CALIBRATION OF AN ULTRASONIC COMPUTED-TOMOGRAPHY SYSTEM

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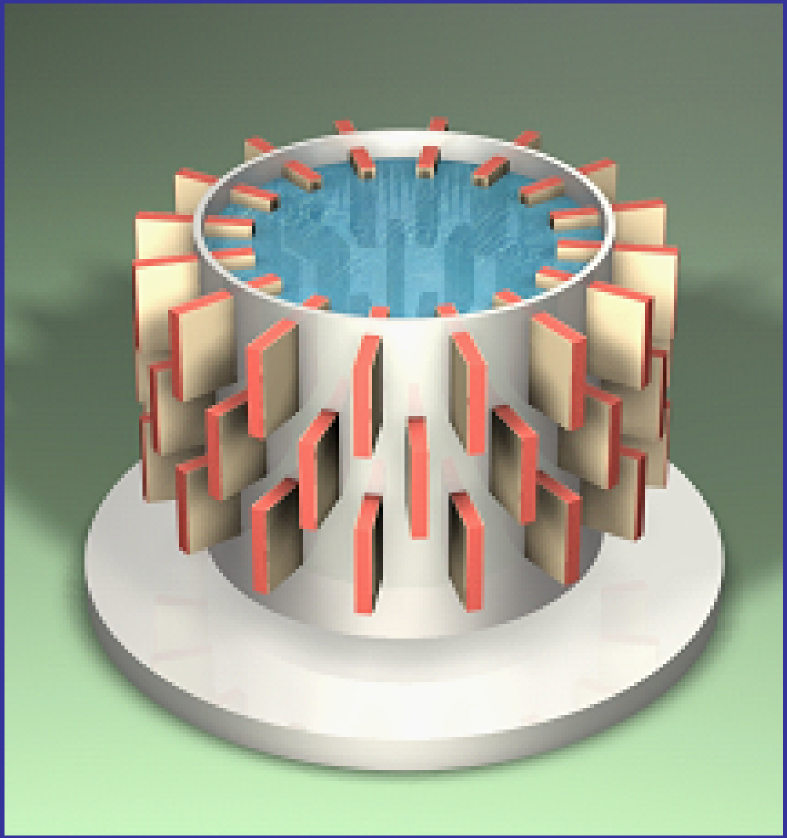
USCT system

consisting of thousands of transducers
in a water tank



ideally:

- all transducers have isotropic radiation
- all transducers are equally effective
- all transducers introduce zero (or at least uniform) delay
- all transducers are situated exactly at their expected positions

not fulfilled in the realistic case, thus calibration needed, determining:

- the (common) directional characteristic of all transducers
- efficiency of individual transducers
- individual signal delay in transducers
- real geometry of the measuring system (within few tenths of **λ,** i.e ~ 0.1 mm)

USCT system calibration:

- direct measurements
  - impractical or even not feasible

- special phantom based calibration
  - expensive, sensitive and fast decaying phantom
  - problematic phantom handling and positioning

- phantom-less calibration

  based only on „empty“ measurements (in homogeneous pure water)

  - first approach: spectral analysis of response signals (2005)

    yielding the directional characteristic and the vector of individual efficiencies
  - second approach: time-delay analysis of response signals (2007)

    yielding the individual transducers‘ delay and real geometry of the system

TRANSDUCER DIRECTIVITY AND EFFICIENCY CALIBRATION

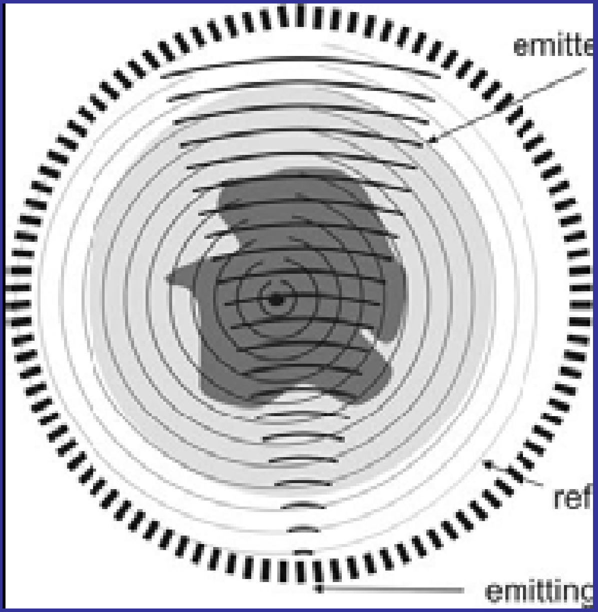
purpose:

to determine (via series of „empty“ measurements)

- angle- and frequency-dependent radiation function
- vector of individual (radiation and reception) efficiencies

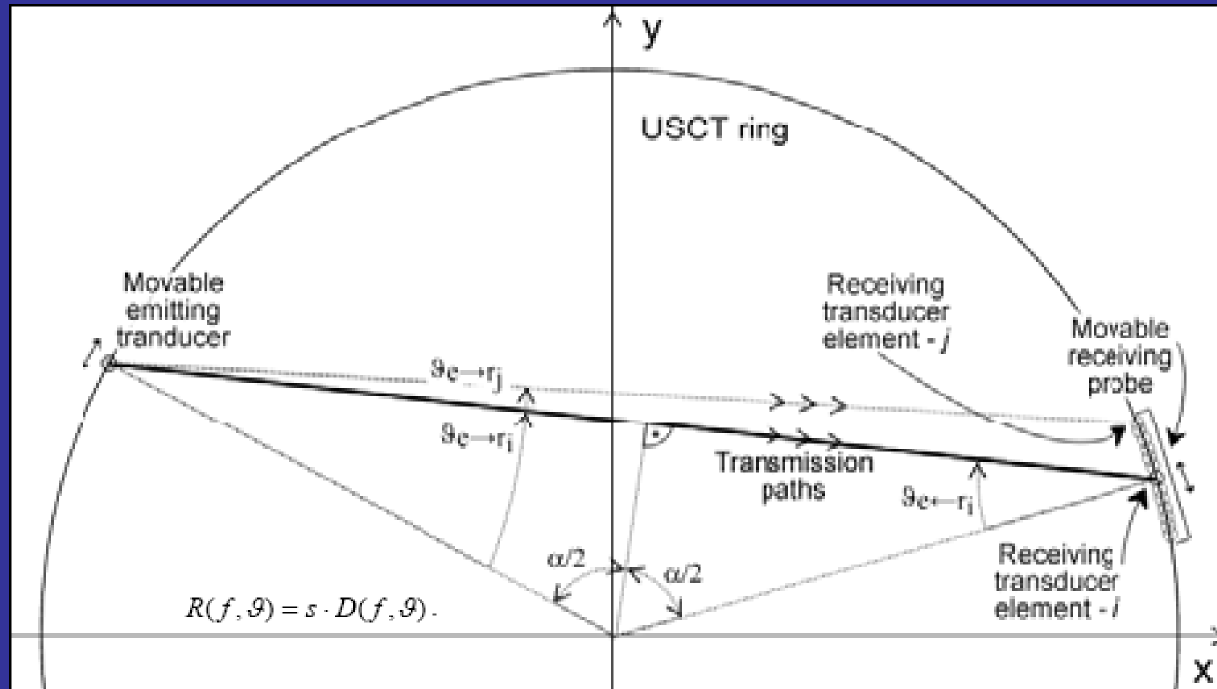
simplifying assumptions:

- linearity
- equal directivity of all transducers for both emitting and receiving,
- radiating efficiency = receiving sensitivity
- 2D (planar) case (so far) →
- limited number of physical transducers
- mirror symmetry of directivity



Equation system formulation

system geometry



Received signal equation

$$S_{e,r}(f) \cong R_e(f,\vartheta_{e\to r})\cdot R_r(f,\vartheta_{e\leftarrow r})$$

$$R(f,\vartheta) = s\cdot D(f,\vartheta).$$

Log-linearised equation system

$$\log\left(S_{e,r}(f)\right) = \log\left(R_e(f,\vartheta_{e\to r})\right) + \log\left(R_r(f,\vartheta_{e\leftarrow r})\right), \forall e,r$$

Available number of independent equations (given limitations of the measuring setup)

$$N_{r-pos}\cdot N_{r-el}\cdot N_{freq} = 91\cdot 16\cdot 64 = 93\,184$$

Number of unknown parameters

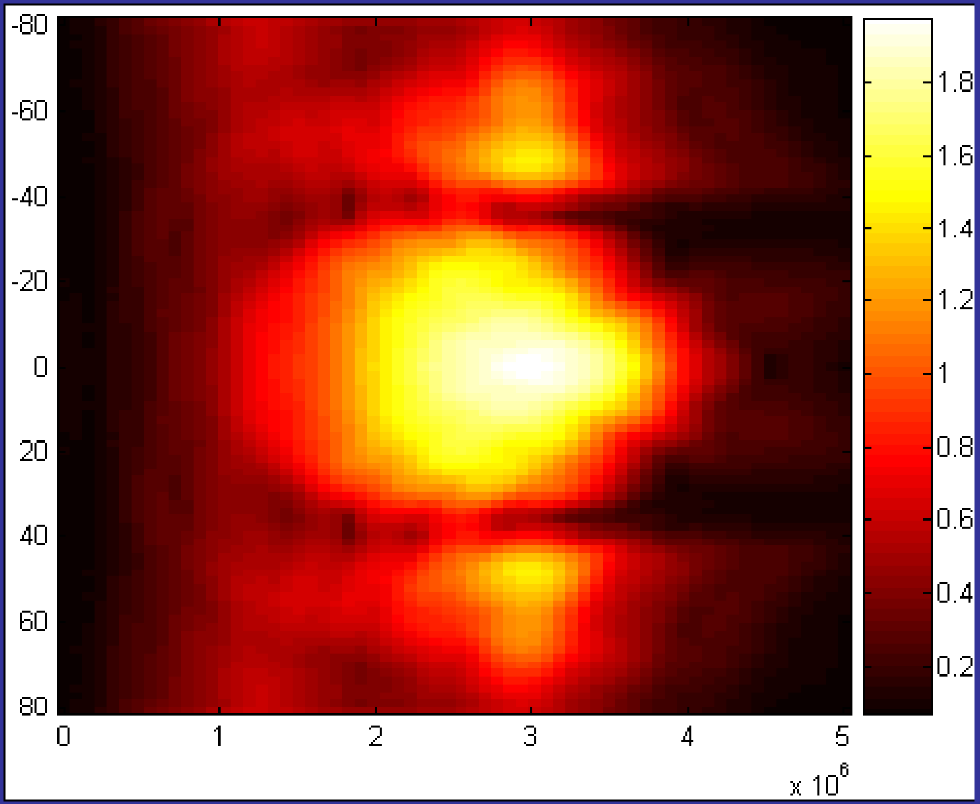
$$(1+N_{r-el})\cdot N_{freq}\cdot N_{ang} = (1+16)\cdot 64\cdot (16\cdot 91) = 1\,584\,128$$

Restricted number of equations thanks to assumptions $1+N_{r-el}+N_{freq}\cdot N_{ang} = 1+16+64\cdot(16\cdot 46) = 47.121$

Results of spectrum-based calibration

radiation function

theta [degrees] ↑

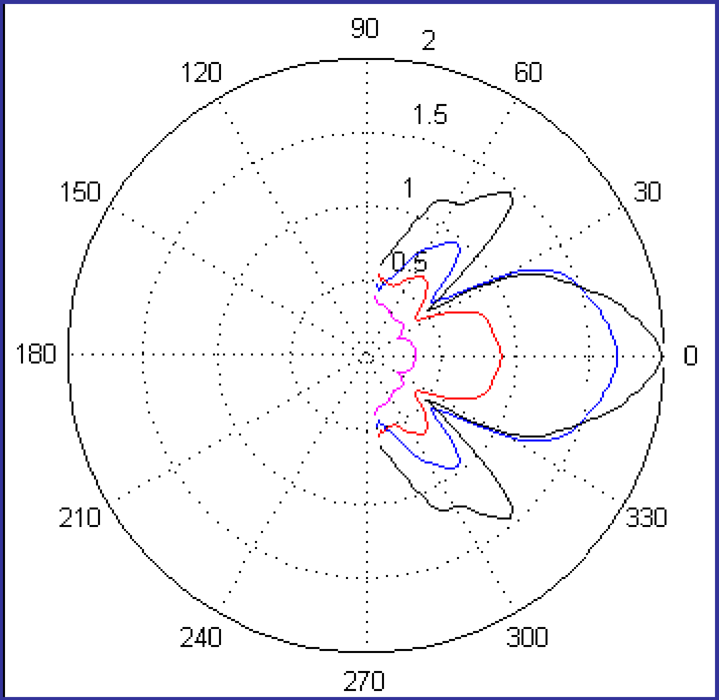


frequency [MHz] →

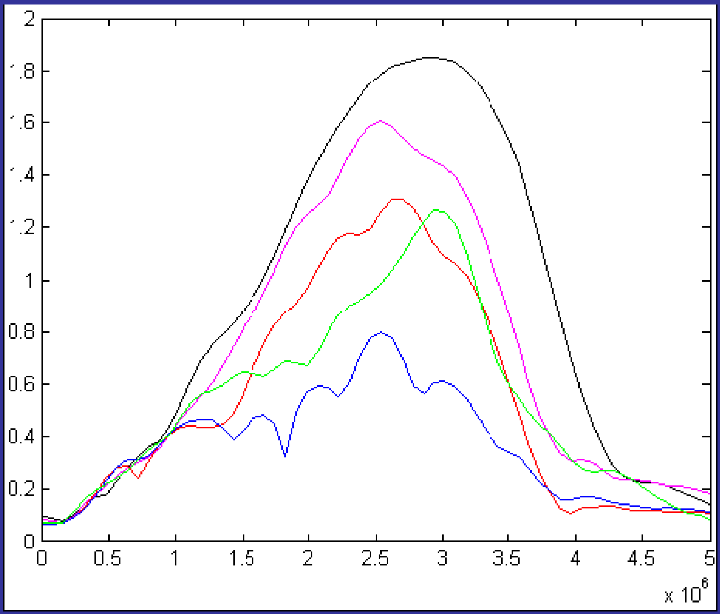
transducer efficiencies (for group of 16 transducers)

0.75 0.95 1.15 0.97 0.99 0.89 1.04 0.95 0.99 1.06 0.98 1.09 1.11 1.15

directional characteristics for different frequencies

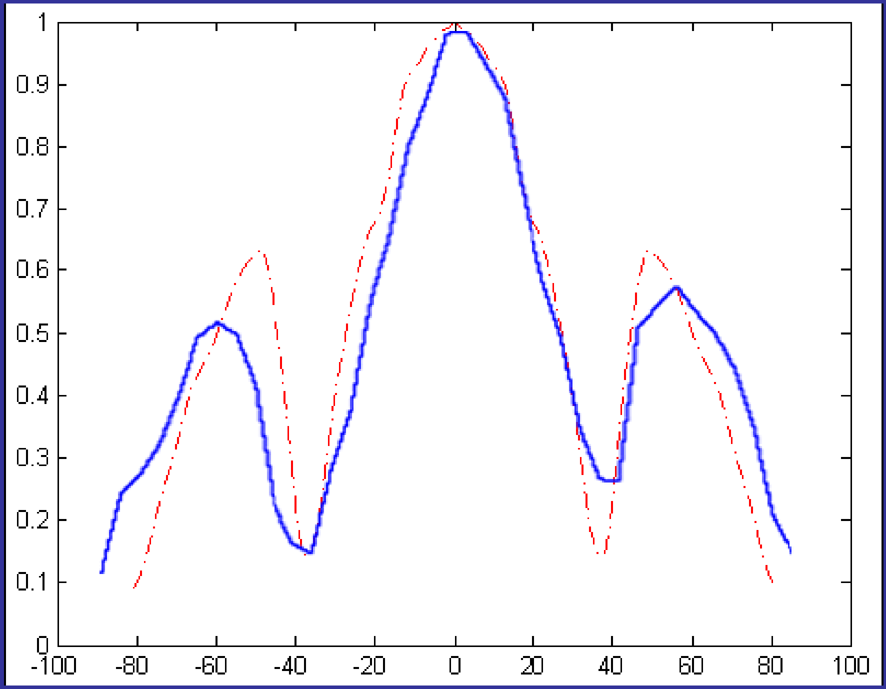


radiation function as dependent on f at different angles



Comparison of the calculated directivity
with the experimental measurement of a transducer field via a hydrophon

(solid – acustic pressure,
dashed – calculation, co-normalized)



theta ⟶

The comparison with the measurement shows a reasonable correlation – the calibration results thus may be acceptable for reconstruction attempts.

However, both the measured and calculated directivity functions violate the physics of an elementary radiator → another branch: exact physical simulation

TRANSDUCER POSITIONING AND DELAY CALIBRATION

purpose:

to determine (via series of „empty“ measurements)

- exact 3D positions of individual transducers
- signal delays of individual transducers

properties and limitations

- neither the sender nor the receiver positions and delays are known
  (only very rough initial estimate of geometry needed)
- the only assumption: the speed of ultrasound is known
- no absolute fixed point – spatial anchoring needed
- only the first response (shortest TOF) to be measured
  (a separate detection problem)
- only sums of both delays on a path available (sufficient)

Equation system formulation

Principle:

formulation of (time-of-flight + delays) equations for all possible paths
(similar to GPS formulation but without a fixed coordinate system)

TOA = TOF + delays:

$$TOA_{s,r} = TOF_{s,r} + \tau_s + \tau_r + \varepsilon_{s,r}$$

TOF itself: $$TOF_{s,r} = r_{s,r}/v = \sqrt{(x_s - x_r)^2 + (y_s - y_r)^2 + (z_s - z_r)^2}\Big/v$$

- a large-scale nonlinear problem
- locally linearised (Taylor series approximation):

$$TOF_{s,r} = \sqrt{(x_{s,0} + \Delta x_s - x_{r,0} - \Delta x_r)^2 + (y_{s,0} + \Delta y_s - y_{r,0} - \Delta y_r)^2 + (z_{s,0} + \Delta z_s - z_{r,0} - \Delta z_r)^2}\Big/v$$

$$\cong TOF_{s,r,0} + \frac{\partial TOF_{s,r}}{\partial x_s}\Delta x_s + \frac{\partial TOF_{s,r}}{\partial y_s}\Delta y_s + \frac{\partial TOF_{s,r}}{\partial z_s}\Delta z_s + \frac{\partial TOF_{s,r}}{\partial x_r}\Delta x_r + \frac{\partial TOF_{s,r}}{\partial y_r}\Delta y_r + \frac{\partial TOF_{s,r}}{\partial z_r}\Delta z_r$$

$$= \frac{d_{s,r,0}}{v} + \frac{x_{s,0} - x_{r,0}}{vd_{s,r,0}}\Delta x_s - \frac{x_{s,0} - x_{r,0}}{vd_{s,r,0}}\Delta x_r + \frac{y_{s,0} - y_{r,0}}{vd_{s,r,0}}\Delta y_s - \frac{y_{s,0} - y_{r,0}}{vd_{s,r,0}}\Delta y_r + \frac{z_{s,0} - z_{r,0}}{vd_{s,r,0}}\Delta z_s - \frac{z_{s,0} - z_{r,0}}{vd_{s,r,0}}\Delta z_r$$

overall difference between the predicted and detected time-of-arrival of each impulse:

$$\Delta TOA_{s,r} = \frac{x_{s0} - x_{r0}}{vd_{s,r,0}} \Delta x_s + \frac{y_{s0} - y_{r0}}{vd_{s,r,0}} \Delta y_s + \frac{z_{s0} - z_{r0}}{vd_{s,r,0}} \Delta z_s + \Delta \tau_s - \frac{x_{e0} - x_{r0}}{vd_{s,r,0}} \Delta x_r - \frac{y_{e0} - y_{r0}}{vd_{s,r,0}} \Delta y_r - \frac{z_{e0} - z_{r0}}{vd_{s,r,0}} \Delta z_r + \Delta \tau_r$$

corresponding over-determined linearised system of equations:

$$\boldsymbol{J}\Delta\boldsymbol{x} = \boldsymbol{r},$$

$$\Delta\boldsymbol{x} = [\Delta x_{s1}, \Delta y_{s1}, \Delta z_{s1}, \Delta\tau_{s1} \ldots \Delta x_{sN}, \Delta y_{sN}, \Delta z_{sN}, \Delta\tau_{sN}, \Delta x_{r1}, \Delta y_{r1}, \Delta z_{r1}, \Delta\tau_{r} \ldots \Delta x_{rM}, \Delta y_{rM}, \Delta z_{rM}, \Delta\tau_{rM}]^T$$

$$\boldsymbol{r} = [\Delta TOA_{s1,r1} \ldots \Delta TOA_{s1,rM}, \Delta TOA_{s2s,r1} \ldots \Delta TOA_{sNs,rM}]^T$$

its LMS solution

$$\Delta\boldsymbol{x}_k = (\boldsymbol{J}_k^T \boldsymbol{J}_k)^{-1} \boldsymbol{J}_k^T \boldsymbol{r}_k$$

(or Levenberg-Marquardt)

and finally update of the position- and delay vector after each iteration step

iteration: $\boldsymbol{x}_{k+1} = \boldsymbol{x}_k + \Delta\boldsymbol{x}_k$ → new residual and system matrix etc.

system is rank deficient – necessity for anchoring some x-components

Experimental verification

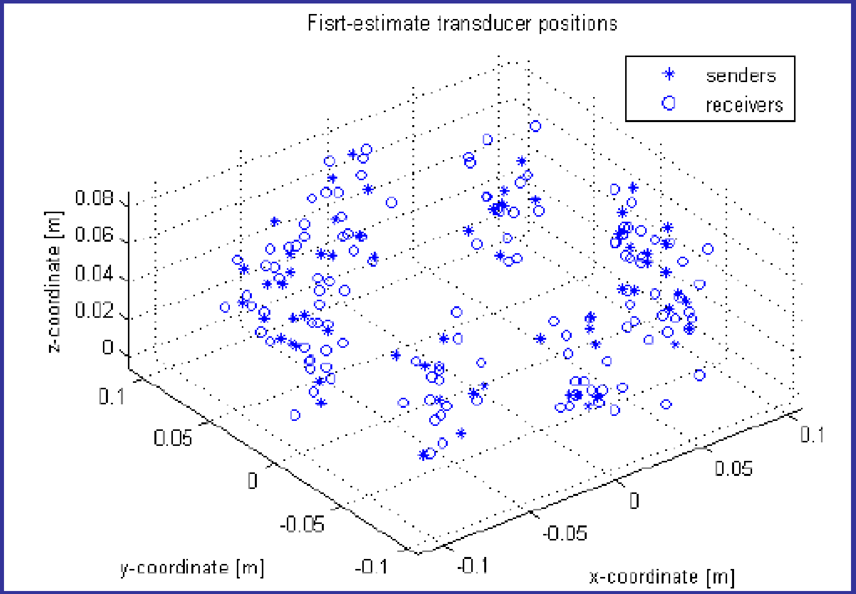
4. identification of simulated imprecise transducer positioning

   64 transmitters + 128 receivers
   → 768 unknowns, 8192 TOA measurements

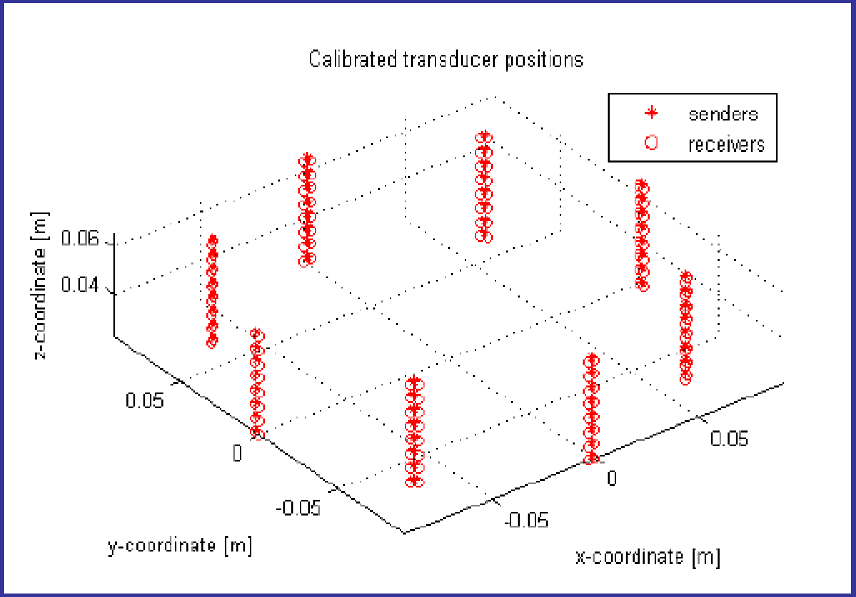
   convergence verified,

   further investigated:

   – range of convergence (related to variance of initial errors)
   – influence of noise

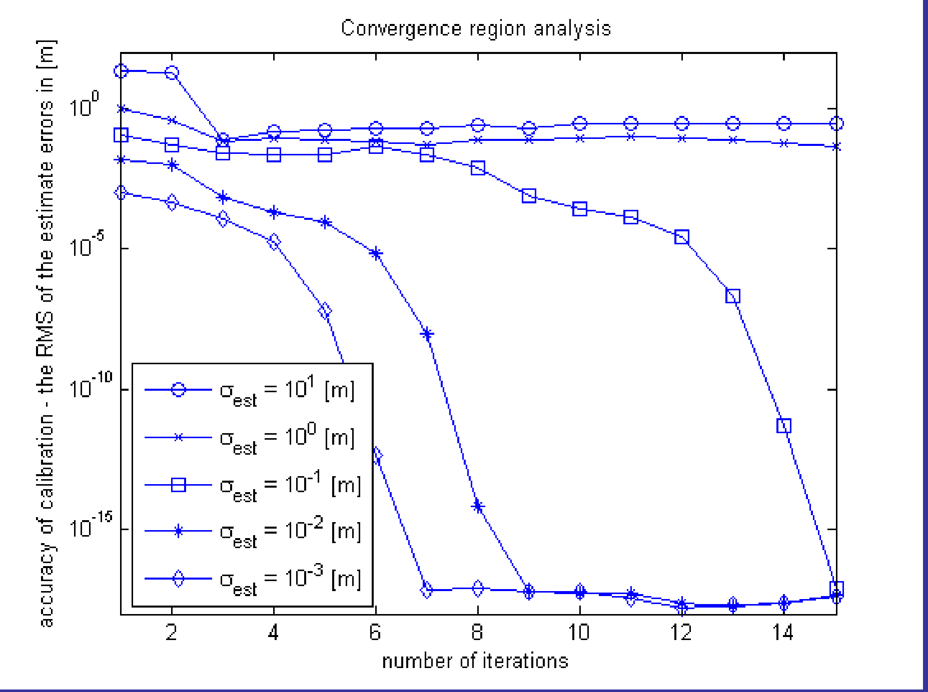


imprecisely measured positions

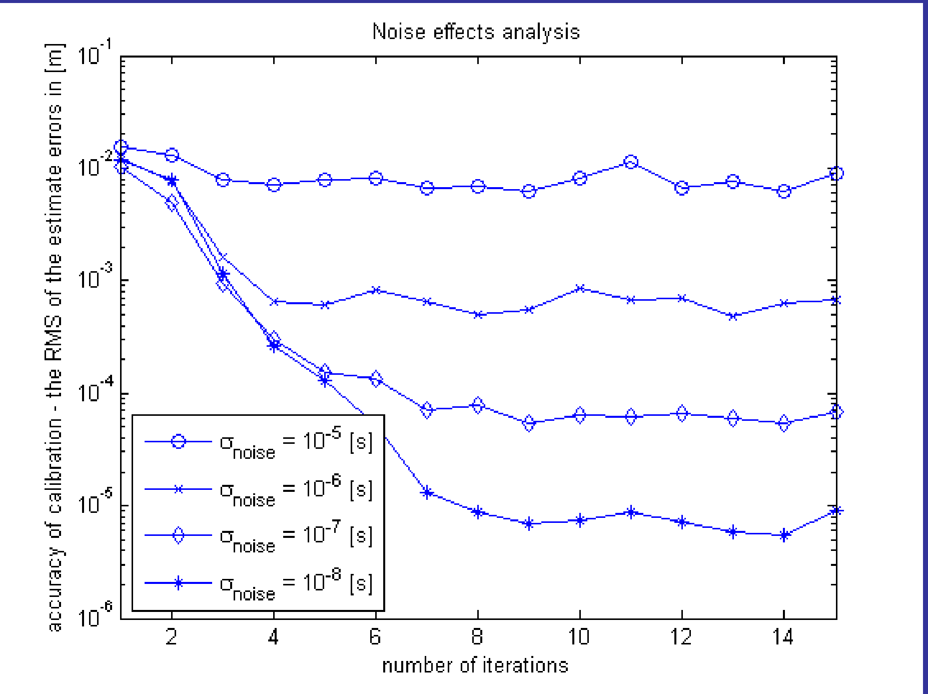


result of iteration ≈ ground truth

Influence of initial error size

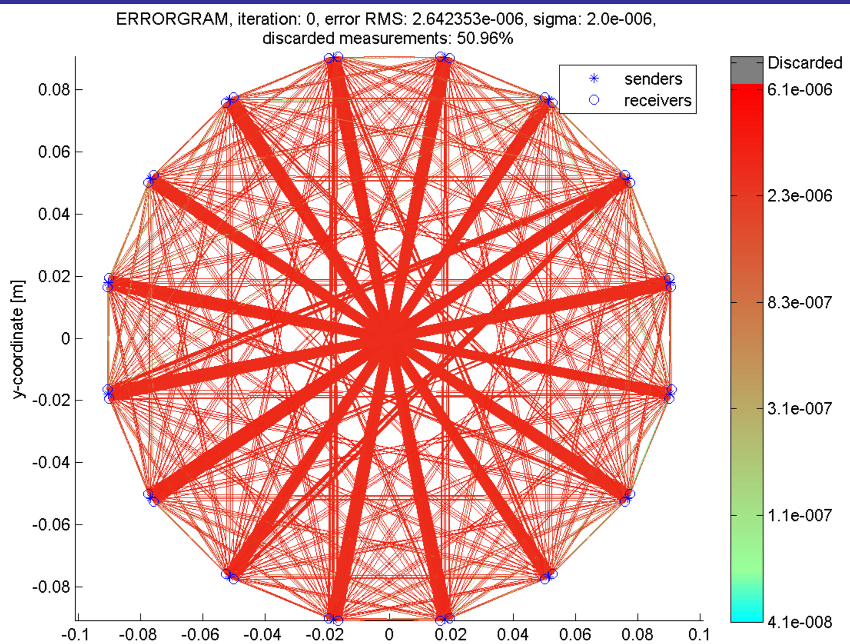


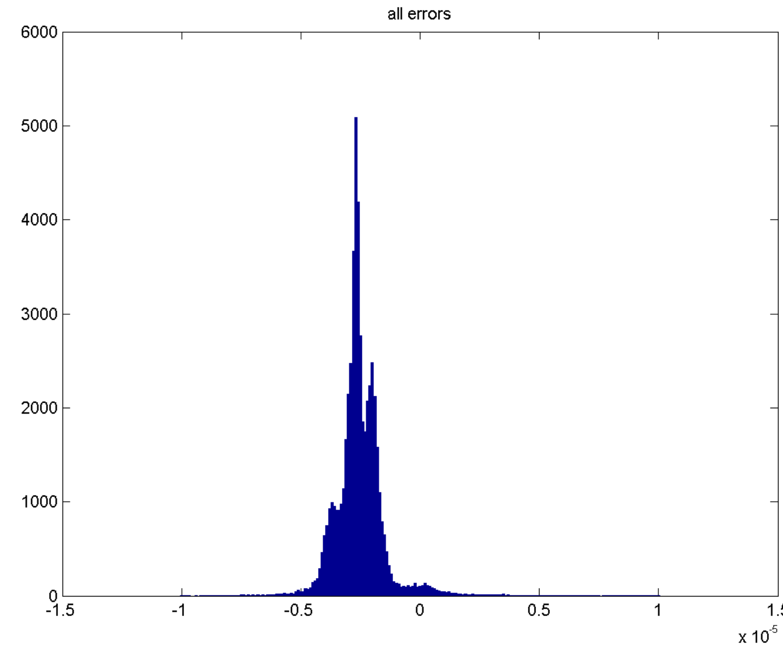
Influence of imprecission of time of arrival measurement



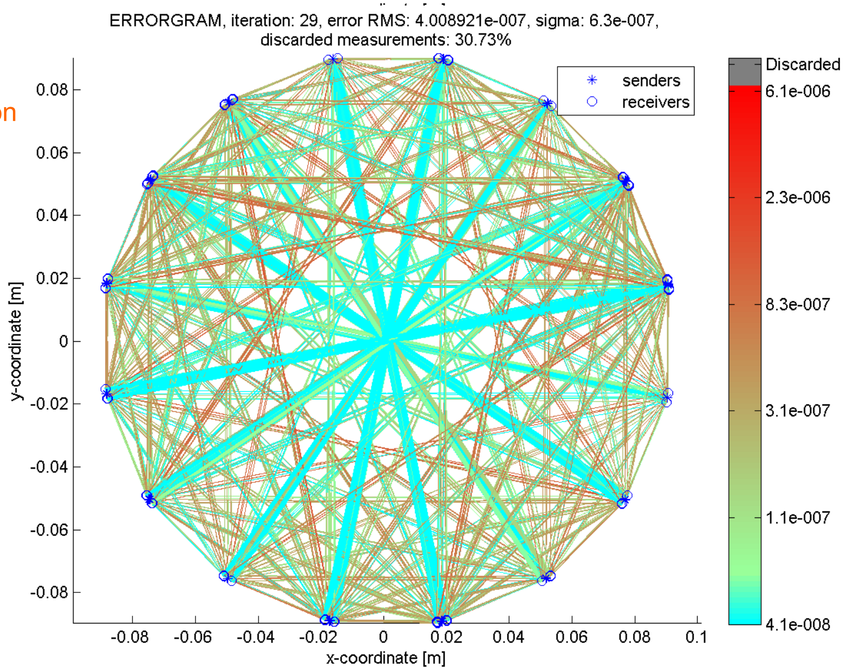
2. Calibration using real data

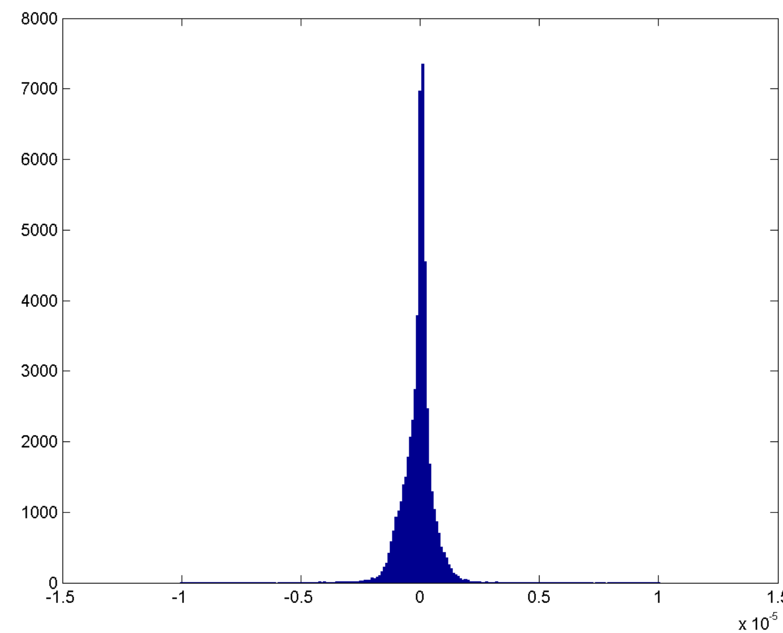
First estimate





After calibration





Conclusions

Two new concepts, enabling to calibrate different parameters of a USCT system, based only on „empty" measurements without any further equipment or phantoms, were presented.

According to experiments, both approaches show ability to provide the calibration data with the required precision (e.g. up to the order of 0.1 mm in position), as needed for successful USCT image reconstruction.

Further development is on the way, with the aim of improving the convergence and final precision.

The investigation (of other authors) in the area of USCT reconstruction shows that the reconstructed image quality is strongly positively influenced by inclusion of the USCT system corrections (i.e. radiation non-isotropic directivity and uneven efficiency as well as geometrical error corrections). Thus, the calibration appears to be a necessary (perhaps regularly exercised) step in use of USCT.

Acknowledgments

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