Bayesian Melding in Urban Simulations

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Joint work with Adrian Raftery and Paul Waddell

2nd International Workshop DAR, Třešť 2006

Outline

- Motivation

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- Description of the computer model UrbanSim

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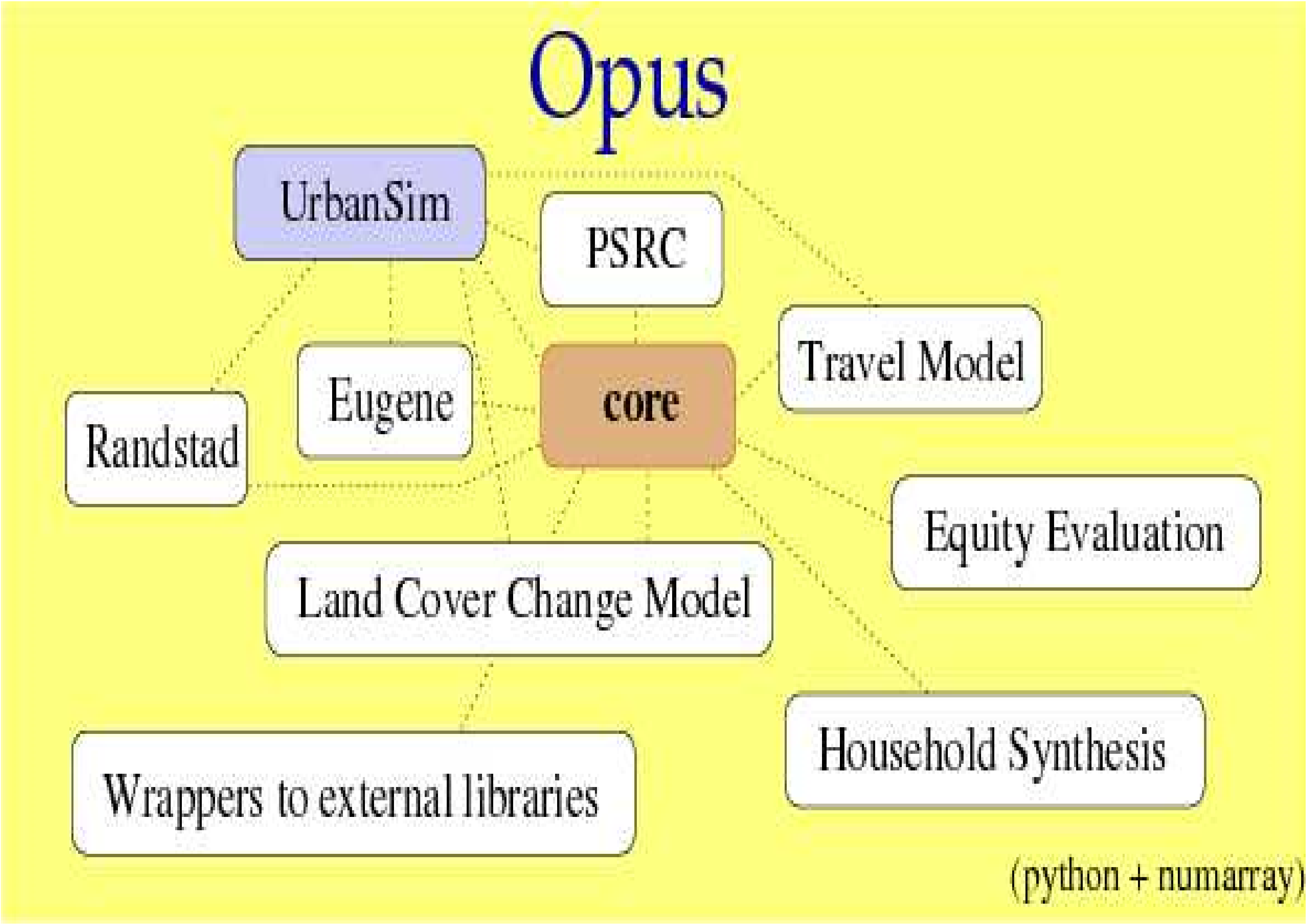
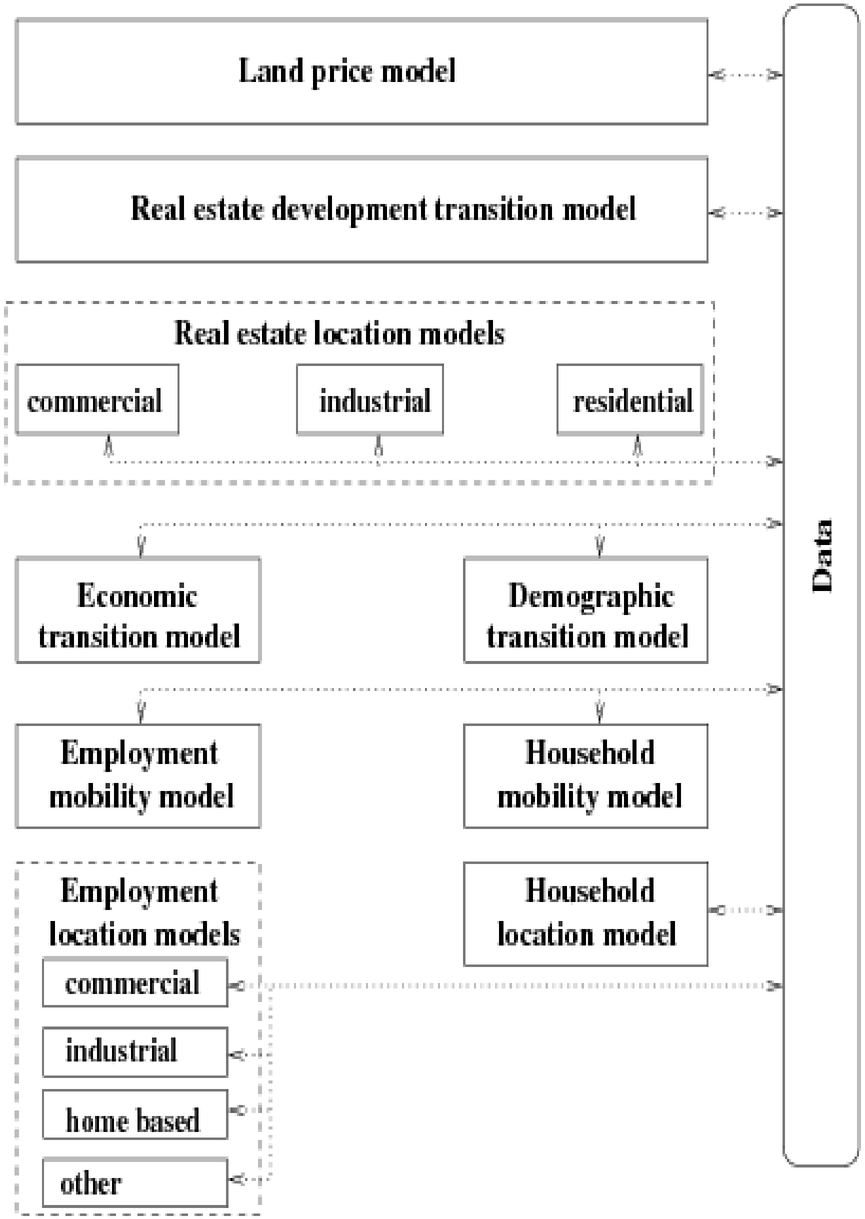
- Method: Bayesian Melding

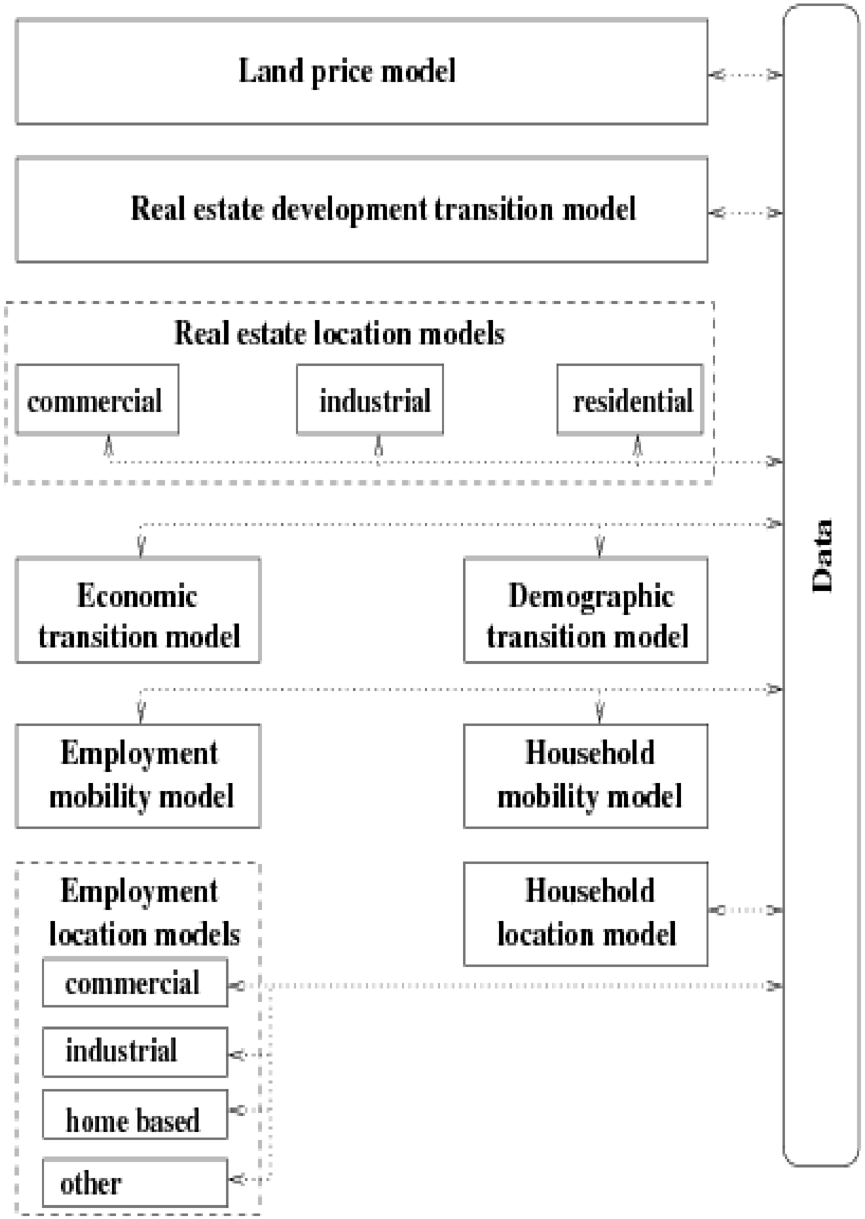
Open Platform for Urban Simulation



UrbanSim


Land price model
Real estate development transition model
Real estate location models
commercial
industrial
residential
Economic transition model
Demographic transition model
Employment mobility model
Household mobility model
Employment location models
commercial
industrial
home based
other
Household location model
Data

UrbanSim



Uncertainty:

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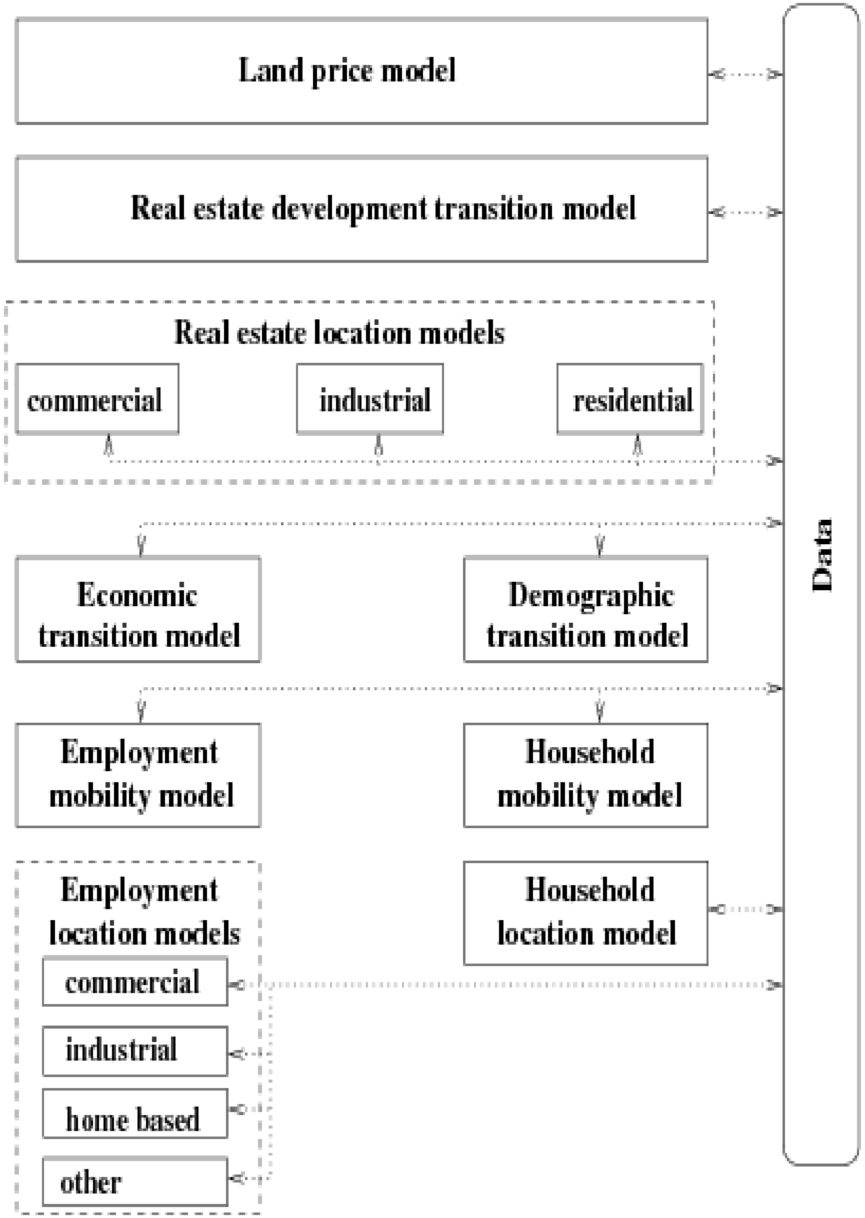

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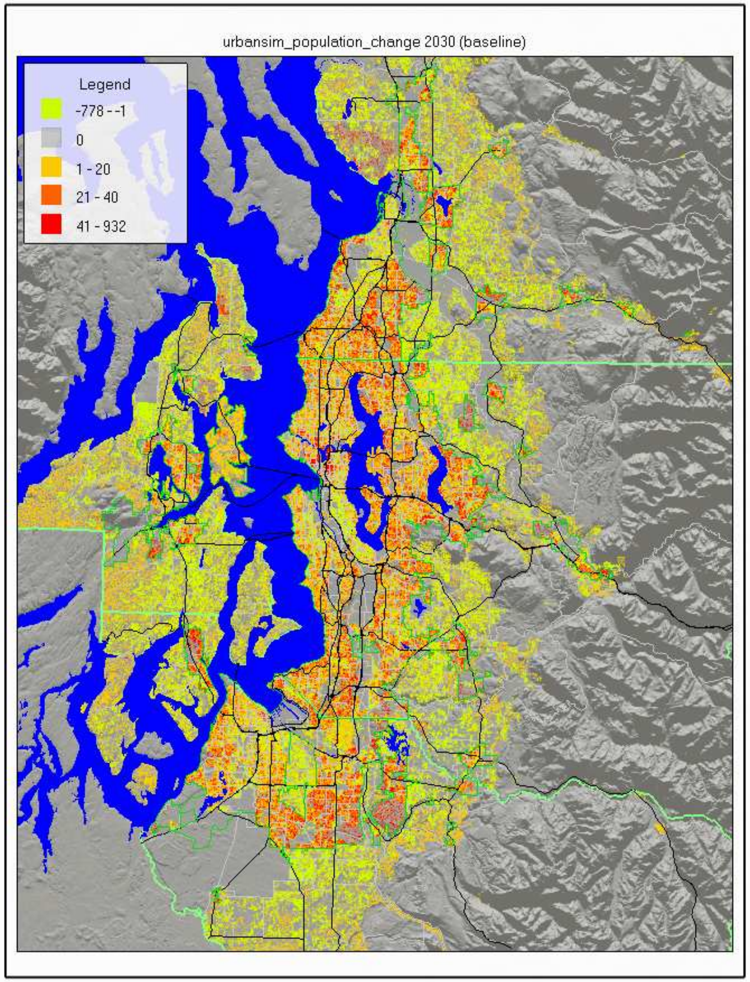
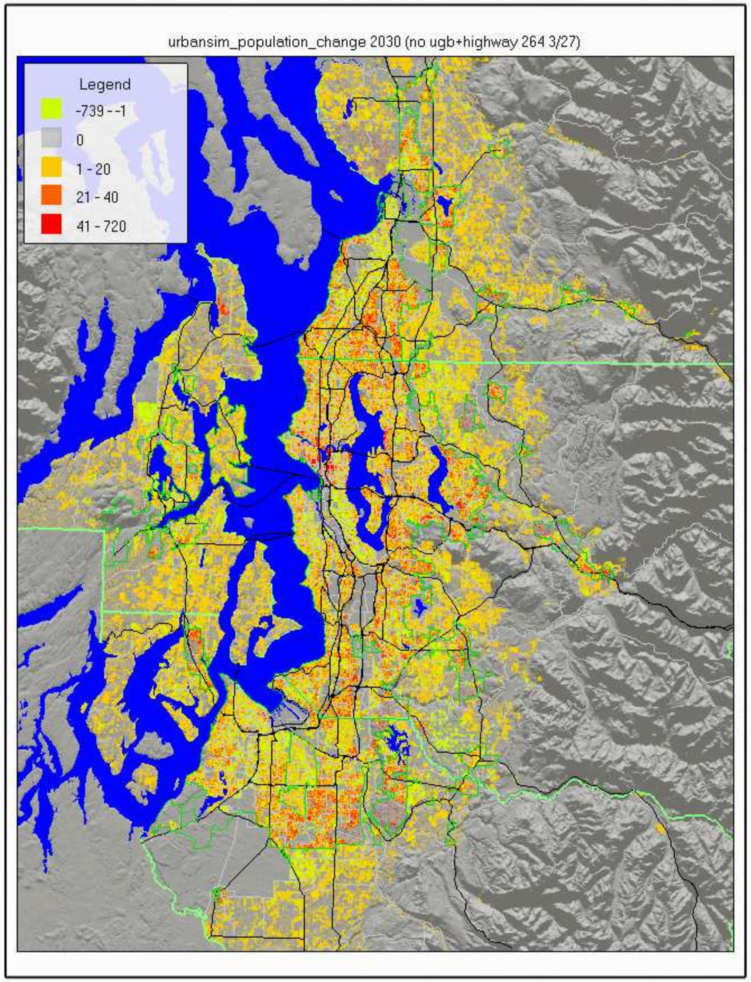
972 input par. for PSRC appl.

Population change in Puget Sound area (30 years prediction)

baseline



no UGB + highway



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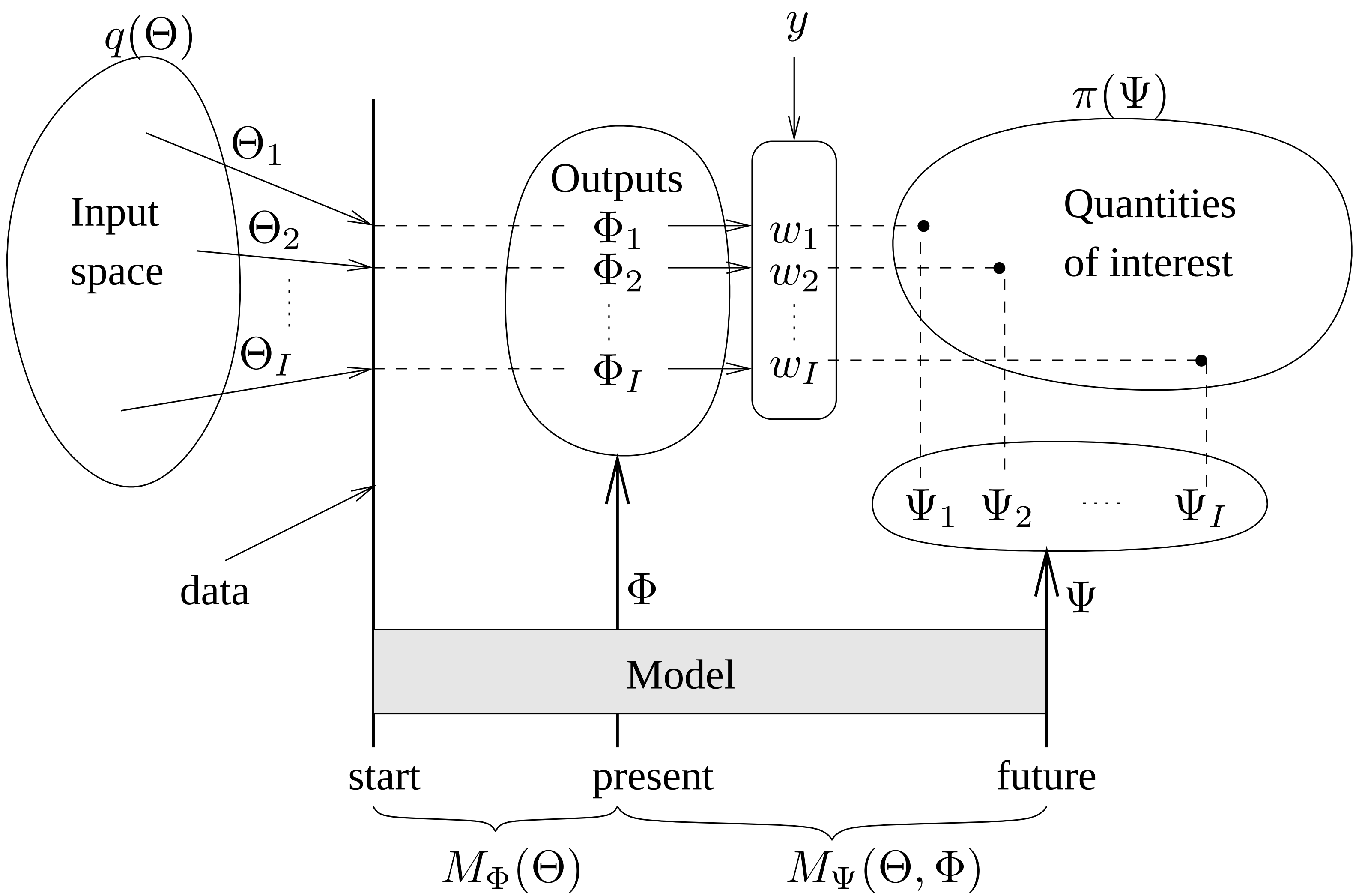

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- start: 1980, present: 1994, future: 2000.

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Estimation of μ_{ik}, σ_δ^2, σ_i^2, and a done by approximate maximum likelihood.

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$$\pi(\Psi_k) = \sum_{i=1}^{I} w_i N(\hat{a} b_a + \frac{1}{J}\sum_{j=1}^{J}\Psi_{ijk}, v_i b_v)\,, \quad k = 1, \ldots, K$$

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$$N(\hat{c}, 20V(\hat{c})), \; V \text{ is estimated variance per year}$$

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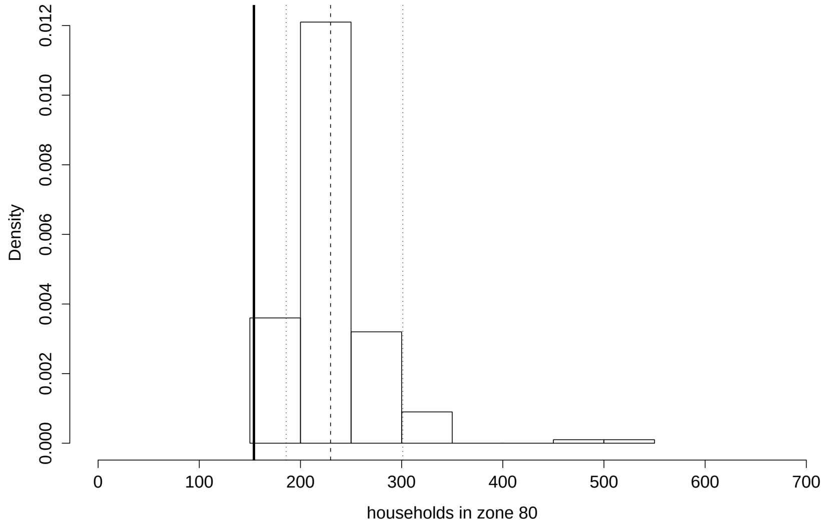
- propagation factors b_a and b_v set to $20/14$ $\left(\frac{2000-1980}{1994-1980}\right)$

Results

Multiple runs



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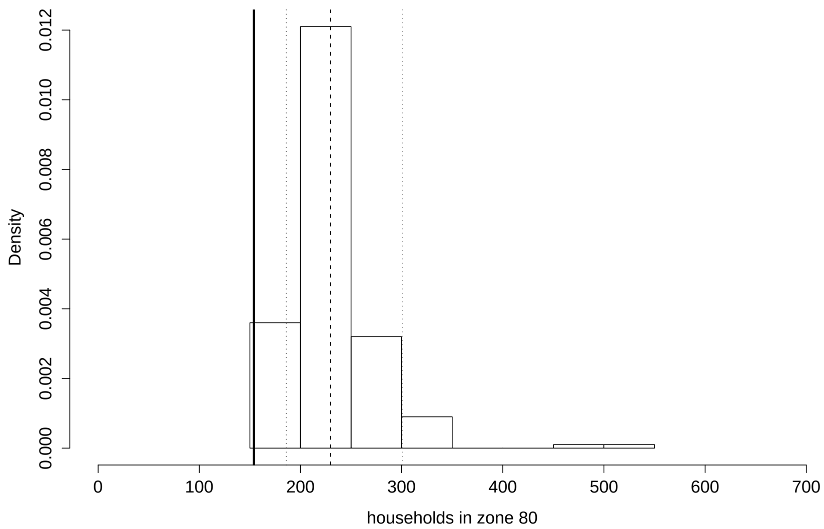
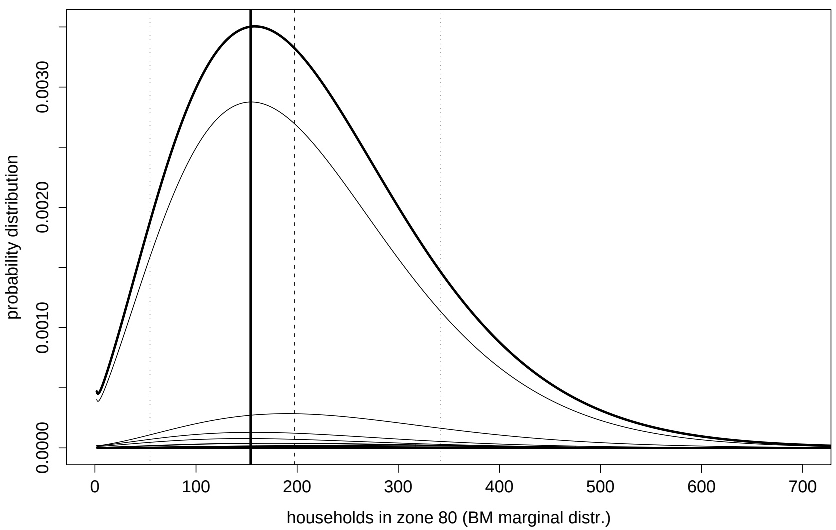


Bayesian melding



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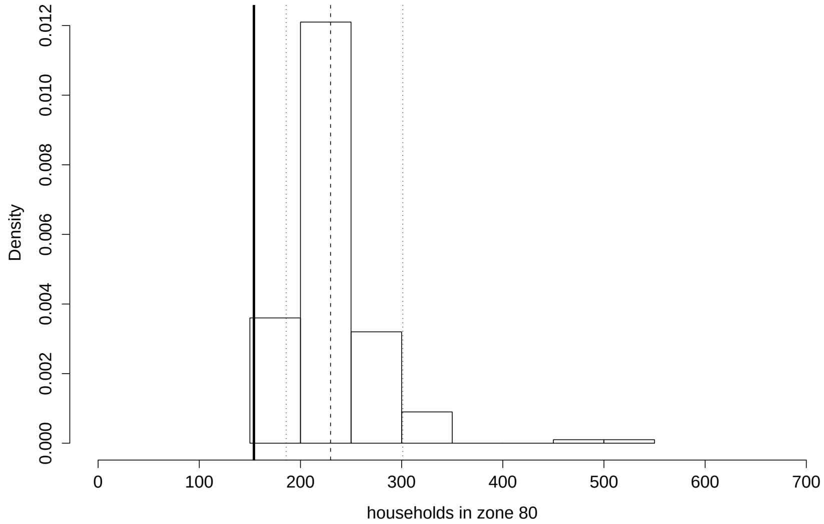

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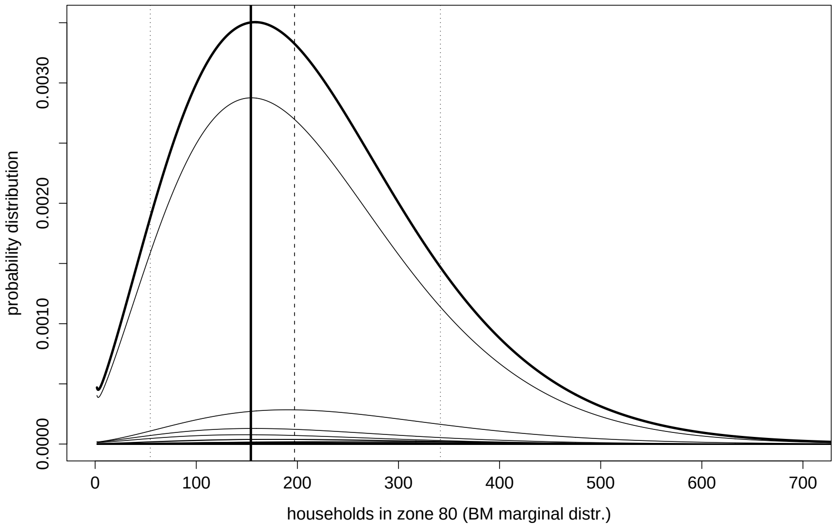

method	missed cases	coverage

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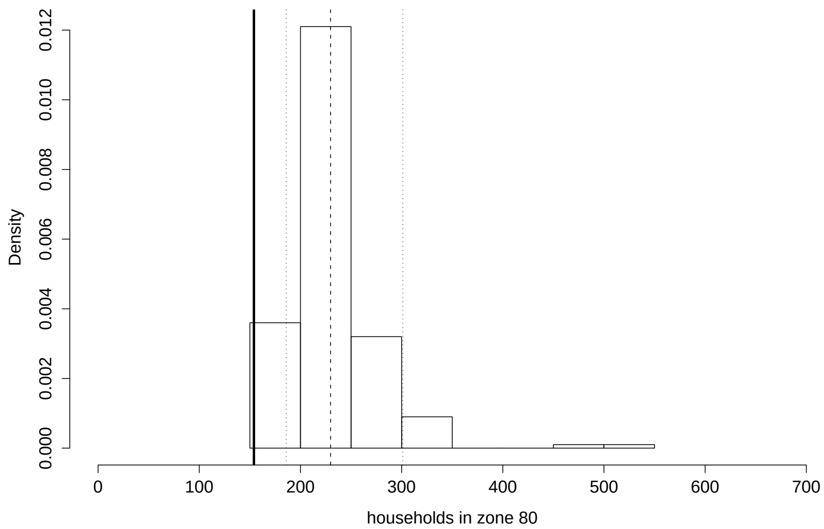

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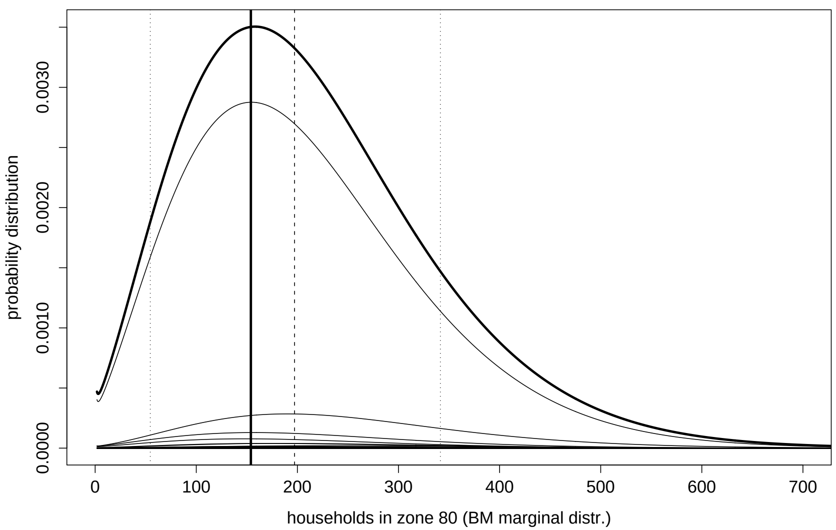

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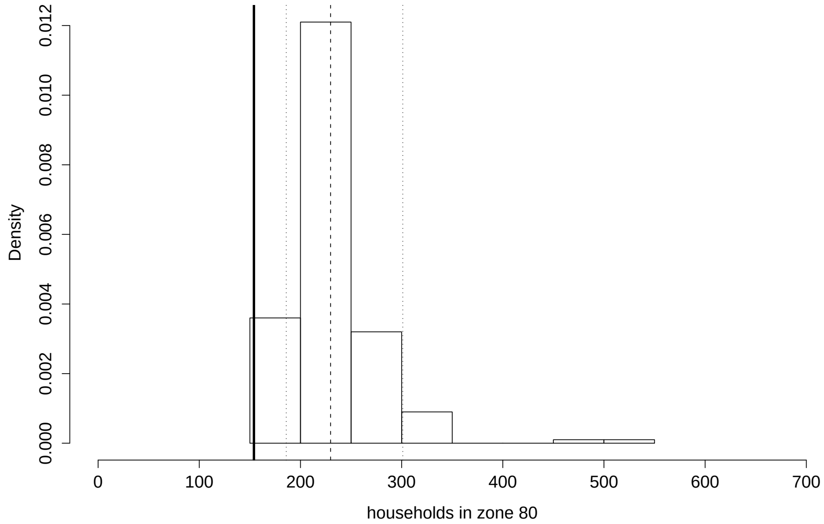	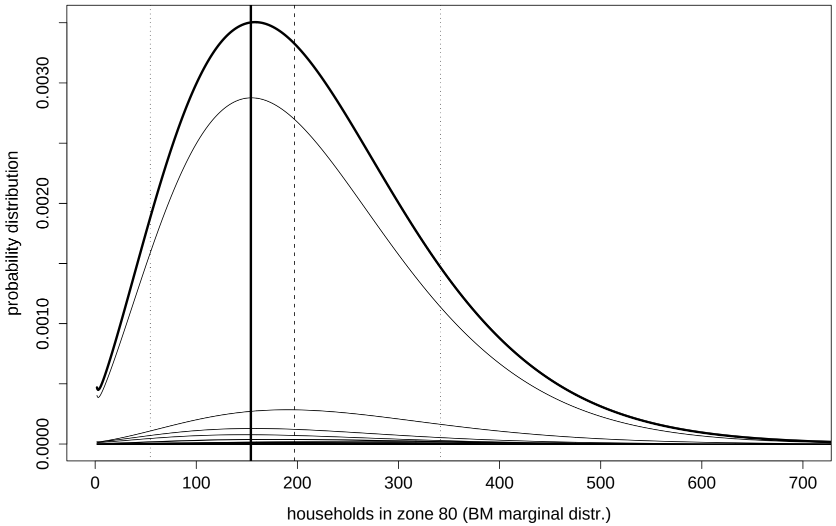

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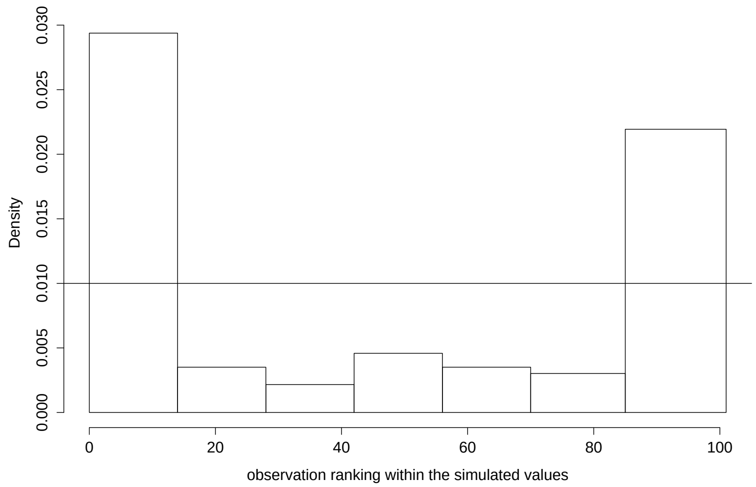
Verification rank histogram

Multiple runs



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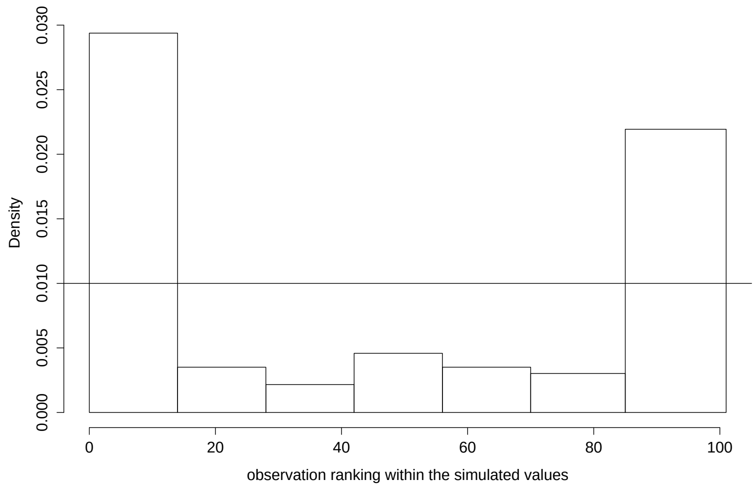

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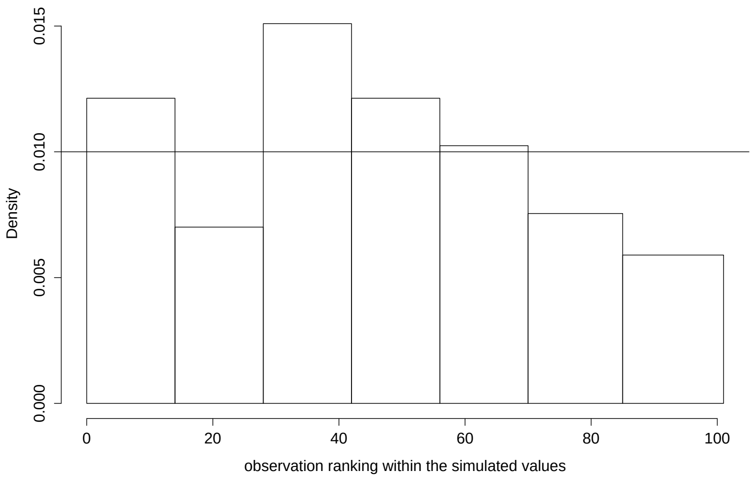

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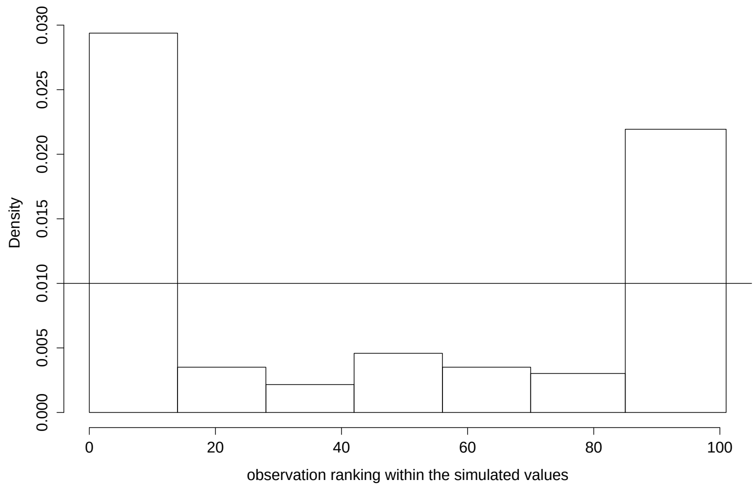

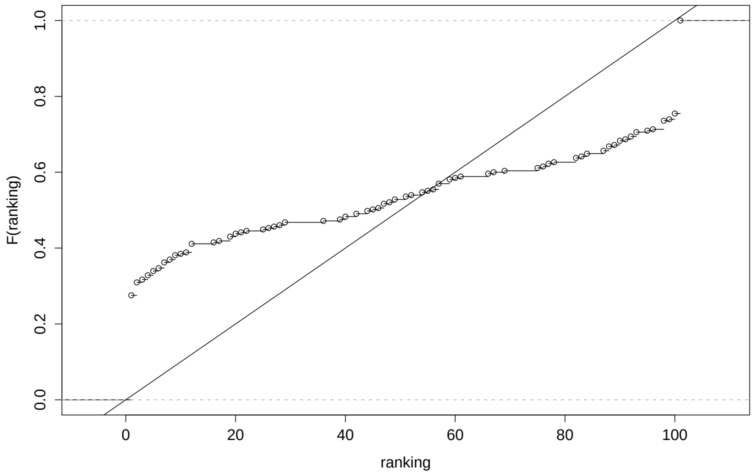

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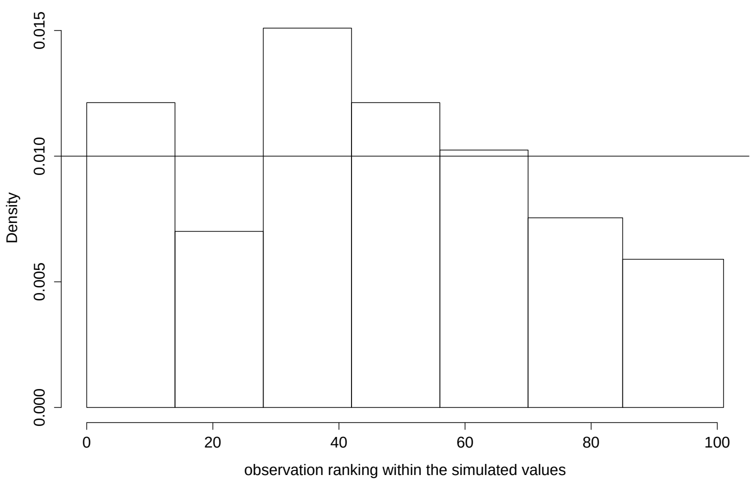
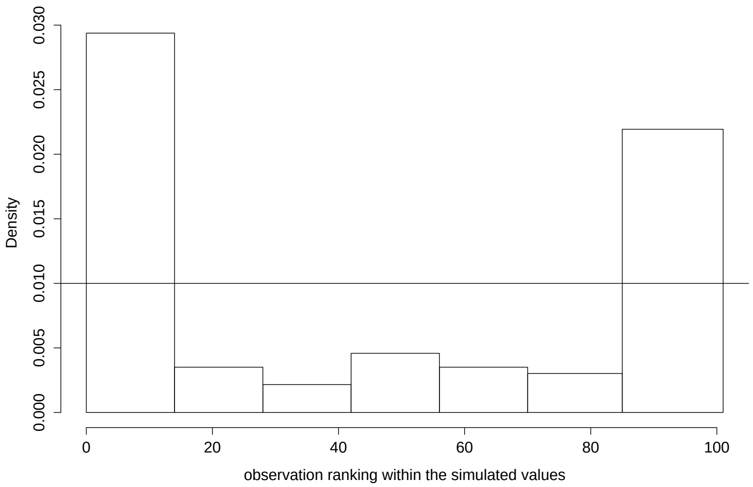
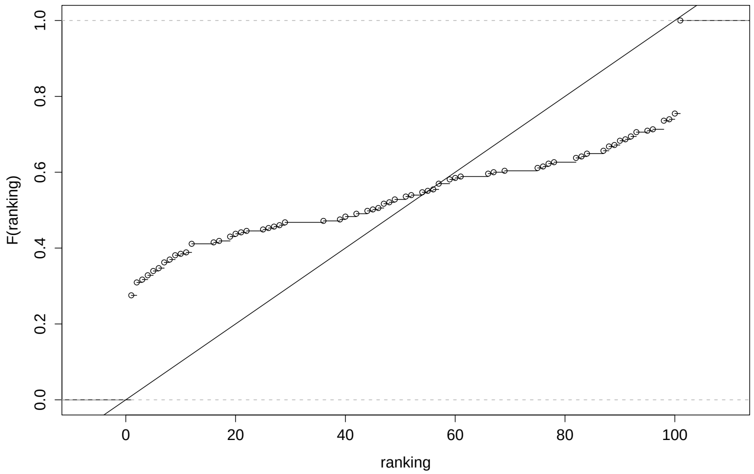


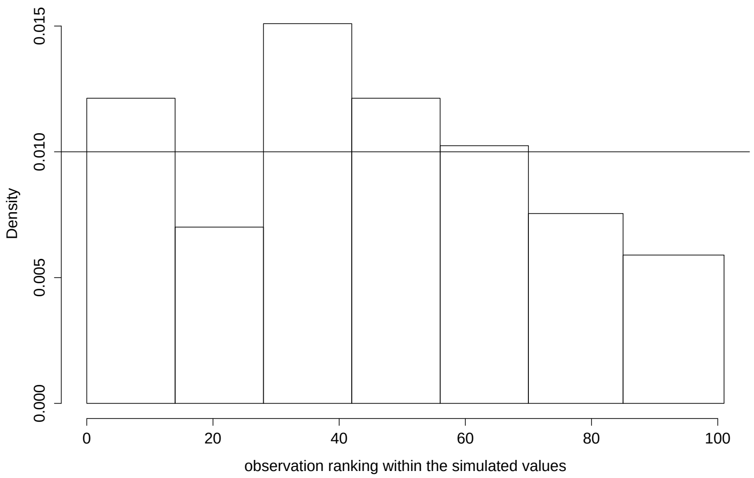
Verification rank histogram

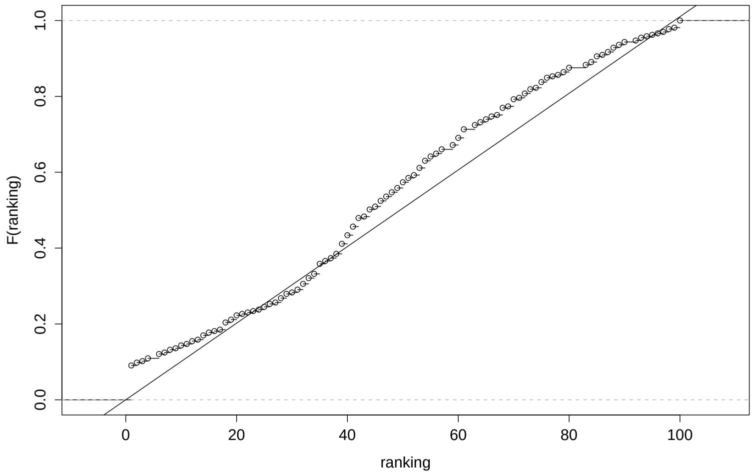
Multiple runs





Bayesian melding





Simulation size

simulation size	100×2		1000×3	
method	missed cases	coverage	missed cases	coverage

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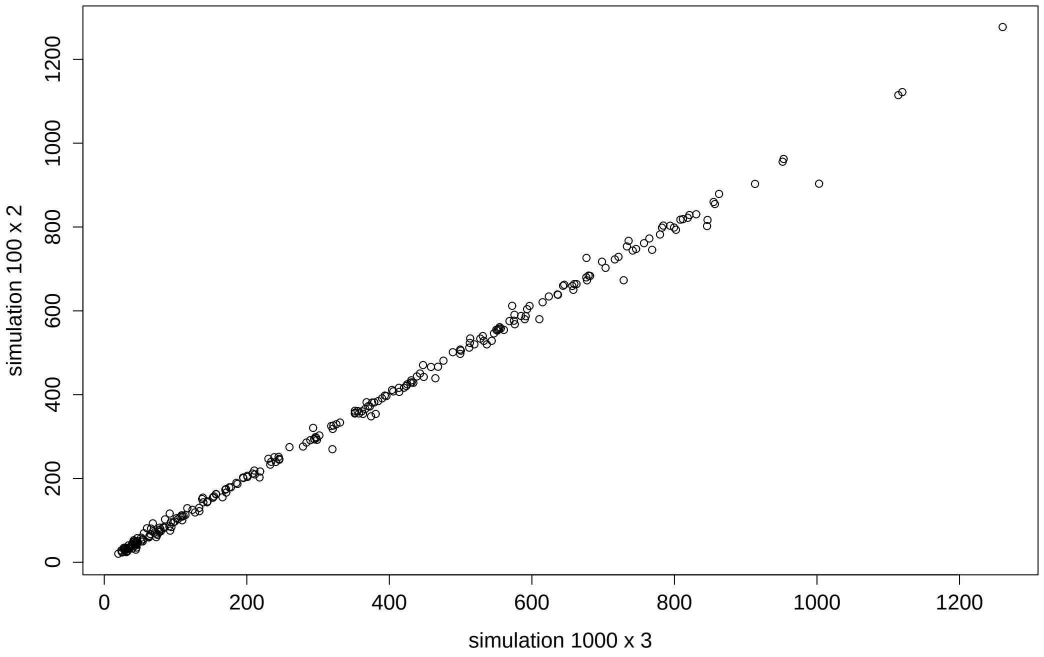

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- Bayesian melding extended for application to a stochastic simulation model.
- Experiment on Eugene, OR, 1980-2000:
  - Simple multiple runs underestimated uncertainty.
  - Bayesian melding provided well calibrated results.