal{M}} \mathrm{E}\left\{ L^d_k(d_k, \mu_k) | \mathbf{y}^k_0, \mathbf{u}^{k-1}_0, d_k \right\} + \min_{\mathbf{u}_k \in \mathcal{U}_k} \mathrm{E}\left\{ V^*_{k+1}(\mathbf{y}^{k+1}_0, \mathbf{u}^k_0) | \mathbf{y}^k_0, \mathbf{u}^k_0 \right\}$$

- Optimal decision d^*_k and optimal input signal $\mathbf{u}^*_k$ are chosen independently, so $\gamma_k(\mathbf{y}^k_0, \mathbf{u}^{k-1}_0, d^k_0) = \gamma_k(\mathbf{y}^k_0, \mathbf{u}^{k-1}_0)$
- It can be proven that $J^{CL}_{ADG} \leq J^{CL}_{ADGG}$

Specifications and solutions of the special cases – cont'd

Case III: Optimal detector and optimal dual controller

- Both the detector $\sigma_k(\mathbf{I}_0^k)$ and dual controller $\gamma_k(\mathbf{I}_0^k, d_k)$ have to be found



- Criterion which has to be minimized

$$J_{ADC}(\sigma_0^F) = \mathrm{E}\left\{\sum_{i=0}^{F} L_i^d(d_i, \mu_i) + \alpha_i L_i^c(\mathbf{x}_i, \mathbf{u}_i)\right\}$$

Specifications and solutions of the special cases – cont'd

Case III: Optimal detector and optimal dual controller

- Backward recursive equation with initial condition $V_{F+1}^* = 0$ and final value of the criterion $J_{ADC}^{CL} = \mathrm{E}\{V_0^*(\mathbf{y}_0)\}$

$$V_k^*(\mathbf{y}_0^k, \mathbf{u}_0^{k-1}) = \min_{d_k \in \mathcal{M}} \mathrm{E}\left\{ L_k^d(d_k, \mu_k) | \mathbf{y}_0^k, \mathbf{u}_0^{k-1}, d_k \right\} + \min_{\mathbf{u}_k \in \mathcal{U}_k} \mathrm{E}\left\{ \alpha_k L_k^c(\mathbf{x}_k, \mathbf{u}_k) + V_{k+1}^*(\mathbf{y}_0^{k+1}, \mathbf{u}_0^k) | \mathbf{y}_0^k, \mathbf{u}_0^k \right\}$$

- Optimal decision d_k^* and optimal input signal $\mathbf{u}_k^*$ are also independent, but optimal input signal is trade-off between control objective and the excitation of the observed system

Specifications and solutions of the special cases – cont'd

Comments on special cases

- The case III: Optimal detector and optimal dual controller describes the most general problem and it includes the previous cases
- A nonlinear filter can provide required pdf's $p(\mu_k|\mathbf{y}_0^k, \mathbf{u}_0^{k-1})$, $p(\mathbf{x}_k|\mathbf{y}_0^k, \mathbf{u}_0^{k-1})$ and $p(\mathbf{y}_{k+1}|\mathbf{y}_0^k, \mathbf{u}_0^k)$
- The function $V_k^*(\cdot)$ can not be expressed analytically in explicit form and the approximations are necessary to obtain a feasible suboptimal solution

Special case – Multiple linear Gaussian model

Specification of the MM framework

- It is special case of the proposed description of the observed system and it simplifies a computation
- The observed system is supposed to be described by finite number of linear Gaussian models, so pdf's of the initial condition $\mathbf{x}_k$ and noises w_k, v_k are Gaussian
- The set $\mathcal{M} = 1, \ldots, N$ is discrete and μ_k denotes scalar index to the model valid at time k
- The state equation $\mu_{k+1} = \mathbf{g}(\mu_k, \mathbf{e}_k)$ is replaced by transition probabilities $P_{i,j} = P(\mu_{k+1} = j | \mu_k = i)$, $i, j \in \mathcal{M}$

Numerical example

Observed system description for $k \in \mathcal{T} = \{0, 1\}$

$$\mu_k = 1: \; x_{k+1} = 0.99x_k + u_k + \sqrt{0.25}w_k$$
$$y_k = 2x_k + \sqrt{0.25}v_k$$
$$\mu_k = 2: \; x_{k+1} = 1.01x_k + 0.99u_k + \sqrt{0.25}w_k$$
$$y_k = 2x_k + \sqrt{0.25}v_k$$
$$\mu_k = 3: \; x_{k+1} = 0.5x_k + 1.5u_k + \sqrt{0.25}w_k$$
$$y_k = 1.5x_k + \sqrt{0.25}v_k$$

$$P_{i,j} = \begin{cases} 0.9 \; \textit{iff} \;\; i = j, & p(w_k) = p(v_k) = \mathcal{N}\{0, 1\}, p(x_0) = \mathcal{N}\{1, 0.1\} \\ 0.05 \; \textit{iff} \;\; i \neq j, & P(\mu_0 = 1) = 0.4, P(\mu_0 = 2) = P(\mu_0 = 3) = 0.3 \end{cases}$$

Numerical example – cont'd

Loss functions for $k \in \mathcal{T}$

$$d_k = \mu_k \Rightarrow L_k^d(d_k, \mu_k) = 0 \qquad L_k^c(x_k, u_k) = x_k^2 + u_k^2$$

$$d_k \neq \mu_k \Rightarrow L_k^d(d_k, \mu_k) = 1 \qquad \alpha = 0.01$$

Input signal generator for $k \in \mathcal{T}$

- Set of input signal $\mathcal{U}_k = \{-1, 1\}$
- Description of the generator for the case I

$$d_k = 1 \vee d_k = 3 \Rightarrow u_k = -1,$$
$$d_k = 2 \Rightarrow u_k = 1$$

Numerical example – cont'd

Approaches used for active detector design

- Case I (Optimal detector for given input signal generator) is solved using OLF (MAP model estimate) and CL strategy
- Case II (Optimal detector and optimal input signal generator) is solved only using CL strategy
- Case III (Optimal detector and optimal dual controller) is solved using certainty equivalence (CE) and CL strategy

Results of Monte Carlo simulations

$\hat{J}^{OLF}_{ADGG}$	$\hat{J}^{CL}_{ADGG}$	$\hat{J}^{CL}_{ADG}$	$\hat{J}^{CE}_{ADC}$	$\hat{J}^{CL}_{ADC}$
1.2647	1.2323	1.0890	1.3022	1.1655

Concluding remarks

Remarks

- The new unified formulation of the active fault detection problem was proposed
- The formulation provides very general framework and the other cases together with corresponding solutions can be easily derived in addition to the presented cases
- In general, CL information processing strategy provides better results than OLF strategy
- In the case II (Optimal detector and optimal input signal generator) it was shown that the optimal decision d_k^* and optimal input signal $\mathbf{u}_k^*$ are independent