Toolbox for Multivariate Adaptive Controller Design

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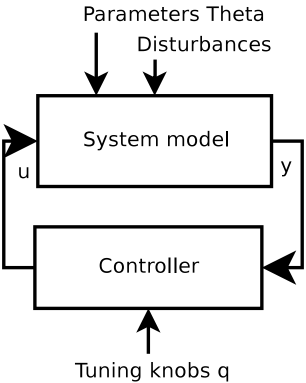
Mixtools-Jobcontrol Toolbox

- Matlab environment for complete controller design
- Mixtools - computational base
- Jobcontrol - connects particular steps of design
  1. Data pre-processing
  2. Structure identification
  3. Parameter estimation
  4. Forgetting factor estimation
  5. Validation of identified model
  6. **Controller design**
  7. Controller verification

Closed Loop

Components

- System model
  - $f(y_t | u_t, \varphi_t)$
  - input u_t, output y_t
  - ARX obtained from identification
- Controller
  - $f(u_t | \varphi_t, q)$
  - use of adaptive LQG controller
  - tuning parameters q to be set



Searched controller - User's aims

- find such value of tuning parameter q
- for which $u \in [u^l, u^u]$
- and output error is minimized

Controller constructed directly for the aims is unavailable

- Requirements on the controller
  - constraints on quantities
  - complicated probabilistic system model - ARX with uncertain parameters
  - adaptive controller
- Possible (approximate) solutions
  - GPC - deterministic models only
  - Bellman function approximation
  - **Controller tuning**

Controller tuning

- Using a simpler controller - matches the task only partially
- Dependent on tuning parameters
- Tuning parameters differs from user's aims
  - constraints kept only through penalization weights
- The simpler controller has good properties even non-tuned
  - stabilizes closed loop

Tuning is difficult for human

- Many tuning parameters — high dimensional tuning space
- Complex dependency of controller behavior on the tuning parameters
- Stochastic behavior of the closed loop

Controller Quality

Closed loop data

- closed loop data $d(T) = (u_1, y_1, u_2, y_2, \ldots, u_T, y_T)$

$$f(d(T)|q) = \prod_{t=1}^{T} f(y_t|u_t, \varphi_t) f(u_t|\varphi_t, q)$$

Controller quality functions

- $Z_c(d(T))$ constraint violation
- $Z_o(d(T))$ output error
- optimal tuning

$$q^{\mathrm{opt}} = \arg \min_{q: EZ_c(q) \leq 0} EZ_o(q)$$

Choice of the controller quality functions

$$Z_c(d(T)) = \frac{1}{T}\sum_{t=1}^{T} \chi_{(-\infty,u^l)\bigcup(u^u,+\infty)} u_t - \alpha$$

- approximates probability of constraints violation

$$Z_o(d(T)) = \frac{1}{T}\sum_{t=1}^{T} \|y_t\|^2$$

- output error
- Mean values $EZ_c(q)$ and $EZ_o(q)$
  - difficult to calculate
  - $f(d(T)|q)$ cannot be found in closed form
  - includes uncertain system model and adaptive LQG controller

Evaluation

Monte Carlo approach

- Stationary case $Z_\bullet(d(T)) \rightarrow EZ_\bullet(q)$ for $T \rightarrow \infty$
- The sample $d(T)$ of $f(d(T)|q)$ is obtained by simulation of length T

Length of Simulation

- Determines estimate precision and computation time
- Long enough to find stabilized estimate of $EZ_\bullet$
- On-line stopping rule

Online Stopping Rule

- Based on Kullback-Leibler divergence of pdf of summed terms v_t in $Z_\bullet = \frac{1}{T}\sum v_t$
- Data d_t and also v_t are correlated
- Modeling the dynamics $f(v_t|v_{t-1},\Theta)$
- Estimating $f(\Theta|v(t)) \quad v(t) = (v_1, v_2, \ldots, v_t)$
- Stopping when pdf of model parameters Θ stabilizes

$$\mathcal{D}_{\mathrm{KL}}[f(\Theta|d(T))\|f(\Theta|d(T-1))] < \varepsilon$$

- Then assuming $Z(d(T)) \sim EZ(q)$

Stopping for output error Z_o

- Modeled variable $v_t = \|y_t\|^2$
- ARX model $v_{t+1} = av_t + c + e_t, \quad e_t \sim \mathbf{N}(0, \sigma^2)$
- Static property $Ev_t = p \doteq EZ_o$
- Pdf of p stabilizes faster than pdf of parameter a, c, σ

  $\mathcal{D}_{\mathrm{KL}}[f(p|d(T))\|f(p|d(T-1))] \leq \mathcal{D}_{\mathrm{KL}}[f(a, c, \sigma|d(T))\|f(a, c, \sigma|d(T-1))] \leq \varepsilon$
- Thus using p as stabilized estimate of EZ_o

Stopping for constraints violation Z_c

- Markov chain model
- Modeled variable $v_t = \begin{cases} 1 & u_t > u^u \\ 0 & u_t \in [u^l, u^u] \\ -1 & u_t < u^l \end{cases}$
- MC model $p(v_{t+1}|v_t) = P_{v_{t+1}|v_t}$

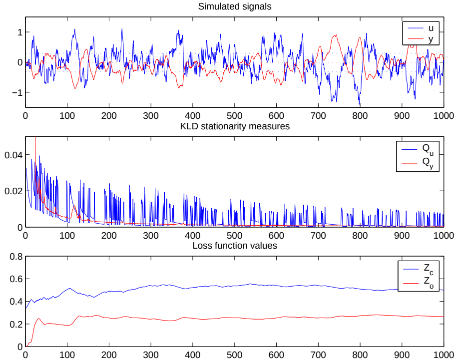
Stopping Rule Experiment

Experiment

- SISO system of 2nd order
- Noise compensation task
- Input constraints $[-0.3, 0.3]$

Diagrams

1. Simulated signals
   - Input BLUE
   - Output RED
2. K-L stationarity measure Q
   - Interpolation of Q for MC
3. Quality function values



- Suitable threshold 0.0015
- Stopping after around 350 steps

Tuned vs. Non-tuned Controller with Constraints

Experiment

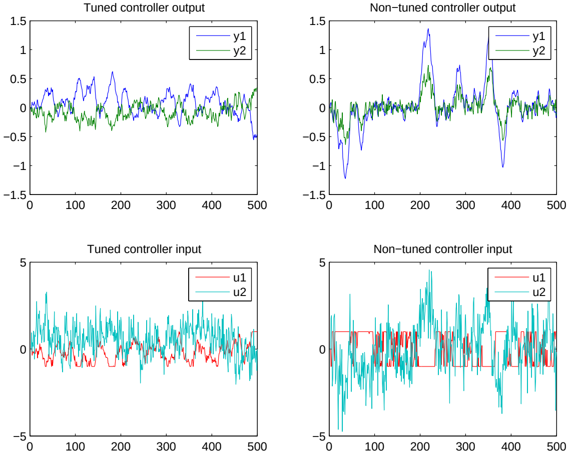
MIMO system
2 inputs 2 outputs

aims:

$\min \sum \|y\|^2$

$u_1 \in [-1, 1]$

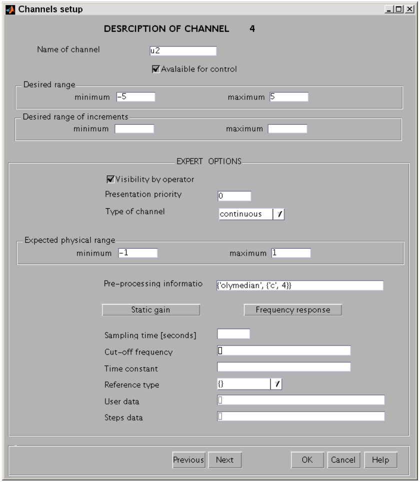
$u_2 \in [-5, 5]$



Conclusion

- Jobcontrol toolbox was implemented
- Method of adaptive LQG controller design was given
  - Replaces manual tuning by automated one
  - Multidimensional controller
- Online stopping rules
  - Speeding up the Monte Carlo evaluation

Jobcontrol GUI - Channel Description



Jobcontrol GUI - Results Display