Active fault detection and dual control in multiple model framework

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Introduction

- The simultaneous problem of active fault detection and control is discussed.
- The problem is formulated as an extension of active fault detection.
- The computational complexity is reduced using the multiple models approach and rolling horizon technique.

From passive fault detection to active fault detection and control

Passive fault detection

- Available data $\mathbf{z}_k = [\mathbf{u}_k, \mathbf{y}_k]$ are used passively.
- Design methods are well established and quite simple.
- The quality of decision is influenced just by the quality of model and used method.

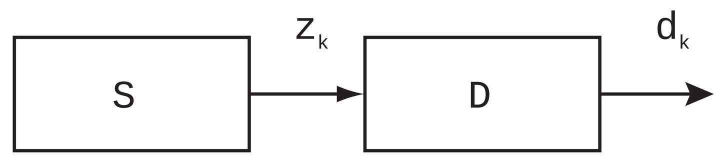

Figure: Passive fault detection

Active fault detection

- The input signal is designed to improve detection.
- The quality of decision is better due to probing.
- The design of active detector is more complex and the control is not incorporated.

Active fault detection and control

- The observed system is actively probed and controlled.
- The quality of decision is better with respect to passive detection, but it is usually worse with respect to active detection because of control aim.
- Computational complexity is further increased.

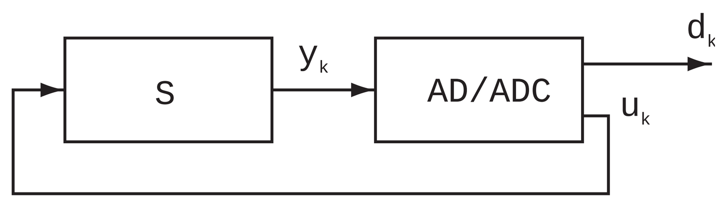

Figure: Active fault detection and possibly control

Problem formulation

Description of the system S on the finite horizon F

- The problem is considered on the finite horizon F.
- The system is described at each time step $k \in \mathcal{T} = \{0, 1, \ldots, F\}$ by the discrete-time linear Gaussian model

$$\mathbf{x}_{k+1} = A(\mu_k)\mathbf{x}_k + B(\mu_k)\mathbf{u}_k + G(\mu_k)\mathbf{w}_k, \quad (1a)$$
$$\mathbf{y}_k = C(\mu_k)\mathbf{x}_k + H(\mu_k)\mathbf{v}_k. \quad (1b)$$

- Switching between models is described by the Markov chain with transition probabilities

$$P_{i,j} = P(\mu_{k+1} = j | \mu_k = i). \quad (2)$$

- $\mathbf{y}_k \in \mathcal{R}^{n_y}$ is the output
- $\mathbf{u}_k \in \mathcal{U}_k \subseteq \mathcal{R}^{n_u}$ is the input
- $\bar{\mathbf{x}}_k = [\mathbf{x}_k, \mu_k]$ is the state
  - $\mathbf{x}_k \in \mathcal{R}^{n_x}$ is the common state of Gaussian models
  - $\mu_k \in \mathcal{M} = \{1, \ldots, N\}$ is the index denoting Gaussian model in effect at time step k
- $\mathbf{w}_k \in \mathcal{R}^{n_x}$ and $\mathbf{v}_k \in \mathcal{R}^{n_y}$ are mutually independent zero-mean white Gaussian noises with identity covariance matrices
- Initial condition $\mathbf{x}_0$ has Gaussian distribution with mean-value $\hat{\mathbf{x}}_k'$ and covariance matrix $P'_{x,0}$. Initial index of model μ_0 is described by the probability function $P(\mu_0)$.

Active detector and controller

$$\begin{bmatrix} d_k \\ \mathbf{u}_k \end{bmatrix} = \rho_k\left(\mathbf{I}_0^k\right), \quad k \in \mathcal{T}, \quad (3)$$

ρ_k is unknown function, $\mathbf{I}_0^k = [\mathbf{y}_0^k, \mathbf{u}_0^{k-1}, d_0^{k-1}]$ is available information and d_k is decision telling which model most likely describes the behavior of the system **S**. Note that $\mathbf{y}_0^k$ represents the whole sequence of the variable from time step 0 to k.

Criterion

$$J(\rho_0^F) = \mathrm{E}\left\{\sum_{k=0}^{F} L_k^{\mathrm{d}}(\mu_k, d_k) + \alpha_k L_k^{\mathrm{c}}(\mathbf{x}_k, \mathbf{u}_k)\right\}, \quad (4)$$

- $L_k^{\mathrm{d}}(\mu_k, d_k)$ is a scalar non-negative function that penalizes wrong decisions
- $L_k^{\mathrm{c}}(\mathbf{x}_k, \mathbf{u}_k)$ is a scalar non-negative function that penalizes the state and input
- α_k is a weighting coefficient

Information processing strategies

- Open loop – uses only a priori information.
- Open loop feedback – uses a priori information and data received up to the current time step.
- Closed loop – besides a priori information and data received up to the current time step takes into account that future data will be obtained.

Aim

Find active fault detector and controller (i.e. functions ρ_0^F) that minimizes criterion (4) given constraints (1) and (2) using closed loop information processing strategy.

Optimal active fault detector and controller

Backward recursive equation

$$V_k^*\left(\mathbf{y}_0^k, \mathbf{u}_0^{k-1}\right) = \min_{d_k \in \mathcal{M}} \mathrm{E}\left\{L_k^{\mathrm{d}}(d_k, \mu_k) | \mathbf{y}_0^k, \mathbf{u}_0^{k-1}, d_k\right\} + \min_{\mathbf{u}_k \in \mathcal{U}_k} \mathrm{E}\left\{\alpha_k L_k^{\mathrm{c}}(\mathbf{x}_k, \mathbf{u}_k) + V_{k+1}^*\left(\mathbf{y}_0^{k+1}, \mathbf{u}_0^k\right) | \mathbf{y}_0^k, \mathbf{u}_0^k\right\}, \quad (5)$$

The initial condition is $V_{F+1}^* = 0$ and the value of criterion is $J^* = J\left(\rho_0^{F*}\right) = \mathrm{E}\left\{V_0^*(\mathbf{y}_0)\right\}$.

Optimal active fault detector and controller

$$d_k^* = \sigma_k^*\left(\mathbf{y}_0^k, \mathbf{u}_0^{k-1}\right) = \arg\min_{d_k \in \mathcal{M}} \mathrm{E}\left\{L_k^{\mathrm{d}}(d_k, \mu_k) | \mathbf{y}_0^k, \mathbf{u}_0^{k-1}, d_k\right\}, \quad (6)$$

$$\mathbf{u}_k^* = \gamma_k^*\left(\mathbf{y}_0^k, \mathbf{u}_0^{k-1}\right) = \arg\min_{\mathbf{u}_k \in \mathcal{U}_k} \mathrm{E}\left\{\alpha_k L_k^{\mathrm{c}}(\mathbf{x}_k, \mathbf{u}_k) + V_{k+1}^*\left(\mathbf{y}_0^{k+1}, \mathbf{u}_0^k\right) | \mathbf{y}_0^k, \mathbf{u}_0^k\right\}, \quad (7)$$

Remarks on the optimal solution

- The optimal decision d_k^* and optimal input $\mathbf{u}_k^*$ are independent at time step k.
  - The optimal decision d_k^* minimizes average cost at time step k.
  - The optimal input $\mathbf{u}_k^*$ minimizes average future costs incurred by wrong decisions.

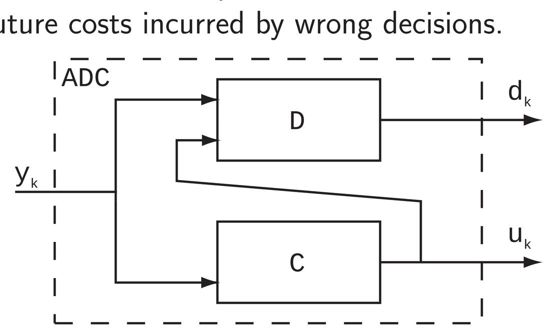

Figure: Internal structure of active fault detector and controller

- The conditional probability $P\left(\mu_k | \mathbf{y}_0^k, \mathbf{u}_0^{k-1}\right)$ and conditional probability density function (pdf) $p\left(\mathbf{y}_{k+1} | \mathbf{y}_0^k, \mathbf{u}_0^k\right)$ have to be computed using a nonlinear filter before the expectations in (5) can be evaluated.
- A solution of backward recursive equation can not be expressed in a closed form and approximative techniques for state estimation and solution of backward recursive equation have to be used.

Suboptimal active fault detector and controller

Nonlinear state estimation

- The optimal nonlinear filter consists of exponentially growing number of Kalman filters.
- The complexity can be reduced e.g. using merging pdf's that correspond to model sequences μ_0^k with the same terminal model sequence μ_{k-l}^k

$$P\left(\mu_{k-l}^k | \mathbf{y}_0^k, \mathbf{u}_0^{k-1}\right) = \sum_{\mu_0^{k-l-1}} P\left(\mu_0^k | \mathbf{y}_0^k, \mathbf{u}_0^{k-1}\right), \quad (8)$$

$$p\left(\mathbf{x}_k | \mathbf{y}_0^k, \mathbf{u}_0^{k-1}, \mu_{k-l}^k\right) = \sum_{\mu_0^{k-l-1}} \beta\left(\mu_0^k\right) p\left(\mathbf{x}_k | \mathbf{y}_0^k, \mathbf{u}_0^{k-1}, \mu_0^k\right), \quad (9) \qquad \beta\left(\mu_0^k\right) = \frac{P\left(\mu_0^k | \mathbf{y}_0^k, \mathbf{u}_0^{k-1}\right)}{P\left(\mu_{k-l}^k | \mathbf{y}_0^k, \mathbf{u}_0^{k-1}\right)}. \quad (10)$$

Solution of backward recursive equation

- Approximative solution is based on the rolling horizon technique.
- Numerical optimization is performed over a shorter horizon F_a and the Bellman function $V_{k+F_a+1}^*$ is approximated by $\bar{V}_{k+F_a+1}^* = 0$.

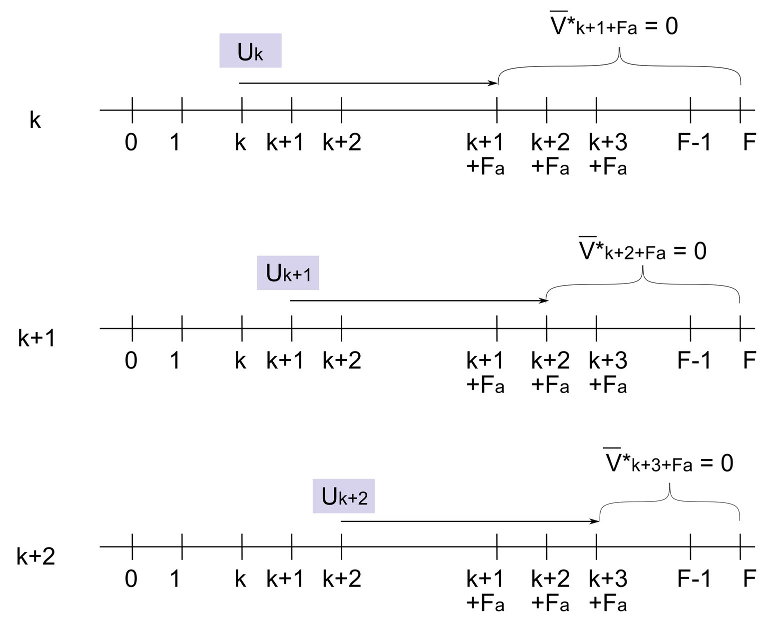

Figure: Rolling horizon technique

Illustrative example of active fault detection and control

- The time horizon is $F = 40$ and the observed/controlled system is described as follows

$$\mu_k = 1: \; x_{k+1} = \begin{bmatrix} 0.0707 & -0.4826 \\ 0.8579 & 0.4996 \end{bmatrix} x_k + \begin{bmatrix} 0.2145 \\ 0.2224 \end{bmatrix} u_k + \begin{bmatrix} 0.003 & 0 \\ 0 & 0.003 \end{bmatrix} w_k,$$
$$y_k = \begin{bmatrix} 0 & 2.25 \end{bmatrix} x_k + 0.005 v_k,$$
$$\mu_k = 2: \; x_{k+1} = \begin{bmatrix} 0.0707 & -0.4826 \\ 0.8579 & 0.4996 \end{bmatrix} x_k + \begin{bmatrix} 1220.1973 \\ 1230.2104 \end{bmatrix} u_k + \begin{bmatrix} 0.003 & 0 \\ 0 & 0.003 \end{bmatrix} w_k,$$
$$y_k = \begin{bmatrix} 0 & 2.25 \end{bmatrix} x_k + 0.005 v_k.$$

The initial state x_0 has mean $x_0' = [0 \; 0]^T$ and covariance matrix $P'_{x,0} = 0.1\mathbf{I}$. The initial probabilities of models are $P(\mu_0 = 1) = P(\mu_0 = 2) = 0.5$, the transition probabilities are $P_{1,1} = P_{2,2} = 0.95$ and $P_{1,2} = P_{2,1} = 0.05$. The set of admissible inputs is $\mathcal{U}_k = \{-0.8, -0.7, -0.6, -0.5, -0.4, -0.3, -0.2, -0.15, -0.1, 0, 0.1, 0.15, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8\}$.

- Cost functions $L_k^{\mathrm{d}}(\mu_k, d_k)$ and $L_k^{\mathrm{c}}(\mathbf{x}_k, u_k)$ are given by

$$L_k^{\mathrm{d}}(\mu_k, d_k) = \begin{cases} 0 & if \;\; \mu_k = d_k \\ 1 & if \;\; \mu_k \neq d_k \end{cases}, \quad (11) \qquad L_k^{\mathrm{c}}(\mathbf{x}_k, u_k) = \left[\mathbf{x}_k^{\mathrm{r}} - \mathbf{x}_k\right]^T Q_k \left[\mathbf{x}_k^{\mathrm{r}} - \mathbf{x}_k\right] + r_k \mathbf{u}_k^2, \quad (12)$$

where $Q_k = \mathbf{I}$, $r_k = 0.001$, and $\alpha_k = 8$.

- The reference state $\mathbf{x}_k^{\mathrm{r}} = \left[x_{1,k}^{\mathrm{r}}, x_{2,k}^{\mathrm{r}}\right]^T$ is defined as follows: $x_{1,k}^{\mathrm{r}} = 0$ for all $k \in \mathcal{T}$ and $x_{2,k}^{\mathrm{r}}$ is the rectangular signal with amplitude ± 0.2667 and period 40 steps.
- Example of a simulation run and Monte Carlo simulation results

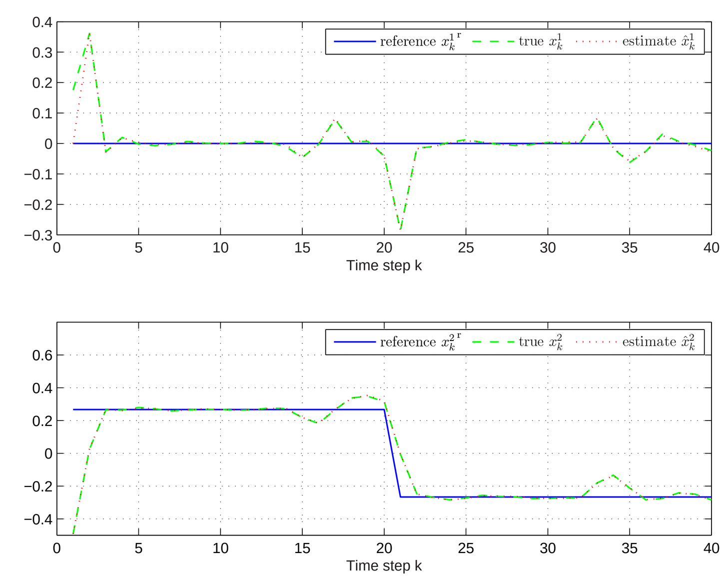

Figure: Reference, true state and its estimate for **ADC**

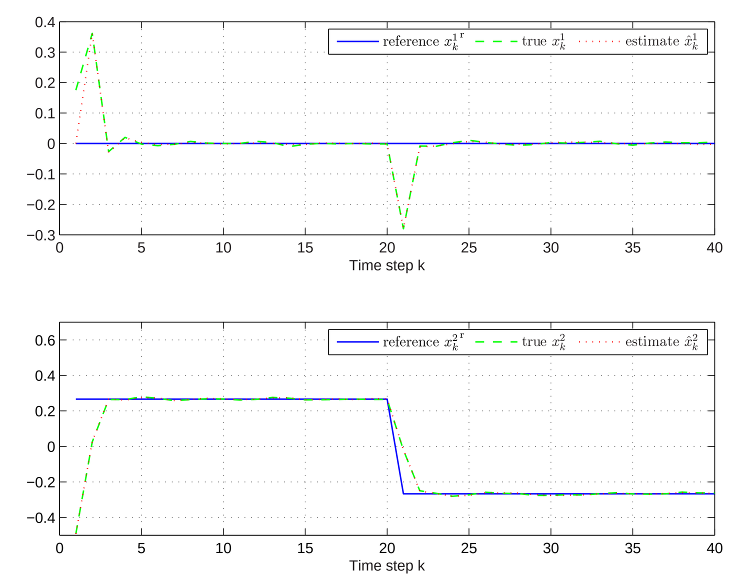

Figure: Reference, true state and its estimate for **HCEC**

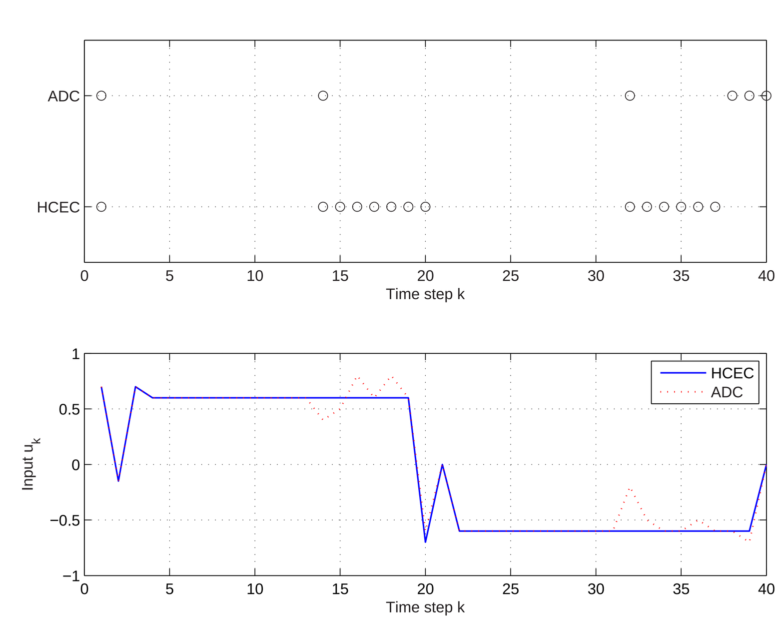

Figure: Upper: Indication of wrong decisions. Bottom: Input trajectories

Table: Monte Carlo simulation results

	N_{WD}	$SE_{x_k^{\mathrm{r}} - x_k}$
ADC	25.47	5.8890
HCEC	30.58	5.9047

N_{WD} is an average number of wrong decisions in percents over the considered horizon and $SE_{x_k^{\mathrm{r}} - x_k}$ is an average square error

- Summary of the example
  - The static gain is the same for both models and thus the control is satisfactory in steady-state.
  - Whenever the active fault detector and controller lacks sufficient information it automatically generates probing signal.
  - The quality of the decision is better and quality of control is just slightly degraded as follows from the table.

Conclusion

- The problem of active fault detection and control was considered in multiple model framework.
- The multiple models can be used simply to describe fault-free and faulty behavior of the system.
- The general solution given by the backward recursive equation was approximated in the state estimation and optimization steps.
- The illustrative example shows a situation where the simultaneous design of active fault detector and controller brings improvement with respect to passive approach and separate design of the detector and controller.