Probabilistic projections of HIV prevalence using Bayesian melding

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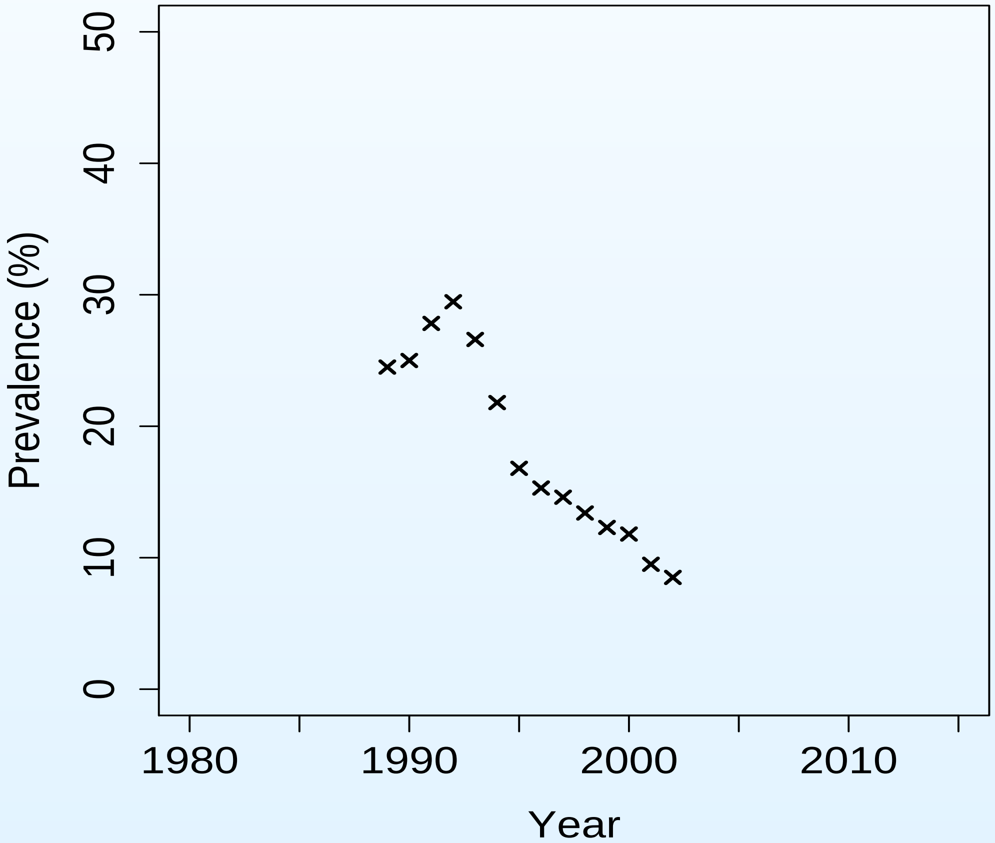
- United Nations program on HIV/AIDS (UNAIDS) produces estimates and projections, using the Estimation and Projection Package (EPP)
- EPP is designed to fit an epidemic curve to various epidemics based on little data

HIV prevalence prediction for Uganda urban

- Observed HIV prevalence in five urban antenatal clinics



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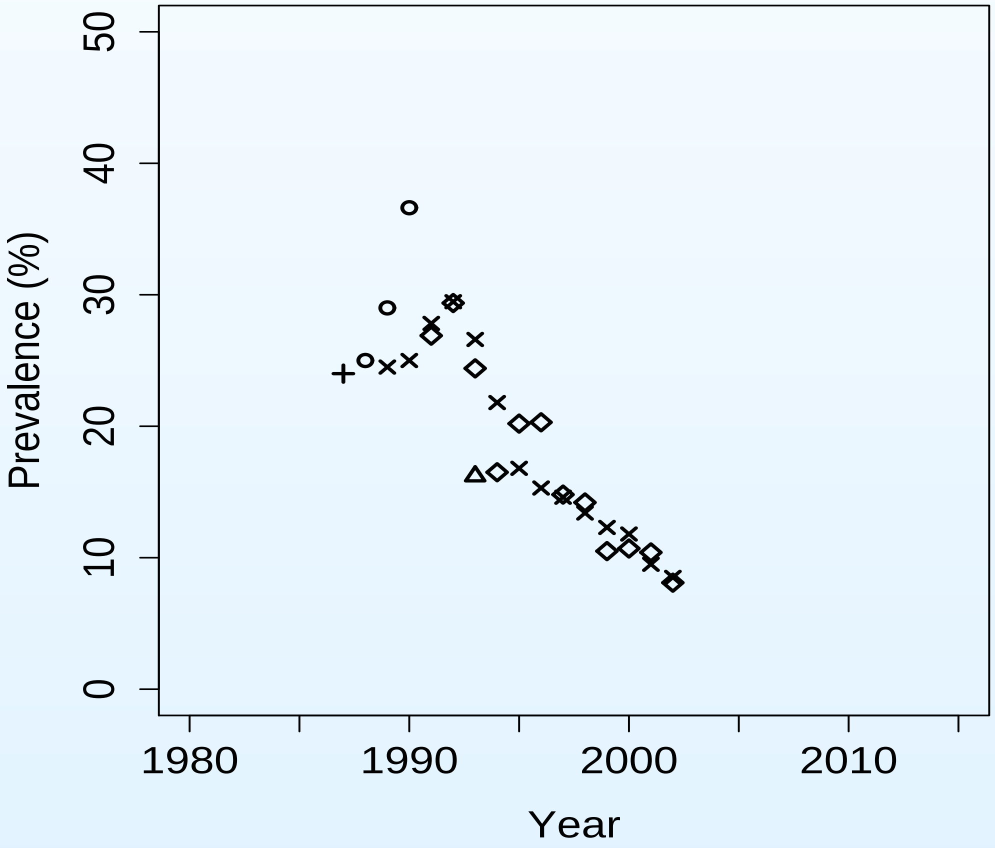

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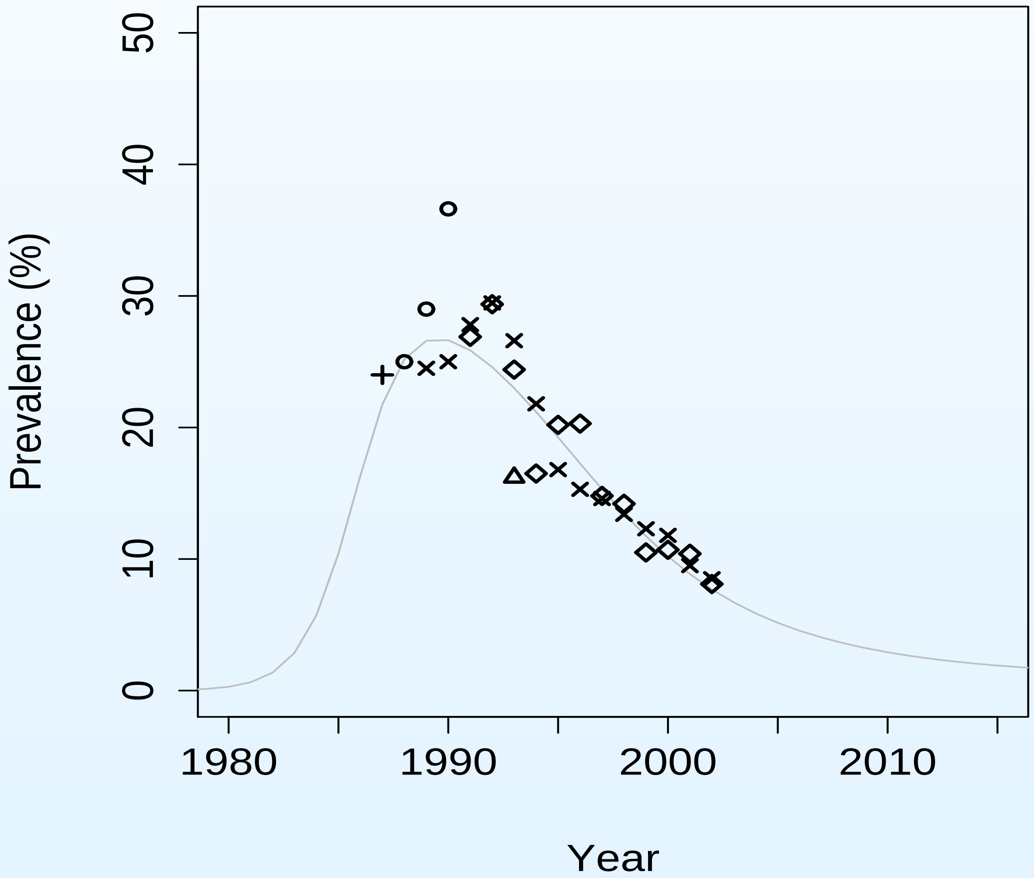
- The EPP model fits an epidemic curve



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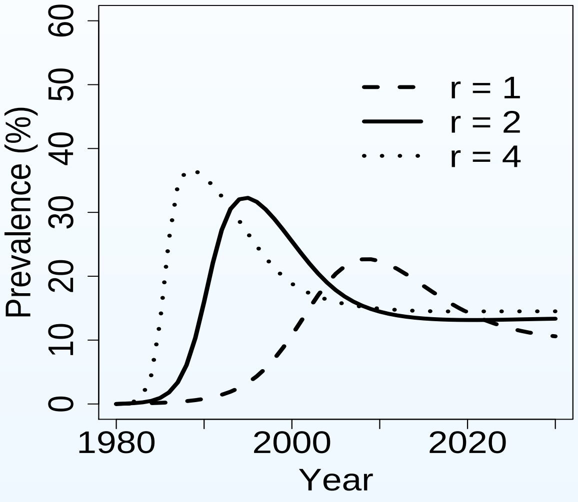
- Three differential equations describe the changes in those groups over time, thus in prevalence over time
- Four parameters determine the shape of the epidemic curve:
  - r = Growth rate of the epidemic
  - f_0 = Fraction of population initially at risk
  - t_0 = Start time epidemic
  - ϕ = Behavioral response

Input parameters (Solid curve: $r = 2, f_0 = 0.4, t_0 = 1980, \phi = 0$)



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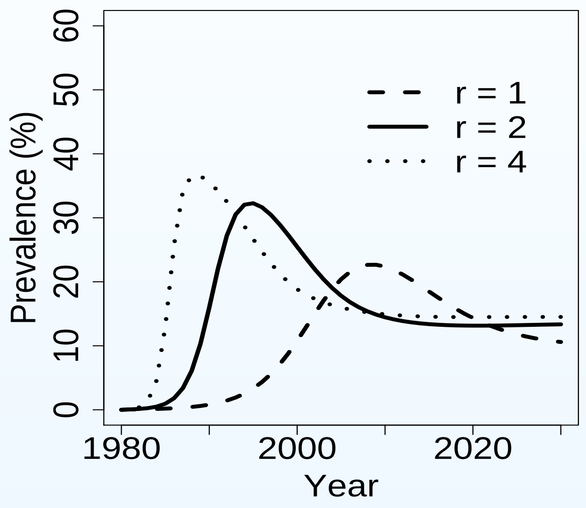

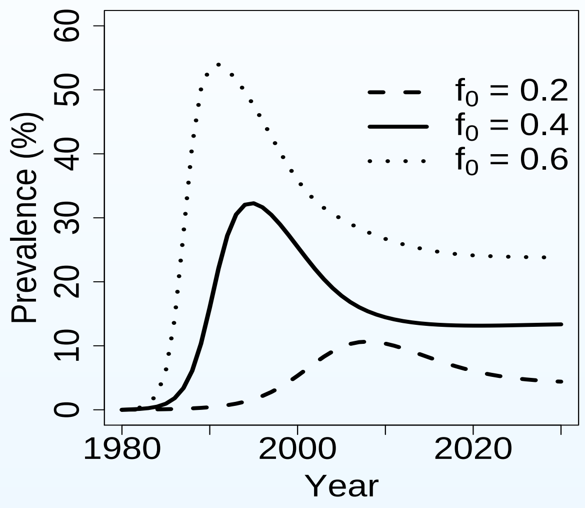

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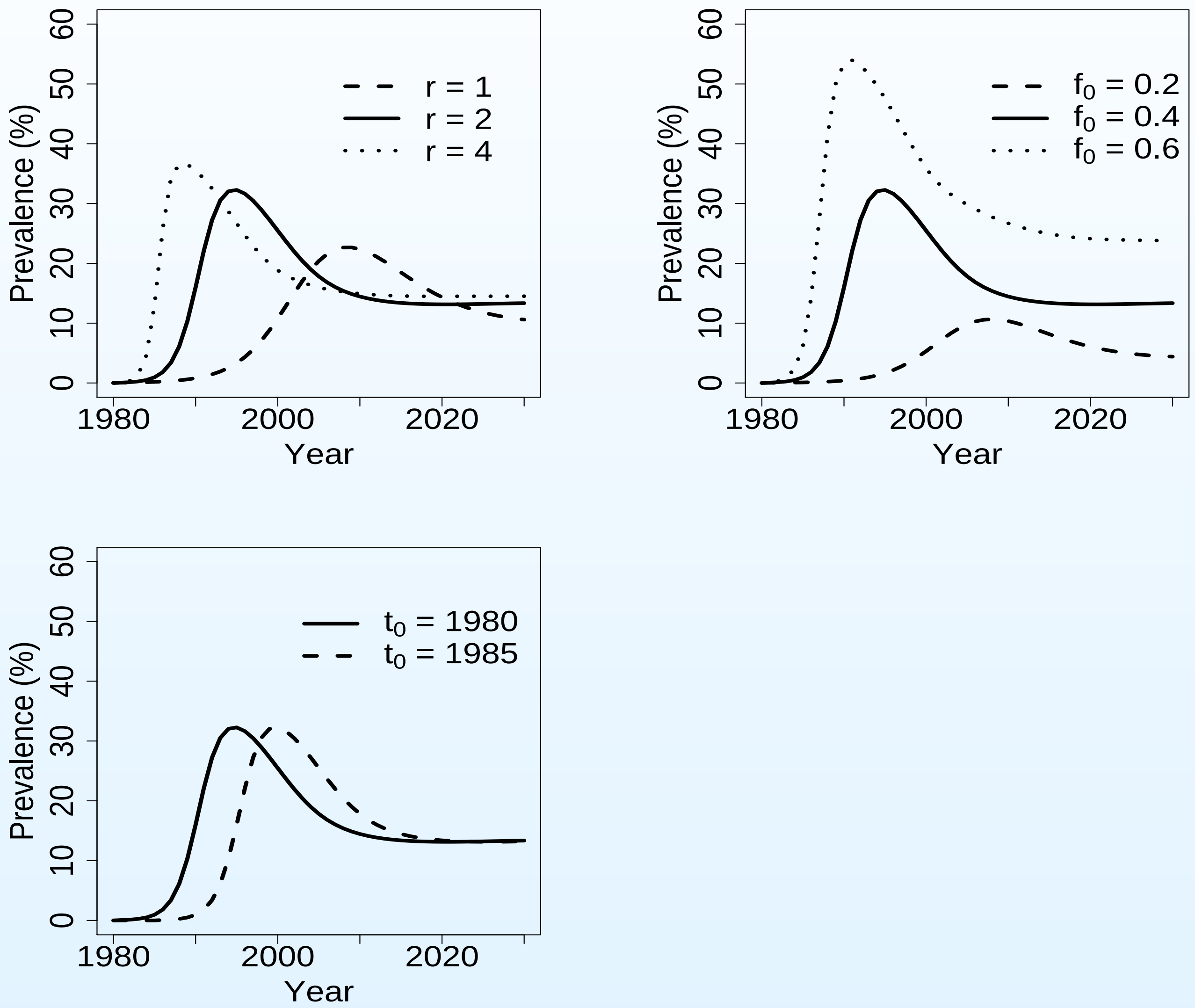

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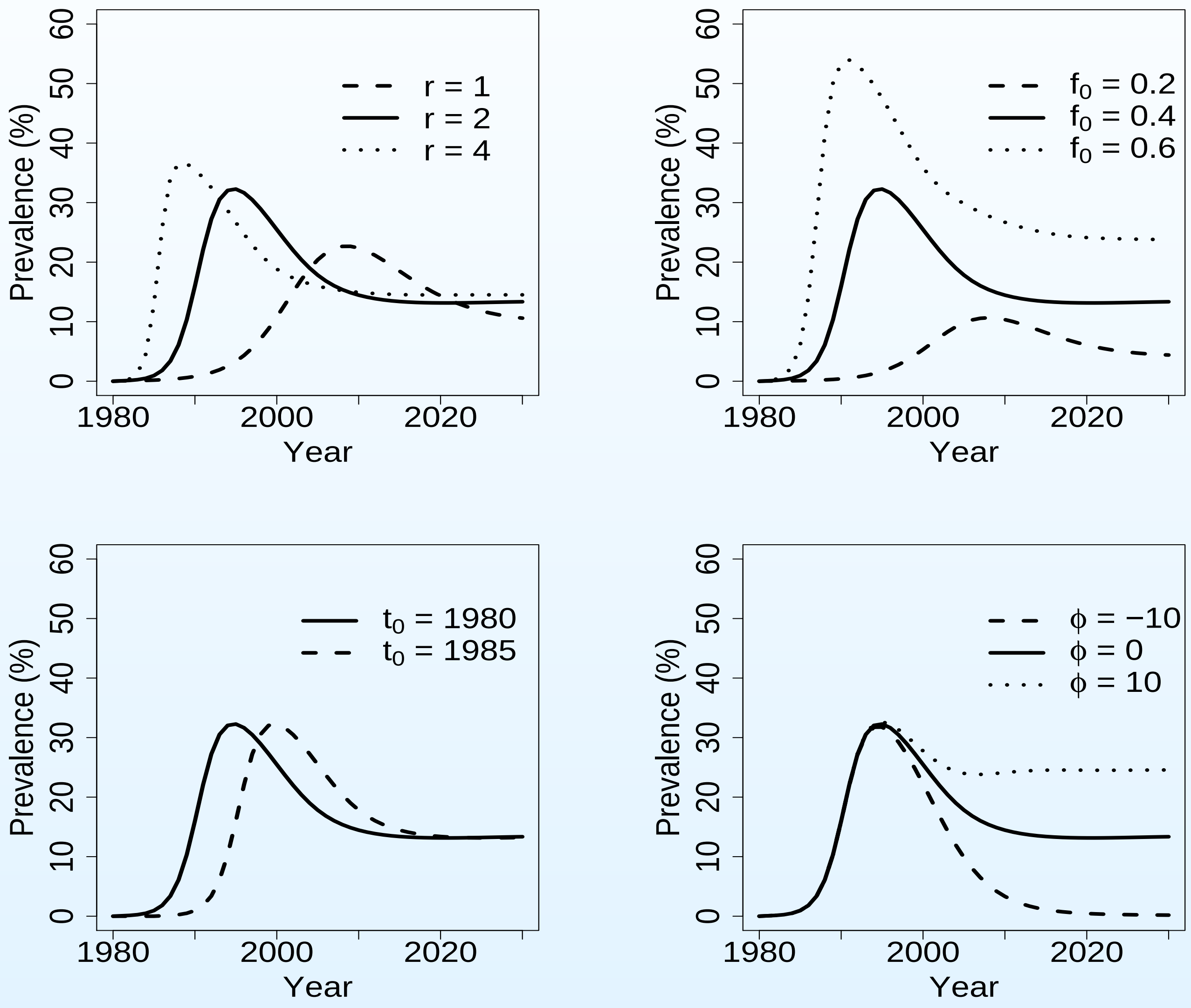

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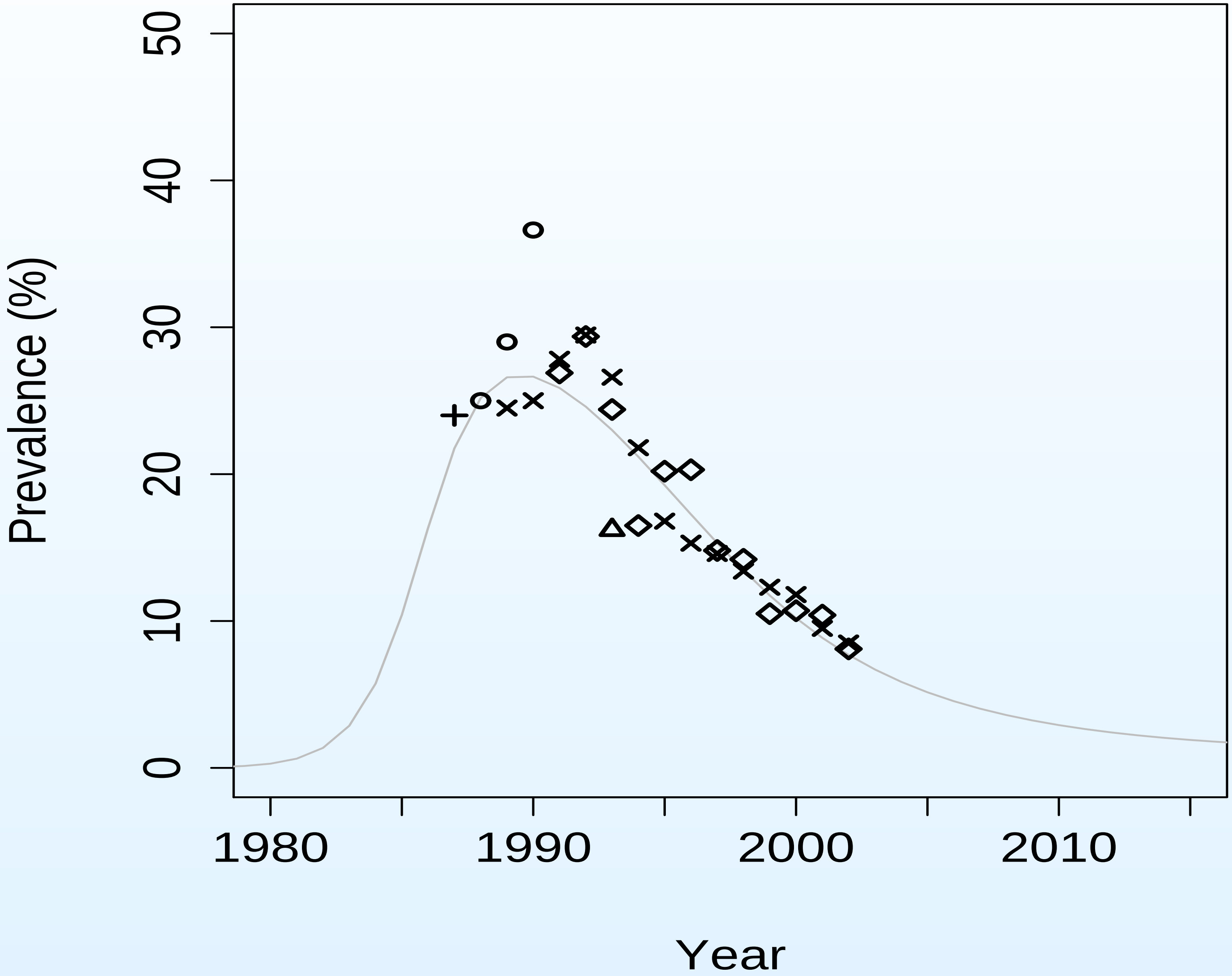

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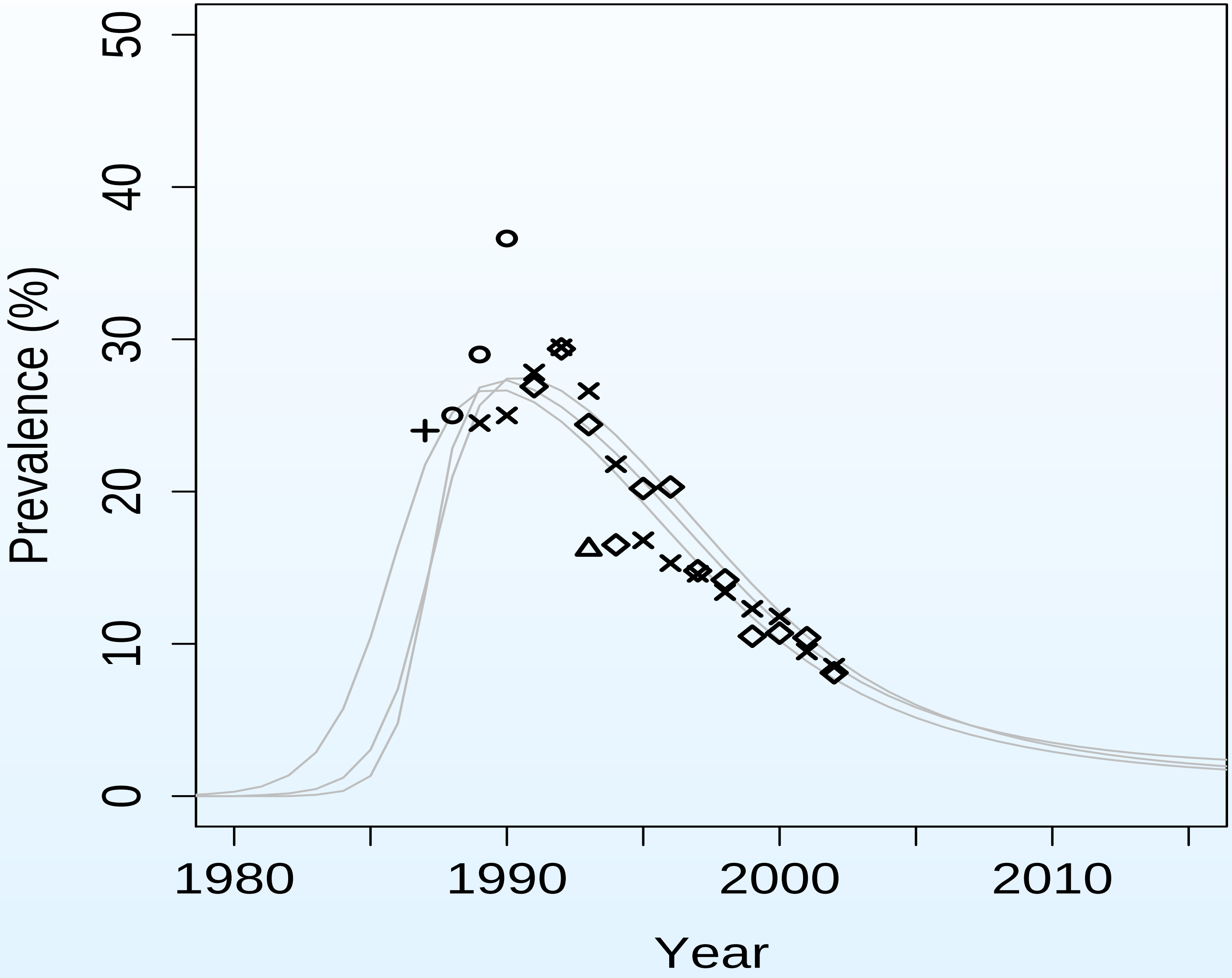

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$$\pi(\rho) \;\propto\; q^*(\rho)L(\rho)$$

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- Sample from the discrete distribution of $\{\theta_1, ..., \theta_n\}$ with probabilities w_i to get the posterior distribution for the inputs, same for the outputs

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Bayesian melding results for Uganda

- Sample inputs from input prior

$$\begin{cases} r & \sim U[0, 15] \\ f_0 & \sim U[0, 1] \\ t_0 & \sim \text{DiscreteU}(1970, 1990) \\ \phi & \sim \text{Logistic}(0, 10) \end{cases}$$

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- Sample sizes:
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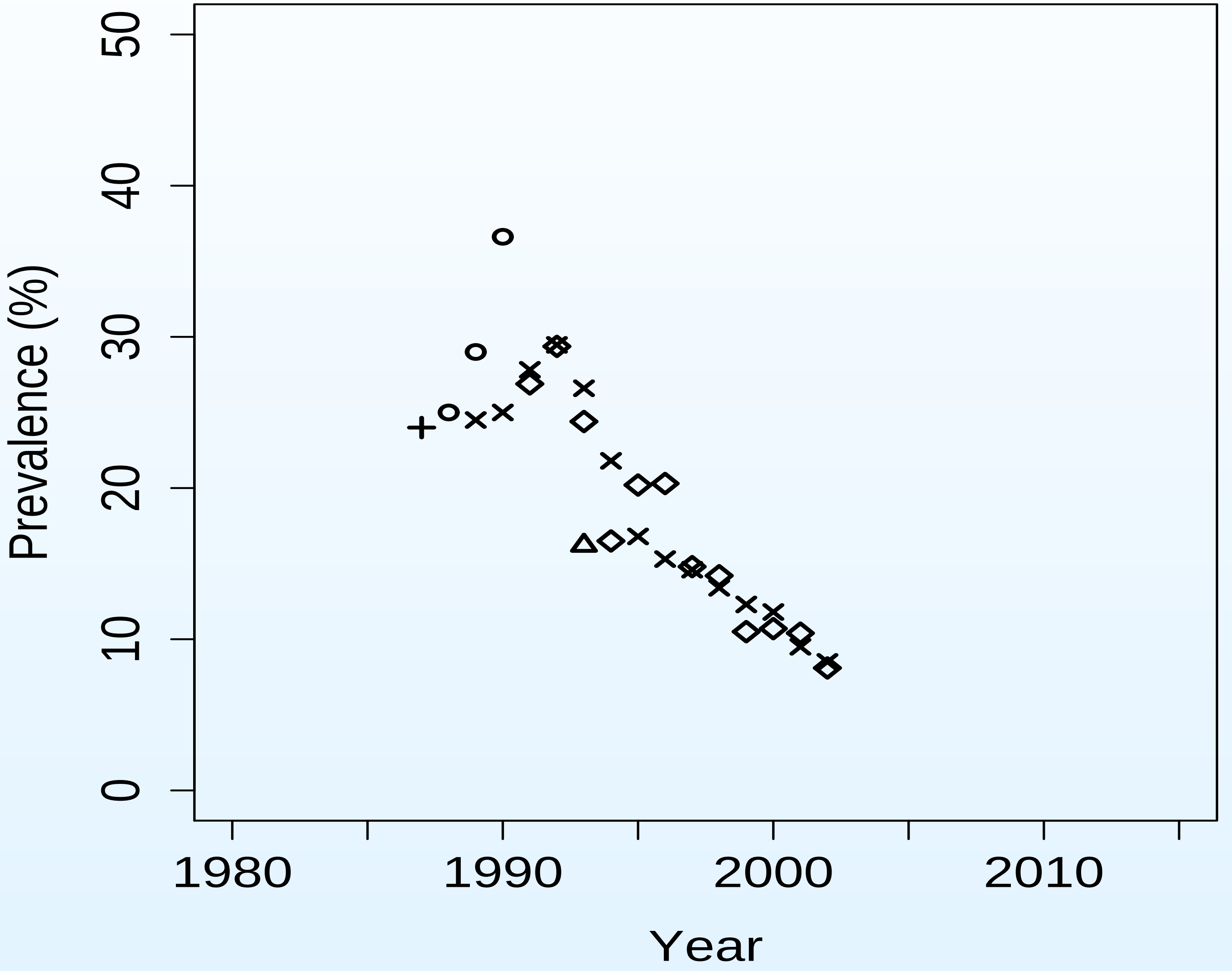
  - resampling 3,000 sets
  - 508 unique combinations

HIV prevalence Uganda



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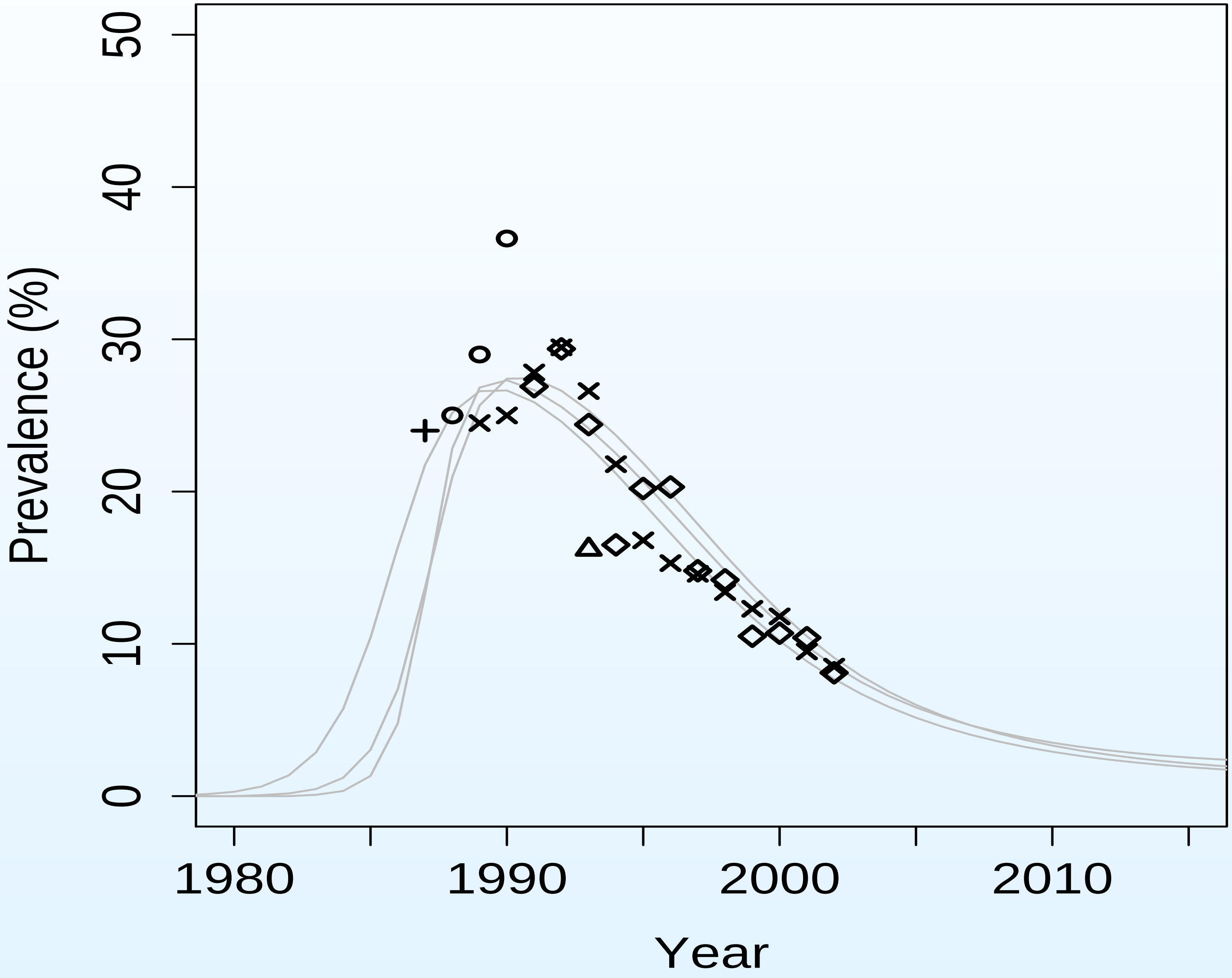

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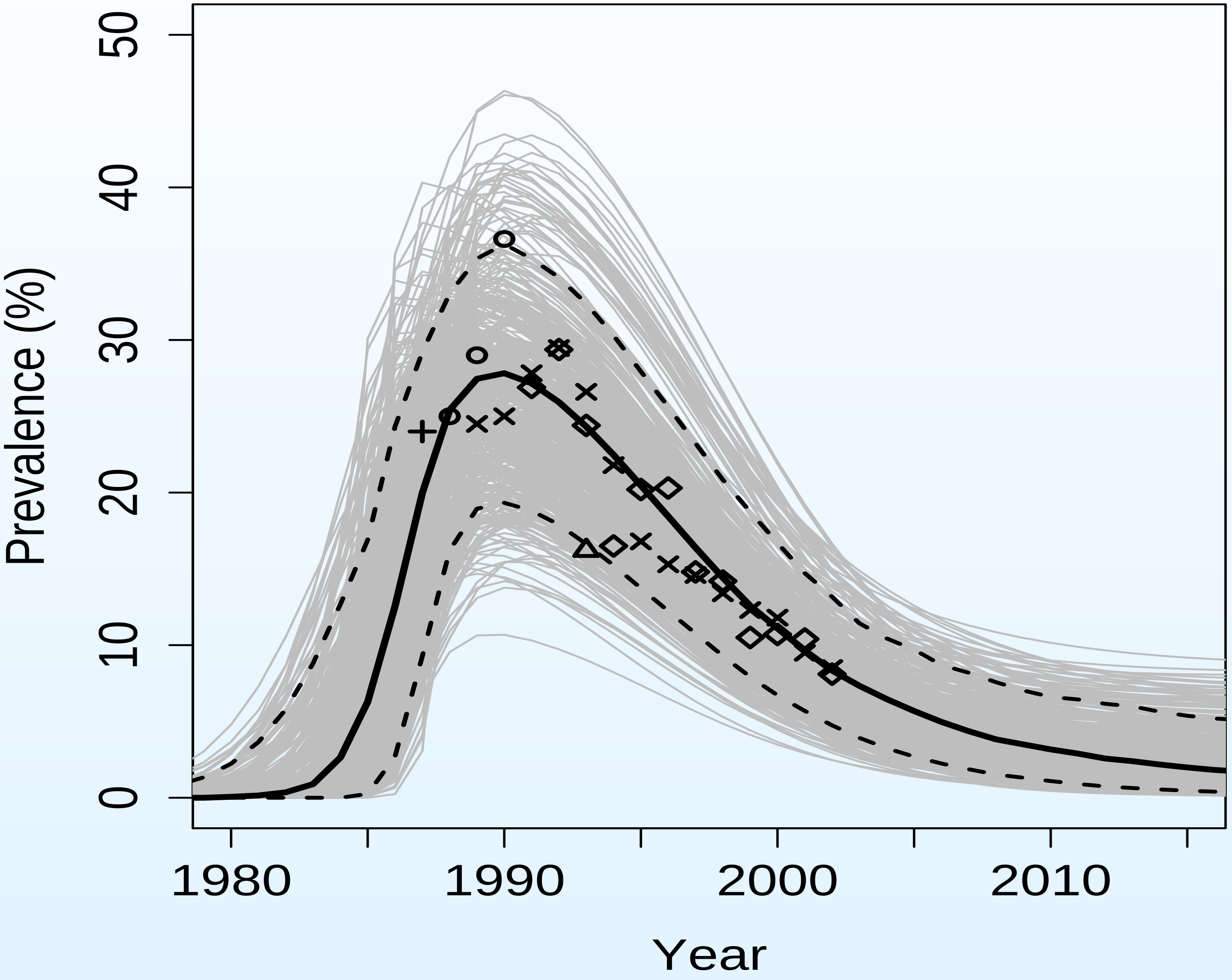
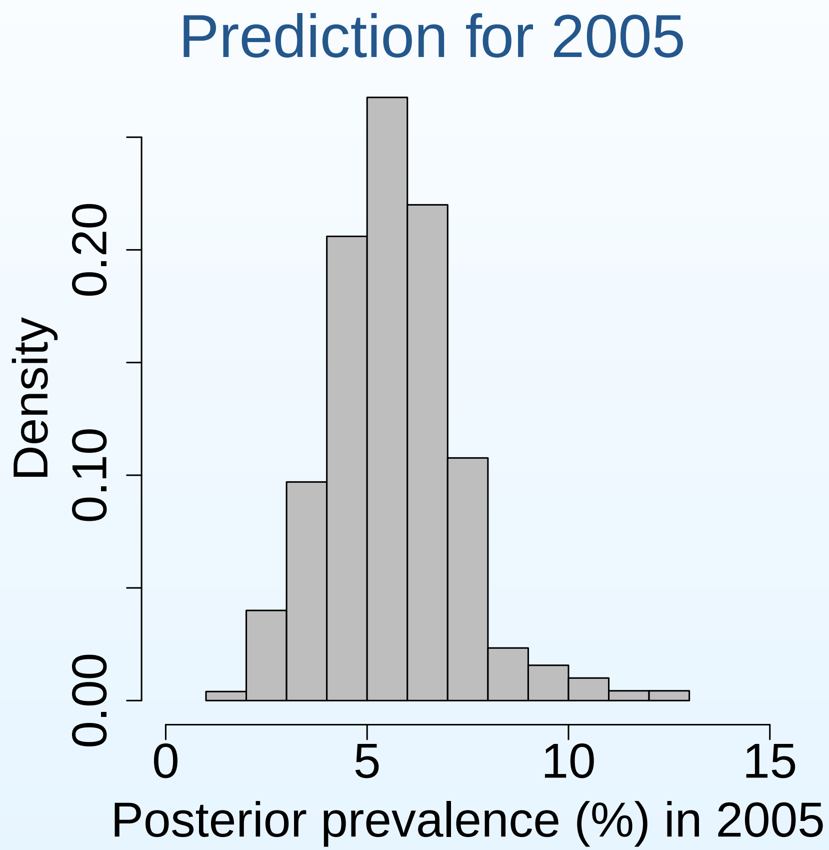


Posterior Predictive Distributions



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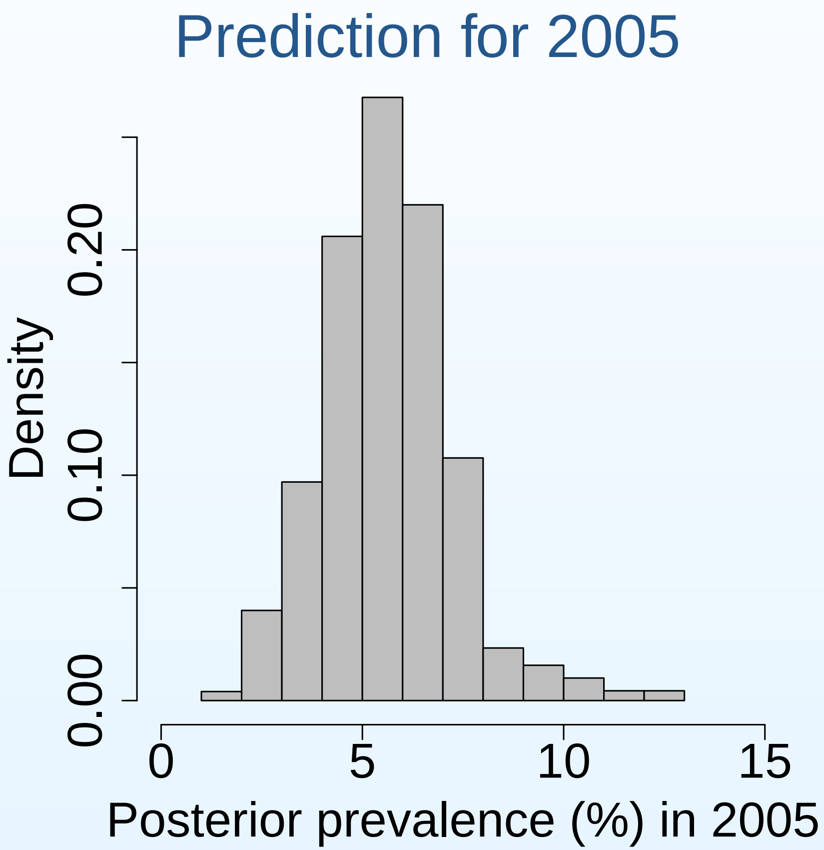
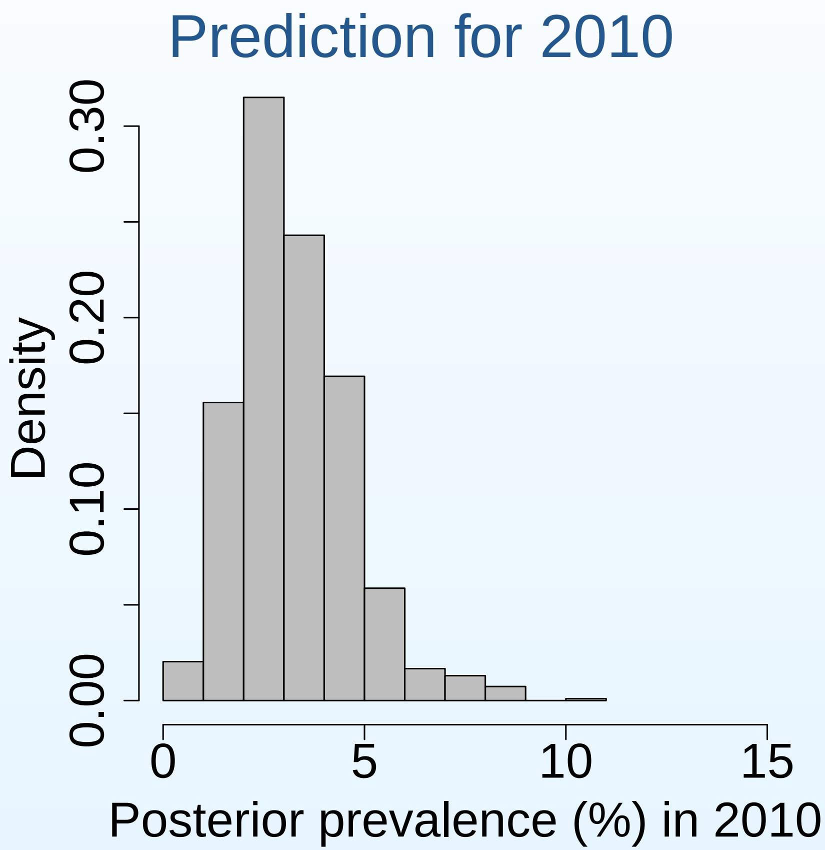
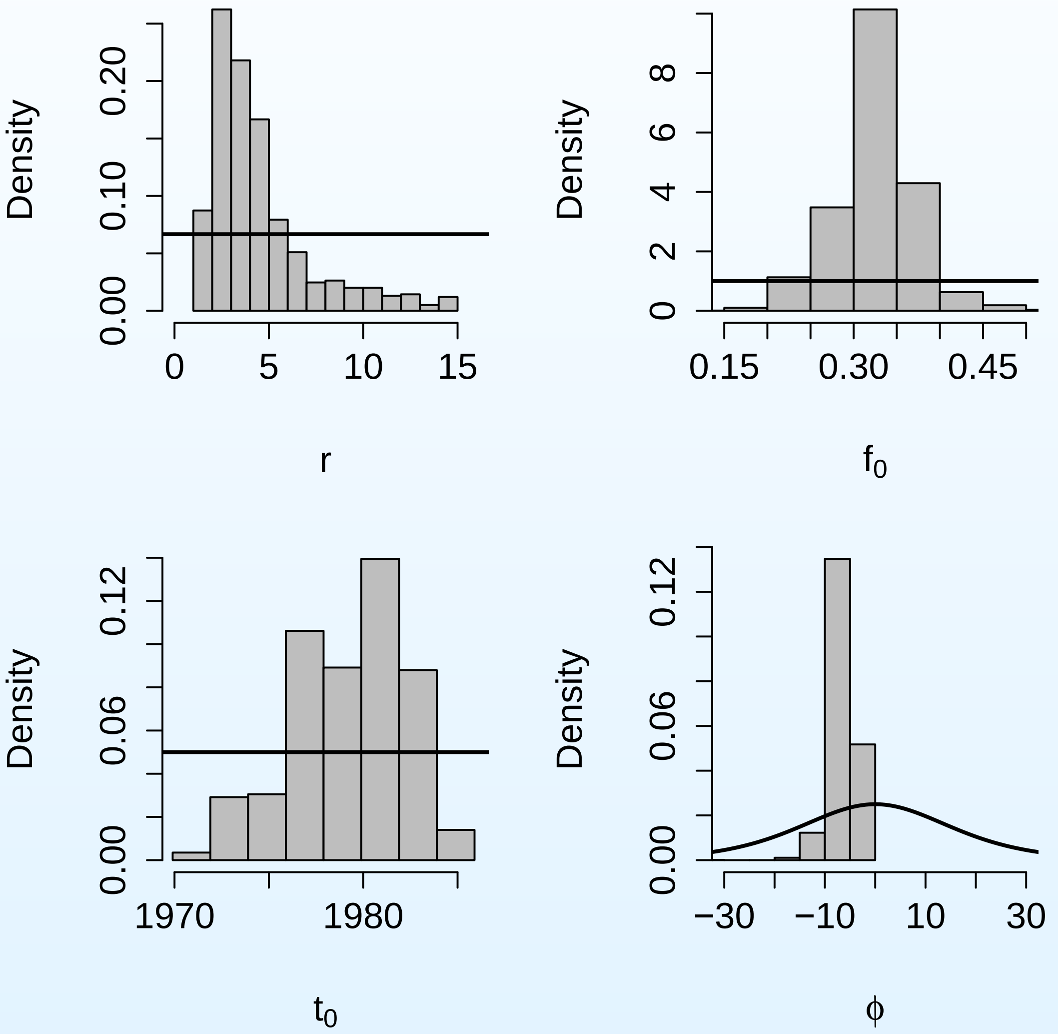
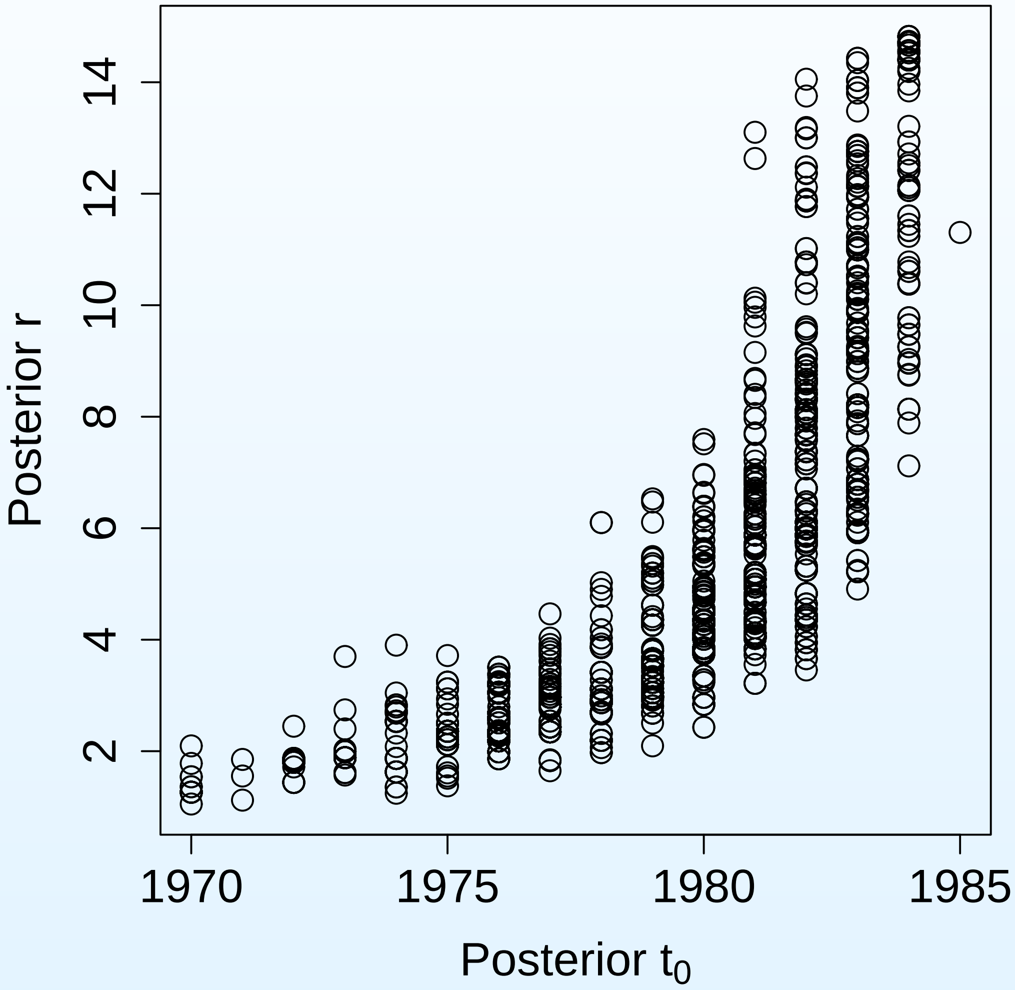




Prior and posterior distribution of inputs

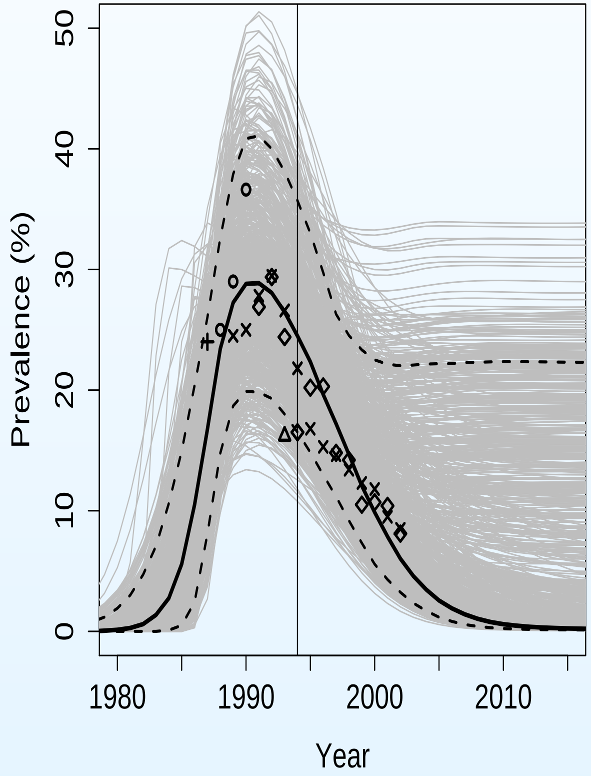


Joint posterior of input parameters r and t_0



Changes in predictions over time

Data through 1994



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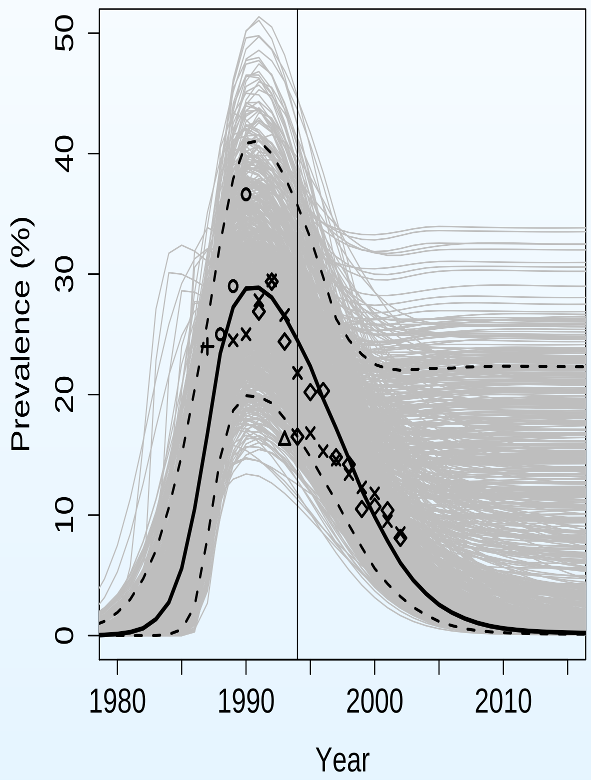

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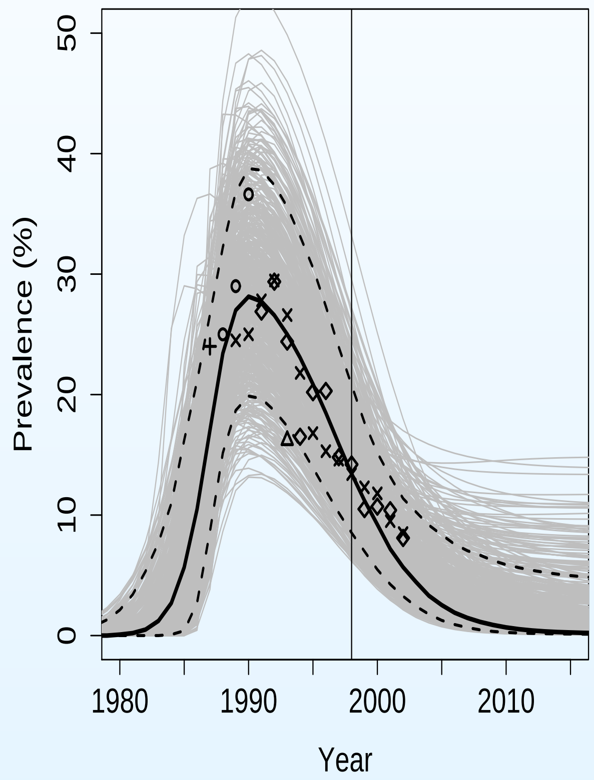

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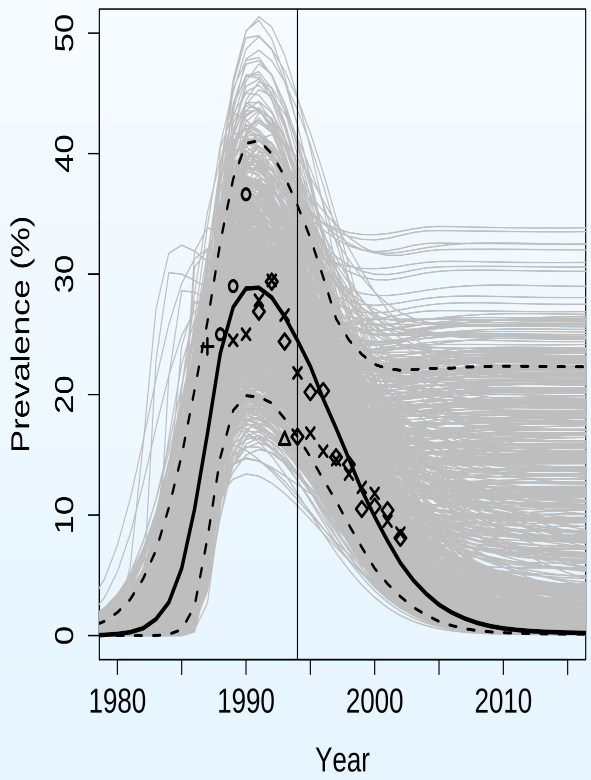

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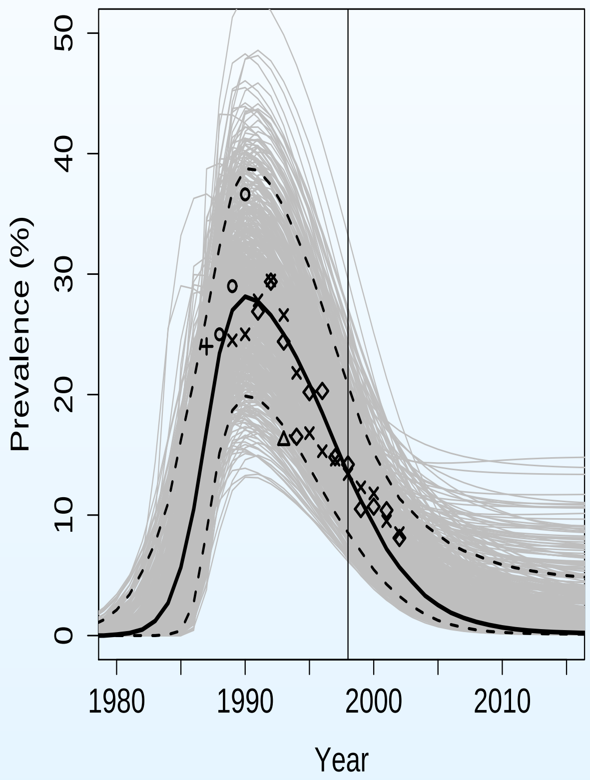
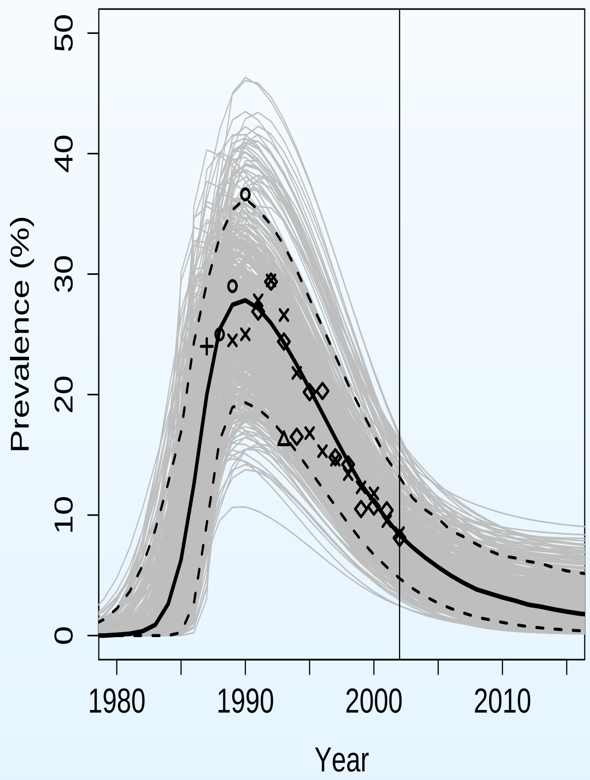


Data through 2002



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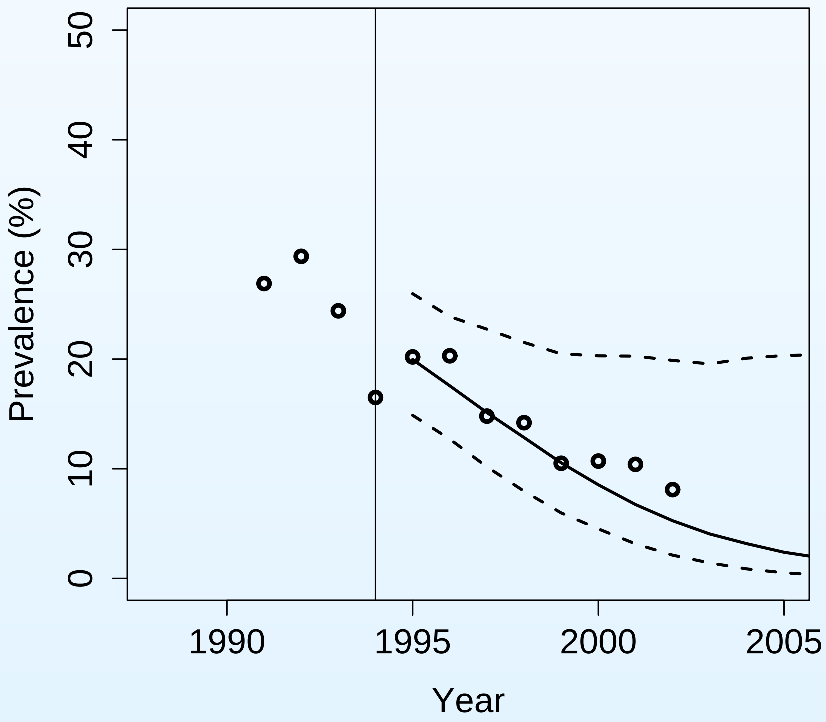

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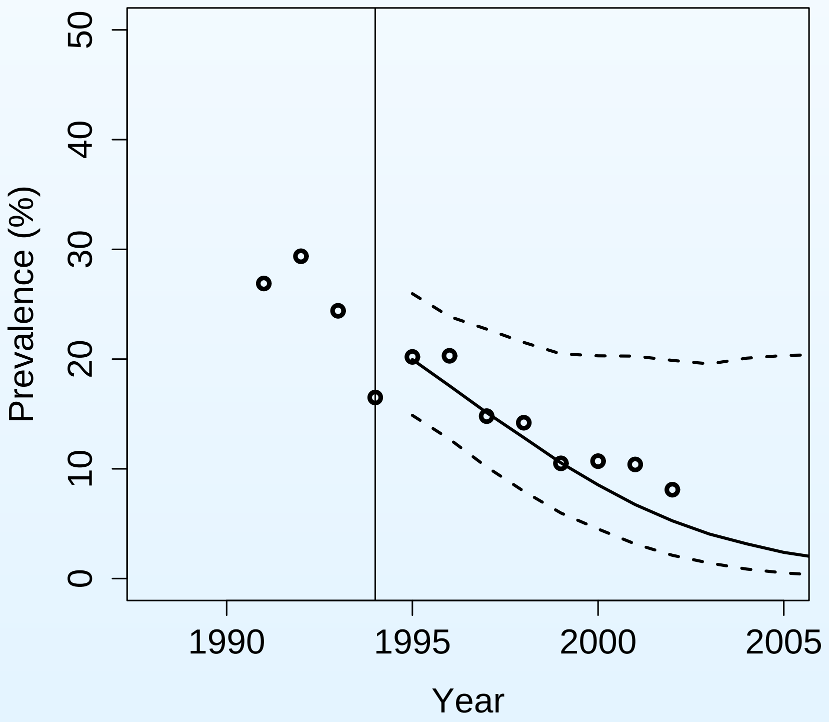

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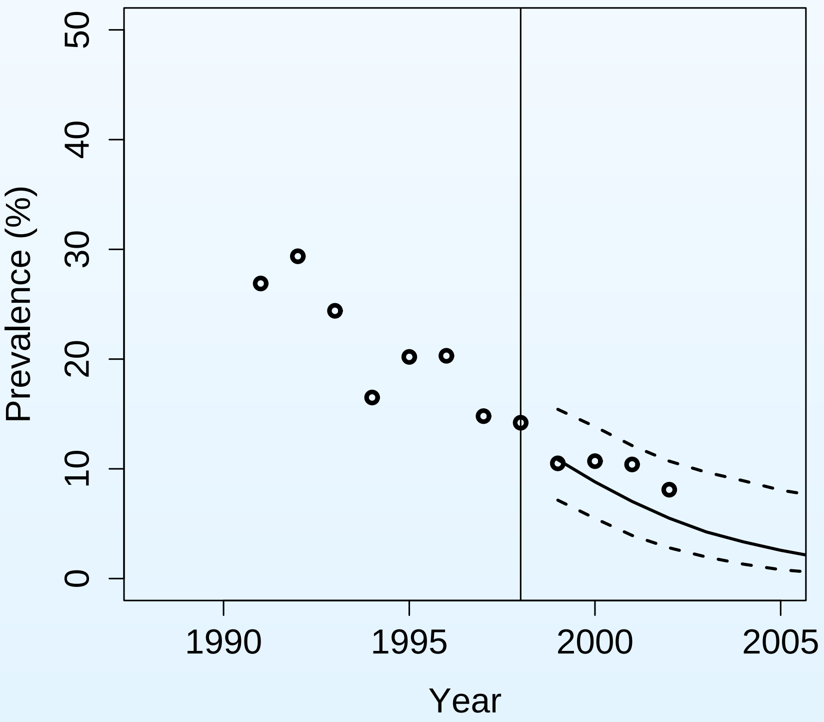

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- Bayesian melding will be available in EPP 2007