A Suboptimal Fault-Tolerant Dual Controller in Multiple Model Framework

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Outline

Introduction

Passive vs. Active fault detection and control

- General formulation of active fault detection and control
- Optimal solution based on closed loop information processing strategy
- The special case - Fault-tolerant dual controller

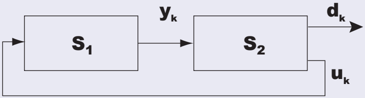
Introduction – cont'd

Goals

- Outline a general formulation of the active fault detection and control problem
- Focus on a special case that can be interpreted as a fault-tolerant dual controller (FTDC)
- Design a suboptimal FTDC using rolling horizon in multiple model framework

General formulation

Description of system S_1 for time steps $k \in \mathcal{T} = \{0, \ldots, F\}$



$$\mathbf{x}_{k+1} = \mathbf{f}_k\left(\mathbf{x}_k, \boldsymbol{\mu}_k, \mathbf{u}_k, \mathbf{w}_k\right)$$
$$\boldsymbol{\mu}_{k+1} = \mathbf{g}_k\left(\mathbf{x}_k, \boldsymbol{\mu}_k, \mathbf{u}_k, \mathbf{e}_k\right)$$
$$\mathbf{y}_k = \mathbf{h}_k\left(\mathbf{x}_k, \boldsymbol{\mu}_k, \mathbf{v}_k\right)$$

$\mathbf{f}_k$, $\mathbf{g}_k$, $\mathbf{h}_k$ – known vector functions
$\bar{\mathbf{x}}_k = \left[\mathbf{x}_k^T, \boldsymbol{\mu}_k^T\right]^T$ – system state, $\mathbf{x}_k \in \mathbb{R}^{n_x}$, $\boldsymbol{\mu}_k \in \mathcal{M} \subseteq \mathbb{R}^{n_\mu}$
$\mathbf{u}_k \in \mathcal{U}_k \subseteq \mathbb{R}^{n_u}$ – input, $\mathbf{y}_k \in \mathbb{R}^{n_y}$ – output
$\mathbf{w}_k$, $\mathbf{e}_k$ – state noises with known pdf's $p(\mathbf{w}_k)$ and $p(\mathbf{e}_k)$
$\mathbf{v}_k$ – output noise with known pdf $p(\mathbf{v}_k)$
$\bar{\mathbf{x}}_0$ – initial condition with known pdf $p(\bar{\mathbf{x}}_0) = p(\mathbf{x}_0)p(\boldsymbol{\mu}_0)$

General formulation – cont'd

Active fault detector and controller S_2

$$\begin{bmatrix} \mathbf{d}_k \\ \mathbf{u}_k \end{bmatrix} = \boldsymbol{\rho}_k\left(\mathbf{I}_0^k\right) = \begin{bmatrix} \boldsymbol{\sigma}_k\left(\mathbf{I}_0^k\right) \\ \gamma_k\left(\mathbf{I}_0^k\right) \end{bmatrix}$$

$\boldsymbol{\sigma}_k$ and γ_k – functions to be designed
$\mathbf{I}_0^k = [{\mathbf{y}_0^k}^T, {\mathbf{u}_0^{k-1}}^T, {\mathbf{d}_0^{k-1}}^T]^T$ – information vector
$\mathbf{d}_k$ – decision (a point estimate of $\boldsymbol{\mu}_k$)

Criterion

$$J\left(\boldsymbol{\rho}_0^F\right) = \mathrm{E}\left\{\sum_{k=0}^{F} \overbrace{\alpha_k L_k^{\mathrm{d}}\left(\boldsymbol{\mu}_k, \mathbf{d}_k\right) + (1-\alpha_k) L_k^{\mathrm{c}}\left(\mathbf{x}_k, \mathbf{u}_k\right)}^{L_k(\mathbf{x}_k, \boldsymbol{\mu}_k, \mathbf{u}_k, \mathbf{d}_k)}\right\}$$

General solution (Closed loop information processing strategy)

Backward recursive relation for time steps $k = F, F-1, \ldots, 0$

$$V_k^*\left(\mathbf{I}_0^k\right) = \min_{\substack{\mathbf{d}_k \in \mathcal{M} \\ \mathbf{u}_k \in \mathcal{U}_k}} \mathrm{E}\left\{L_k\left(\mathbf{x}_k, \boldsymbol{\mu}_k, \mathbf{u}_k, \mathbf{d}_k\right) + V_{k+1}^*\left(\mathbf{I}_0^{k+1}\right) | \mathbf{I}_0^k, \mathbf{u}_k, \mathbf{d}_k\right\}$$

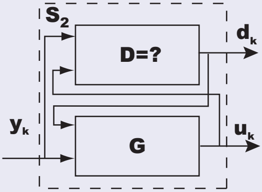
$V_{F+1}^* = 0$ – initial condition, $J(\rho_0^{F*}) = \mathrm{E}\left\{V_0^*\left(\mathbf{y}_0\right)\right\}$ – optimal value

Optimal active fault detector and controller

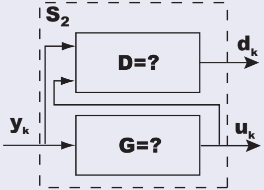
$$\begin{bmatrix} \mathbf{d}_k^* \\ \mathbf{u}_k^* \end{bmatrix} = \arg\min_{\substack{\mathbf{d}_k \in \mathcal{M} \\ \mathbf{u}_k \in \mathcal{U}_k}} \mathrm{E}\left\{L_k\left(\mathbf{x}_k, \boldsymbol{\mu}_k, \mathbf{u}_k, \mathbf{d}_k\right) + V_{k+1}^*\left(\mathbf{I}_0^{k+1}\right) | \mathbf{I}_0^k, \mathbf{u}_k, \mathbf{d}_k\right\}$$

Some special cases – cont'd

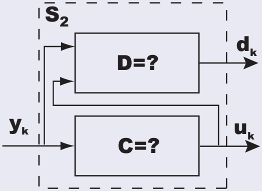
Detector for given input generator



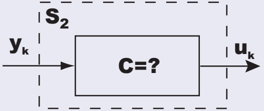
Active detector



Detector and controller



Controller



Fault-tolerant dual controller

General formulation → Optimal fault-tolerant dual controller design

- Only the control aim is considered $\Rightarrow \alpha_k = 0$ and the cost function is

$$L_k(\mathbf{x}_k, \boldsymbol{\mu}_k, \mathbf{u}_k, \mathbf{d}_k) = L_k^c(\mathbf{x}_k, \mathbf{u}_k)$$

- It implies that the function $\boldsymbol{\sigma}_k$ is not designed
- Optimal controller γ_k^*, designed using closed loop information processing strategy, steers the the system to minimize criterion regardless fault signal $\boldsymbol{\mu}_k$

$$\Downarrow$$

Optimal fault-tolerant dual controller

Fault-tolerant dual controller – cont'd

Optimal fault-tolerant dual controller

- Backward recursive equation

$$V_k^*\left(\mathbf{y}_0^k, \mathbf{u}_0^{k-1}\right) = \min_{\mathbf{u}_k \in \mathcal{U}_k} \mathrm{E}\left\{ L_k^{\mathrm{c}}\left(\mathbf{x}_k, \mathbf{u}_k\right) + V_{k+1}^*\left(\mathbf{y}_0^{k+1}, \mathbf{u}_0^k\right) \middle| \mathbf{y}_0^k, \mathbf{u}_0^k \right\}$$

- Optimal input

$$\mathbf{u}_k^* = \gamma_k^*\left(\mathbf{y}_0^k, \mathbf{u}_0^{k-1}\right) = \arg\min_{\mathbf{u}_k \in \mathcal{U}_k} \mathrm{E}\left\{ L_k^{\mathrm{c}}\left(\mathbf{x}_k, \mathbf{u}_k\right) + V_{k+1}^*\left(\mathbf{y}_0^{k+1}, \mathbf{u}_0^k\right) \middle| \mathbf{y}_0^k, \mathbf{u}_0^k \right\}$$

Multiple model framework

System description

$$\begin{aligned}\mathbf{x}_{k+1} &= \mathbf{A}_{\mu_k}\mathbf{x}_k + \mathbf{B}_{\mu_k}\mathbf{u}_k + \mathbf{G}_{\mu_k}\mathbf{w}_k \\ \mathbf{y}_k &= \mathbf{C}_{\mu_k}\mathbf{x}_k + \mathbf{H}_{\mu_k}\mathbf{v}_k\end{aligned}$$

μ_k – a scalar index into set of models $\mathcal{M} = \{1, 2, \ldots, N\}$
$P(\mu_{k+1} = j | \mu_k = i) = P_{ij}$ – transition probabilities
$\mathbf{w}_k$, $\mathbf{v}_k$ – noises with Gaussian distribution $\mathcal{N}\{\mathbf{0}, \mathbf{I}\}$
$\mathbf{x}_0$ – initial state with Gaussian distribution $\mathcal{N}\{\hat{\mathbf{x}}'_0, \mathbf{P}'_0\}$
μ_0 – initial model with probabilities $P(\mu_0)$

State estimation

- Pdf's $p(\mathbf{x}_k|\mathbf{y}_0^k, \mathbf{u}_0^k)$ and $p(\mathbf{y}_{k+1}|\mathbf{y}_0^k, \mathbf{u}_0^k)$ are Gaussian sums
- Merging or pruning of pdf's (IMM, GPB1, ...)

Rolling horizon

Additional assumption

- Quadratic cost function

$$L_k^{\mathrm{c}}\left(\mathbf{x}_k, \mathbf{u}_k\right) = \left[\mathbf{x}_k - \mathbf{r}_k\right]^T \mathbf{Q}_k \left[\mathbf{x}_k - \mathbf{r}_k\right] + \mathbf{u}_k^T \mathbf{R}_k \mathbf{u}_k$$

  $\mathbf{r}_k$ – known function of time
- Optimization horizon $F_o = 3$ means that $V_{k+3}^{*}(\mathbf{y}_0^{k+3}, \mathbf{u}_0^{k+2})$ is replaced by 0

Rolling horizon – cont'd

Time step $k+2$

- Approximate cost-to-go function

$$V^{\mathrm{a}}_{k+2}\left(\mathbf{y}_0^{k+2},\mathbf{u}_0^{k+1}\right) = \left[\hat{\mathbf{x}}_{k+2}-\mathbf{r}_{k+2}\right]^T \mathbf{Q}_{k+2}\left[\hat{\mathbf{x}}_{k+2}-\mathbf{r}_{k+2}\right] + \mathrm{tr}(\mathbf{Q}_{k+2}\mathbf{P}_{k+2})$$

- Input

$$\mathbf{u}^{\mathrm{a}}_{k+2} = 0$$

Note: Mean value $\hat{\mathbf{x}}_i = \mathrm{E}\{\mathbf{x}_i|\mathbf{y}_0^i,\mathbf{u}_0^{i-1}\}$ and covariance matrix $\mathbf{P}_i = \mathrm{cov}\{\mathbf{x}_i|\mathbf{y}_0^i,\mathbf{u}_0^{i-1}\}$ can be obtained from estimation algorithm.

Rolling horizon – cont'd

Time step $k+1$

- Approximate cost-to-go function
$V^{\mathrm{a}}_{k+1}\left(\mathbf{y}_0^{k+1},\mathbf{u}_0^k\right) = [\hat{\mathbf{x}}_{k+1} - \mathbf{r}_{k+1}]^T\,\mathbf{Q}_{k+1}\,[\hat{\mathbf{x}}_{k+1} - \mathbf{r}_{k+1}] + \mathrm{tr}(\mathbf{Q}_{k+1}\mathbf{P}_{k+1}) + K - \mathbf{D}^T\mathbf{W}^{-1}\mathbf{D}$
- Input $\mathbf{u}^{\mathrm{a}}_{k+1} = -\mathbf{W}^{-1}\mathbf{D}$

Matrix $\mathbf{W}$, column vector $\mathbf{D}$ and scalar K are computed as

$$\mathbf{W}=\mathbf{R}_k+\sum_{\mu_k}\mathbf{B}^T_{\mu_k}\mathbf{Q}_{k+1}\mathbf{B}_{\mu_k}P\left(\mu_k|\mathbf{y}_0^k,\mathbf{u}_0^{k-1}\right)$$

$$\mathbf{D}=\sum_{\mu_k}\mathbf{B}^T_{\mu_k}\mathbf{Q}_{k+1}\left[\mathbf{A}_{\mu_k}\hat{\mathbf{x}}_k(\mu_k)-\mathbf{r}_{k+1}\right]P\left(\mu_k|\mathbf{y}_0^k,\mathbf{u}_0^{k-1}\right)$$

$$K=\sum_{\mu_{k+1}}\Big\{\left[\mathbf{A}_{\mu_{k+1}}\hat{\mathbf{x}}_{k+1}(\mu_{k+1})-\mathbf{r}_{k+2}\right]^T\mathbf{Q}_{k+2}\left[\mathbf{A}_{\mu_{k+1}}\hat{\mathbf{x}}_{k+1}(\mu_{k+1})-\mathbf{r}_{k+2}\right]$$

$$+\mathrm{tr}\left(\mathbf{Q}_{k+2}(\mathbf{A}_{\mu_{k+1}}\mathbf{P}_{k+1}(\mu_{k+1})\mathbf{A}^T_{\mu_{k+1}}+\mathbf{G}_{\mu_{k+1}}\mathbf{G}^T_{\mu_{k+1}})\right)\Big\}P\left(\mu_{k+1}|\mathbf{y}_0^{k+1},\mathbf{u}_0^k\right)$$

Rolling horizon – cont'd

Time step k

- Approximate cost-to-go function

$$V_k^{\mathrm{a}}\left(\mathbf{y}_0^k, \mathbf{u}_0^{k-1}\right) = \min_{\mathbf{u}_k \in \mathcal{U}_k} \mathrm{E}\left\{ L_k^{\mathrm{c}}\left(\mathbf{x}_k, \mathbf{u}_k\right) + V_{k+1}^{\mathrm{a}}\left(\mathbf{y}_0^{k+1}, \mathbf{u}_0^k\right) \middle| \mathbf{y}_0^k, \mathbf{u}_0^k \right\}$$

- Input applied at time step k

$$\mathbf{u}_k^{\mathrm{a}} = \arg\min_{\mathbf{u}_k \in \mathcal{U}_k} \mathrm{E}\left\{ L_k^{\mathrm{c}}\left(\mathbf{x}_k, \mathbf{u}_k\right) + V_{k+1}^{\mathrm{a}}\left(\mathbf{y}_0^{k+1}, \mathbf{u}_0^k\right) \middle| \mathbf{y}_0^k, \mathbf{u}_0^k \right\}$$

- The expectation with respect to $\mathbf{y}_{k+1}$ and minimization over $\mathbf{u}_k$ is performed numerically

Numerical example

Fault-tolerant dual controller for two models

- Parameters of models

μ_k	$\mathbf{A}_{\mu_k}$	$\mathbf{B}_{\mu_k}$	$\mathbf{G}_{\mu_k}$	$\mathbf{C}_{\mu_k}$	$\mathbf{H}_{\mu_k}$
1	0.9	0.1	0.01	1	0.05
2	0.9	-0.098	0.01	1	0.05

- Transition probabilities $P_{1,1} = P_{2,2} = 0.9,\ P_{1,2} = P_{2,1} = 0.1$
- Detection horizon $F = 30$
- Initial conditions $P(\mu_0 = i) = 0.5$, $\hat{\mathbf{x}}'_0 = 1$ and $\mathbf{P}'_{0,x} = 0.01$
- Set of admissible inputs $\mathcal{U}_k = \{-3, -2.9, \ldots, 2.9, 3\}$
- Square wave reference signal, matrices $\mathbf{Q}_k = 1$, $\mathbf{R}_k = 0.001$

Numerical example – cont'd

Cautious controller and certainty equivalent controller

- Cautious controller (CAC) – optimization horizon consists of two steps

$$\mathbf{u}_k^{\mathrm{CAC}} = -\mathbf{W}^{-1}\mathbf{D}, \text{ where}$$

$$\mathbf{W}{=}\mathbf{R}_k{+}\sum_{\mu_k}\mathbf{B}_{\mu_k}^T\mathbf{Q}_{k+1}\mathbf{B}_{\mu_k}P\big(\mu_k|\mathbf{y}_0^k,\mathbf{u}_0^{k-1}\big)$$

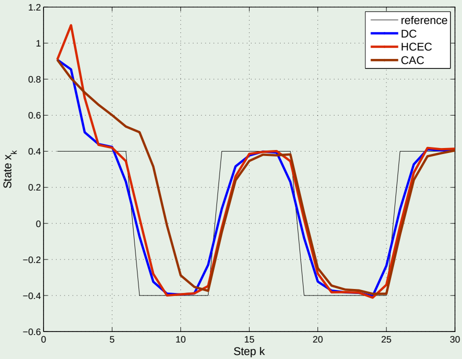
$$\mathbf{D}{=}\sum_{\mu_k}\mathbf{B}_{\mu_k}^T\mathbf{Q}_{k+1}\big[\mathbf{A}_{\mu_k}\hat{\mathbf{x}}_k(\mu_k){-}\mathbf{r}_{k+1}\big]P\big(\mu_k|\mathbf{y}_0^k,\mathbf{u}_0^{k-1}\big)$$

- Heuristic certainty equivalence controller (HCEC) – random variables are set to most probable values

$$\mathbf{u}_k^{\mathrm{HCEC}} = \mathbf{K}_k\hat{\mathbf{x}}_k, \text{ where } \mathbf{K}_k \text{ solves t-variant LQ optimal control}$$

Numerical example – cont'd

A typical state trajectories for various controllers



Numerical example – cont'd

Results of M = 1000 MC simulations for horizon F = 30

Controller	$\hat{J}$	$\mathrm{var}\{\hat{J}\}$	$\mathrm{var}\{L\}$
HCEC	3.2126	0.0164	3.2885
CAC	7.2186	0.0068	1.3194
FTDC	2.3131	0.0109	2.0889

$$J = \mathrm{E}\left\{ \overbrace{\sum_{k=0}^{F} L_k^{c}\left(\mathbf{x}_k, \mathbf{u}_k\right)}^{L} \right\} \qquad \hat{J} = \frac{1}{M}\sum_{i=1}^{M} L^i$$

$$\mathrm{var}\left\{\hat{J}\right\} = \text{bootstrap}\left\{L^i\right\} \qquad \mathrm{var}\{L\} = \frac{1}{M-1}\sum_{i=1}^{M}\left(L^i - \hat{J}\right)^2$$

Conclusion remarks

- The general formulation of active change/fault detection and control
- The suboptimal fault-tolerant dual controller based on rolling horizon technique
- The numerical example illustrating a benefit of dual approach