PARTICLE FILTER ADAPTATION BASED ON EFFICIENT SAMPLE SIZE

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Outline

Motivation

- The problem how to specify a suitable sample size of the particle filter is usually overlooked.
- Particle filter usually considers unvarying sample size while estimate quality varies.

- It would be advantageous to know that certain number of samples of the particle filter provide the same estimate quality as a given number of samples drawn from the filtering pdf directly. *(although the filtering pdf is unknown and being searched)*

State estimation

Consider a discrete time stochastic system:

$$\mathbf{x}_{k+1} = \mathbf{f}_k(\mathbf{x}_k, \mathbf{w}_k), \quad k = 0, 1, 2, \ldots$$
$$\mathbf{z}_k = \mathbf{h}_k(\mathbf{x}_k, \mathbf{v}_k), \quad k = 0, 1, 2, \ldots$$

- $\mathbf{x}_k$ is nx dimensional state vector with $p(\mathbf{x}_0)$
- $\mathbf{z}_k$ is nz dimensional measurement vector
- $\mathbf{w}_k$ is white noise with known $p(\mathbf{w}_k)$
- $\mathbf{v}_k$ is white noise with known $p(\mathbf{v}_k)$
- $\mathbf{f}_k(\mathbf{x}_k, \mathbf{w}_k)$ and $\mathbf{h}_k(\mathbf{x}_k, \mathbf{v}_k)$ are known vector functions

The aim of state estimation here is to find the filtering pdf $p(\mathbf{x}_k|\mathbf{z}^k)$, where $\mathbf{z}^k = [\mathbf{z}_0^{\mathrm{T}}, \ldots \mathbf{z}_k^{\mathrm{T}}]^{\mathrm{T}}$

Particle filter

- General solution of the filtering problem is given by the Bayesian Recursive Relations (BRR).
- Closed form solution of the BRR is available for a few special cases only (e.g. linear Gaussian systems).
- Thus an *approximate* solution of the BRR is usually searched.
- Solution of the BRR by the particle filter is based on approximating the filtering pdf by a set of samples (particles) and corresponding weights as

$$r_{N_k}(\mathbf{x}_k|\mathbf{z}^k) = \sum_{i=1}^{N_k} \omega_k^{(i)} \delta(\mathbf{x}_k - \mathbf{x}_k^{(i)}),$$

$\mathbf{x}_k^{(i)}$ - samples, $\omega_k^{(i)}$ - normalized weights,
δ - the Dirac function ($\delta(\mathbf{x}) = 0$ for $\mathbf{x} \neq 0$, $\int \delta(\mathbf{x})\mathrm{d}\mathbf{x} = 1$).

Initialization

draw $\{\mathbf{x}_0^{(i)}\}_{i=1}^{N_0}$ from $p(\mathbf{x}_0|\mathbf{z}^{-1})$ and set $\omega_0^{(i)} \propto p(\mathbf{z}_0|\mathbf{x}_0^{(i)})$

$k = 0$

Resampling

generate $\{\mathbf{x}_k^{*(i)}\}_{i=1}^{N_k}$ by resampling with replacement from $\{\mathbf{x}_k^{(i)}\}_{i=1}^{N_k}$ according to $P(\mathbf{x}_k^{*(i)} = \mathbf{x}_k^{(i)}) = \omega_k^{(i)}$

$k = k + 1$

Sampling

draw new samples $\{\mathbf{x}_k^{(i)}\}_{i=1}^{N_k}$ from $\pi(\mathbf{x}_k|\mathbf{x}_{k-1}^{*(1:N_{k-1})}, \mathbf{z}_k)$

Weighting

compute the weights

$$\{\omega_k^{(i)}\}_{i=1}^{N_k} \propto \frac{p(\mathbf{z}_k|\mathbf{x}_k^{(i)})p(\mathbf{x}_k^{(i)}|\mathbf{x}_{k-1}^{*(i)})}{\pi(\mathbf{x}_k^{(i)}|\mathbf{x}_{k-1}^{*(i)},\mathbf{z}_k)}$$

Sampling pdf's

prior sampling pdf (Gordon et al. 1993)

$$\pi(\mathbf{x}_k|\mathbf{x}_{k-1}^{(1:\nu)},\mathbf{z}_k)=\sum_{i=1}^{\nu}\frac{1}{\nu}p(\mathbf{x}_k|\mathbf{x}_{k-1}^{(i)})$$

optimal sampling pdf (Liu Chen 1998)

$$\pi(\mathbf{x}_k|\mathbf{x}_{k-1}^{(1:\nu)},\mathbf{z}_k)=\sum_{i=1}^{\nu}\frac{1}{\nu}p(\mathbf{x}_k|\mathbf{x}_{k-1}^{(i)},\mathbf{z}_k)$$

auxiliary sampling pdf (Pitt and Shephard 1999)

$$\pi(\mathbf{x}_k|\mathbf{x}_{k-1}^{(1:\nu)},\mathbf{z}_k)=\sum_{i=1}^{\nu}\mathrm{v}(\mathbf{x}_{k-1}^{(i)},\mathbf{z}_k)p(\mathbf{x}_k|\mathbf{x}_{k-1}^{(i)}),$$
$$\mathrm{v}(\mathbf{x}_{k-1}^{(i)},\mathbf{z}_k)\propto p(\mathbf{z}_k|\mu_k^{(i)})$$

Sample size

- a key parameter significantly affecting estimate quality
- usually time invariant, i.e. $N_k = N$
- suitable specification of time-invariant sample size addressed in

Šimandl M. and Straka O.
Nonlinear estimation by particle filters and Cramér Rao bound.
Proceedings of the 15th triennial world congress of IFAC, 79-84, 2002.

sample size is set according to a distance between the mean square error and the Cramér Rao bound

Current adaptive approaches

- Fox D.
  KLD sampling: Adaptive particle filter for mobile robot localization.
  Advances in Neural Information Processing Systems, 2001.

probability that the KL distance between the true filtering pdf and the approximate filtering pdf is lower than ε is studied

- Koller D. and Fratkina R..
  Using learning for approximation in stochastic processes.
  Proc. of 15th Int. Conf. on Machine Learning, 287–295, 1998.

sum of likelihoods instead of sample size is kept constant

- Straka O. and Šimandl M.
  Sample size adaptation for particle filters.
  Proceedings of the 16th IFAC symposium on Automatic Control in Aerospace, 437–442, 2003.

assessing position of the generated samples according to the current mesurement and sampling until a criterion is met

Sample size adaptation based on ESS

Idea:

to preserve **Efficient sample size (ESS)** and to adapt sample size accordingly

Efficient sample size (ESS):

The ESS describes the number of samples drawn from the filtering pdf necessary to attain the same estimate quality as N_k samples drawn from the sampling pdf.

$$\mathrm{ESS}_k(N_k) = N_k \frac{1}{1 + d(\pi, p)}$$

$d(\pi, p)$ is the χ^2 distance between $\pi(\mathbf{x}_k | \mathbf{x}_{k-1}^{(1:N_{k-1})}, \mathbf{z}_k)$ and $p(\mathbf{x}_k | \mathbf{z}^k)$

Sample size adaptation based on ESS (cont)

ESS can be derived in the following form

$$\mathrm{ESS}_k(N_k) = N_k \frac{1}{\int \frac{p(\mathbf{x}_k|\mathbf{z}_k)^2}{\pi(\mathbf{x}_k|\mathbf{x}_{k-1}^{(1:N_{k-1})},\mathbf{z}_k)} \mathrm{d}\mathbf{x}_k}$$

Sample size can be then specified as

$$N_k = \lceil N_k^* \int \frac{[p(\mathbf{x}_k|\mathbf{z}^k)]^2}{\pi(\mathbf{x}_k|\mathbf{x}_{k-1}^{(1:N_{k-1})})} \mathrm{d}\mathbf{x}_k \rceil$$

N_k^* is a prespecified ESS

Sample size adaptation based on ESS (cont)

The filtering pdf $p(\mathbf{x}_k|\mathbf{z}^k)$ is unknown but can be computed using the BRR as:

$$p(\mathbf{x}_k|\mathbf{z}^k) = C^{-1} p(\mathbf{z}_k|\mathbf{x}_k) \sum_{i=1}^{N_{k-1}} \frac{1}{N_{k-1}} p(\mathbf{x}_k|\mathbf{x}_{k-1}^{(i)})$$

Sample size for the prior sampling pdf is

$$N_k = N_k^* \left\lceil \frac{\int [p(\mathbf{z}_k|\mathbf{x}_k)]^2 \sum_{i=1}^{N_{k-1}} \frac{1}{N_{k-1}} p(\mathbf{x}_k|\mathbf{x}_{k-1}^{(i)}) \mathrm{d}\mathbf{x}_k}{[\int p(\mathbf{z}_k|\mathbf{x}_k) \sum_{i=1}^{N_{k-1}} \frac{1}{N_{k-1}} p(\mathbf{x}_k|\mathbf{x}_{k-1}^{(i)}) \mathrm{d}\mathbf{x}_k]^2} \right\rceil$$

the integrals - usually intractable for nonlinear or nongaussian systems
- can be approximated using the MC method
- as $N_k \geq N_k^*$, the first N_k^* samples can be used for approximation

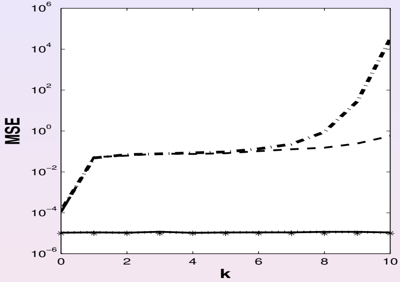
System

$$x_{k+1} = x_k - 0.2 \cdot x_k^2 + e_k \qquad p(e_k) = \mathcal{N}\{e_k : 0, 0.1\}$$
$$z_k = x_k + v_k \qquad p(v_k) = \mathcal{N}\{v_k : 0, 0.0001\}$$
$$p(x_0) = \mathcal{N}\{x_0 : 0, 0.001\}$$

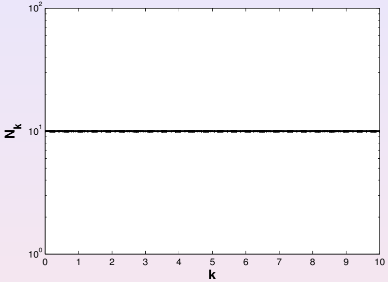
Particle filter

- Sampling pdf
  - optimal sampling pdf
  - prior sampling pdf
  - auxiliary sampling pdf
  - filtering pdf
- $k = 0, 1, \ldots 10$, $N_k^* = 10$
- Criterion - MSE $\Pi_k = \mathrm{E}[x_k - \hat{x}_k]^2$, 1000 simulations

Results without sample size adaptation



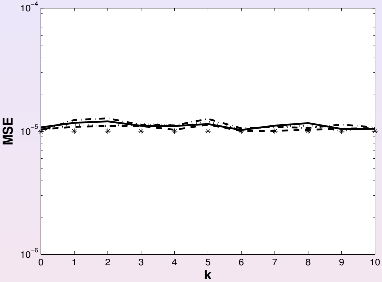
Mean square error



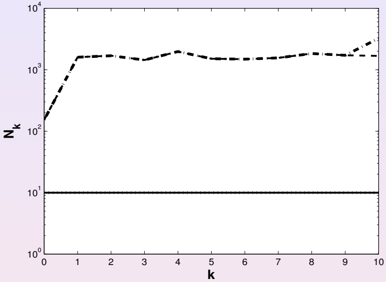
Sample size

solid - **optimal** sampling pdf
dashed - **prior** sampling pdf
dot-dashed - **auxiliary** sampling pdf
dotted - **filtering** pdf
stars - **Cramér Rao bound**

Results with sample size adaptation



Mean square error



Sample size

solid - **optimal** sampling pdf

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Conclusion

- A new sample size adaptation technique was proposed
- It is based on preserving the Efficient sample size.
- The technique achieves the same MSE for all sampling pdf's.
- Enumeration of the adapted sample size introduces almost no extra computational overheads.