Human reasoning about uncertain conditionals

Pfeifer, N., & Kleiter, G. D.

Department of Psychology, University of Salzburg (Austria)

http://www.users.sbg.ac.at/˜pfeifern/

Motivation

- Traditional normative framework in psychology:
  - Deductive reasoning: classical logic
  - Judgment: classical probability theory

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- Promising synthesis: framing human inference by coherence based probability logic
- Main goal: building a competence theory of human reasoning

Contents

- Probabilistic approaches in the literature
- Mental probability logic
  - Example 1: Modus ponens
- Studies on nonmonotonic conditionals
  - Example 2: Premise strengthening
  - Example 3: Contraposition
  - Example 4: Hypothetical syllogism

Probabilistic approaches to human deductive reasoning

Postulated interpretation of the "IF A, THEN B"

$P(A \supset B)$

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Probabilistic extension of the *mental model* theory

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e.g., from IF A, THEN B infer IF A AND C, THEN B

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The material implication is not a genuine conditional

$$(A \supset B) \quad \Leftrightarrow \quad (\neg A \vee B)$$

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The conditional event $B|A$ is a genuine conditional

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Empirical Result:
$P(B|A)$ best predictor
for "IF A, THEN B"
Evans, Over et al.
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- Deductive relation between premise(s) and conclusion
  Mental probability logic
  Pfeifer & Kleiter
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Mental probability logic

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  - A, normally B iff $P(B|A) = high$

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- coherence

Coherence

- de Finetti, Lad, Walley, Scozzafava, Coletti, Gilio

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- In logic
  from $\boxed{A \text{ and } A \supset B}$ infer $\boxed{B}$

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  from $\boxed{P(A) = x \text{ and } P(B|A) = y}$

  infer $\boxed{P(B) \in [xy, xy + (1 - x)]}$

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  from $\boxed{P(A) = x \text{ and } P(B|A) = y}$

  infer $\boxed{P(B) \in [\underbrace{xy}_{at\ least}, \underbrace{xy + (1-x)}_{at\ most}]}$

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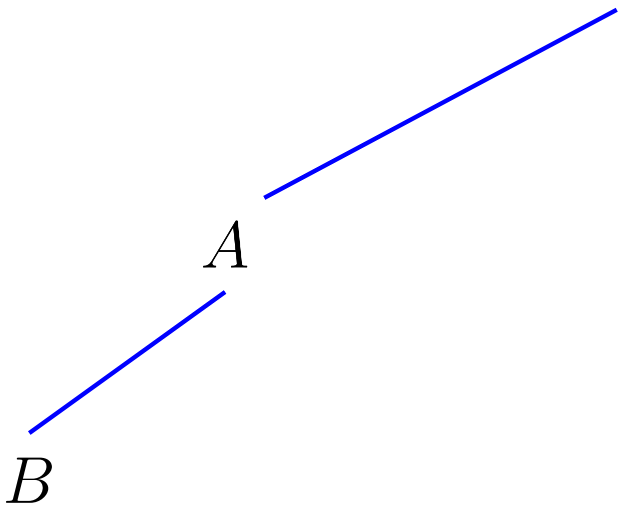

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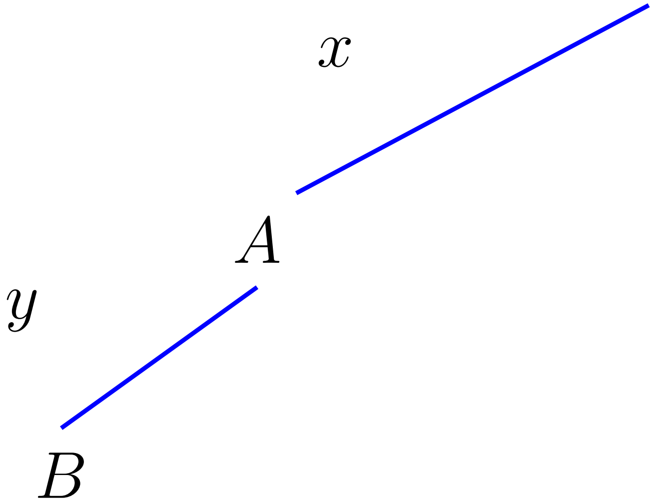

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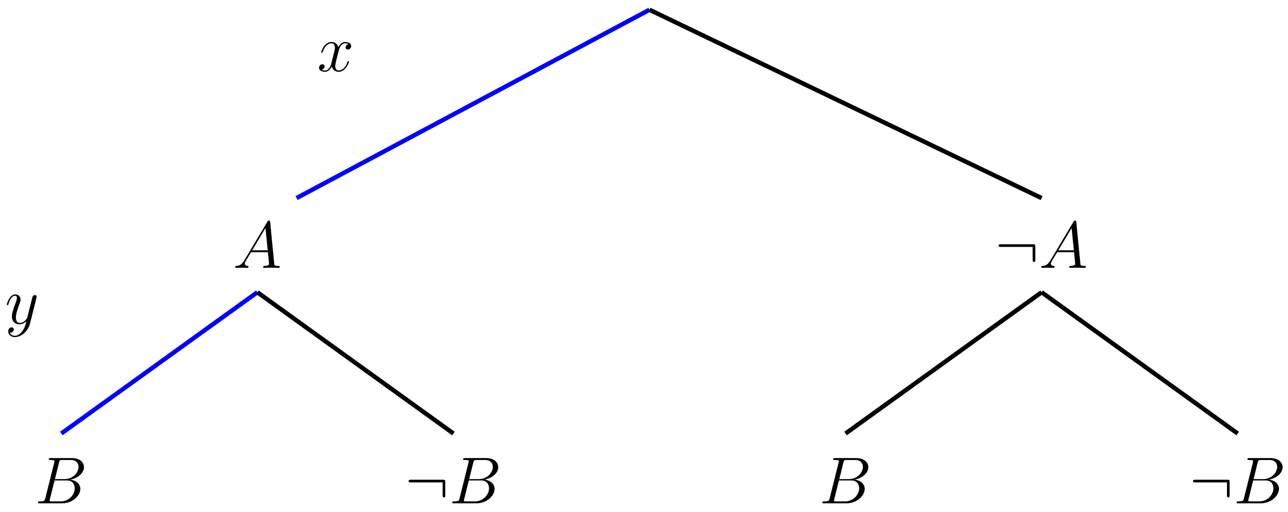

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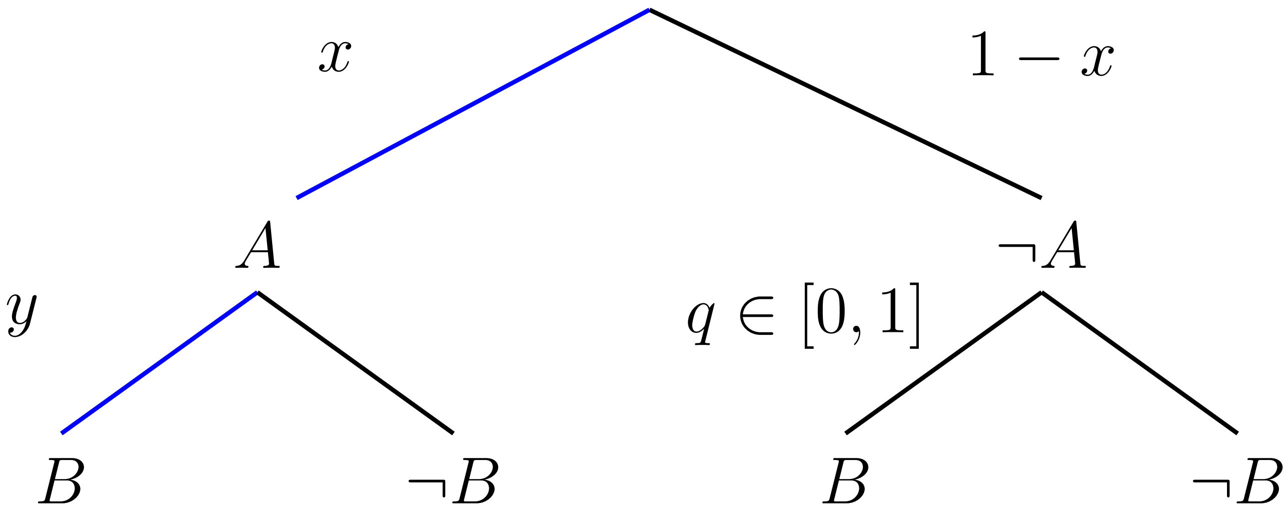

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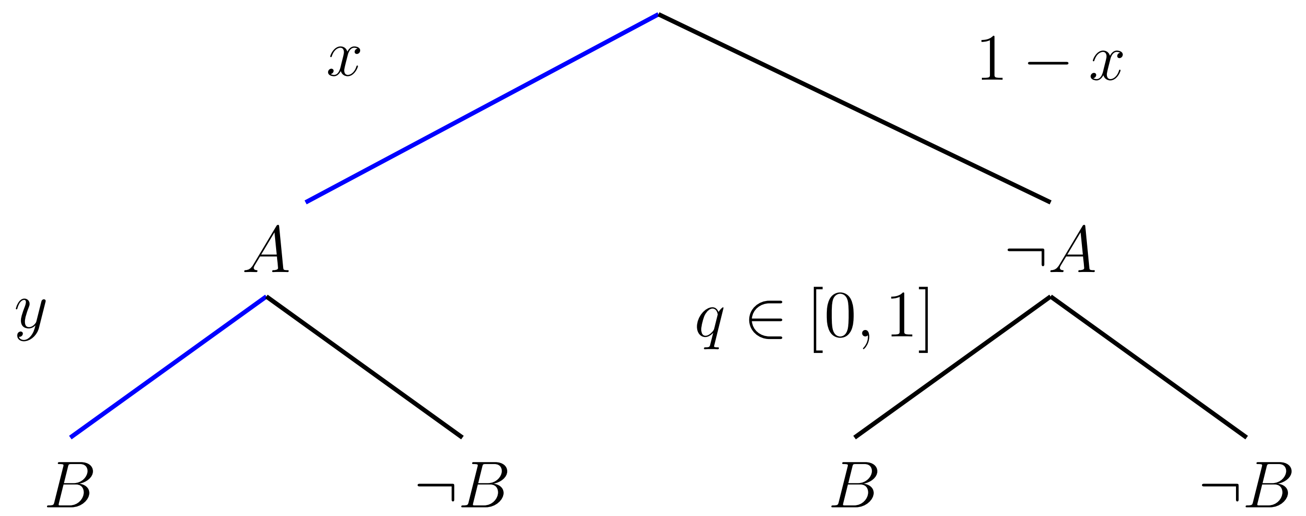

$$P(B) = \underbrace{P(A)}_{x}\,\underbrace{P(B|A)}_{y} + \underbrace{P(\neg A)}_{1-x}\,\underbrace{P(B|\neg A)}_{q\in[0,1]}$$

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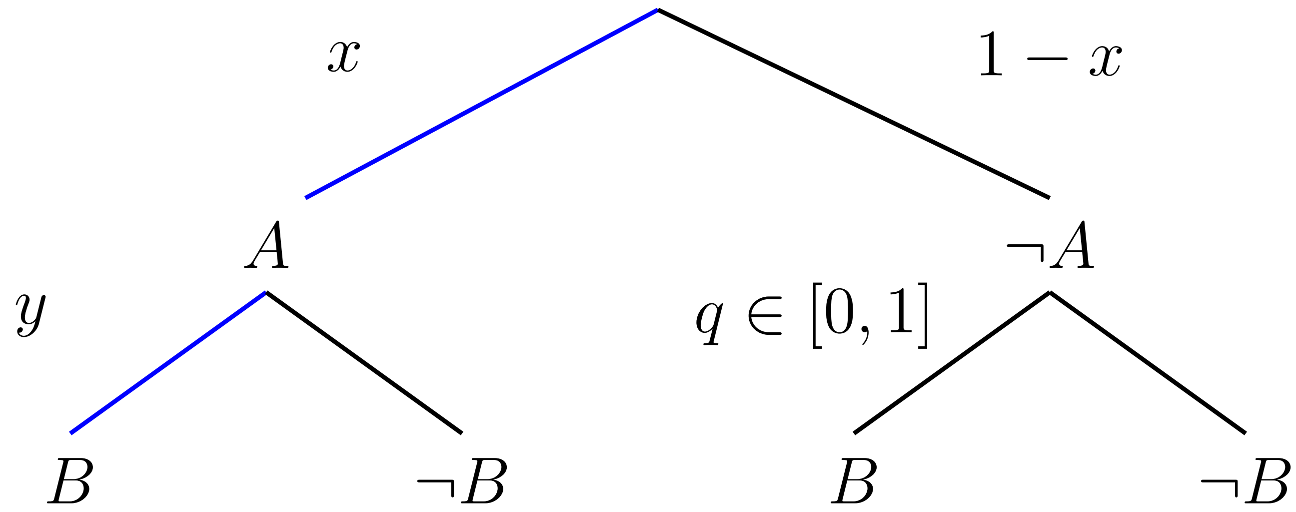
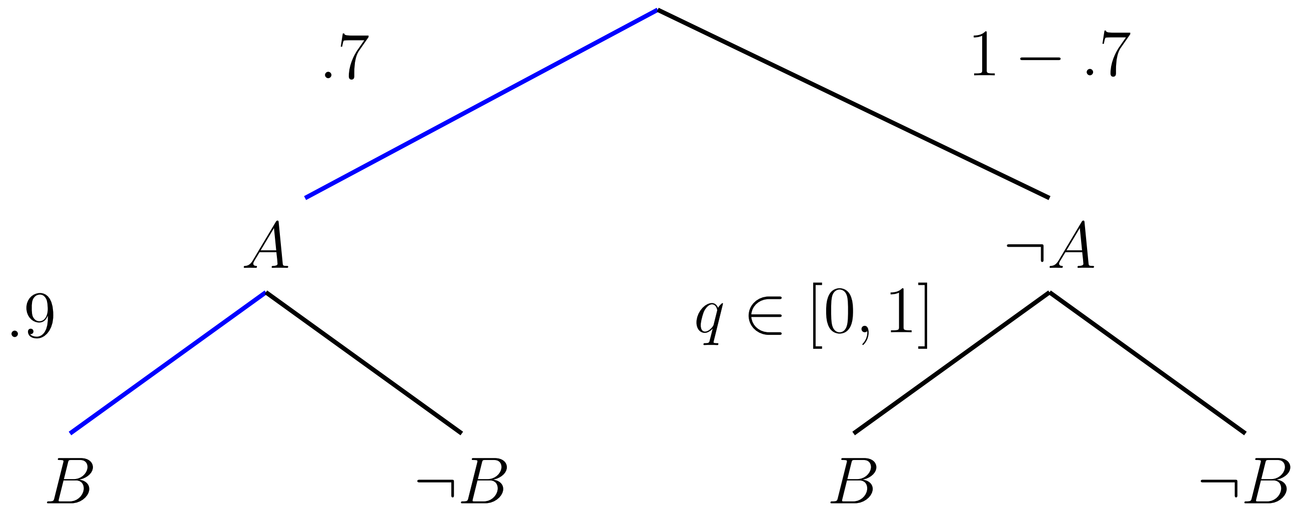


$$P(B) = \underbrace{P(A)}_{x}\,\underbrace{P(B|A)}_{y} + \underbrace{P(\neg A)}_{1-x}\,\underbrace{P(B|\neg A)}_{q \in [0,1]}$$

$$\underbrace{xy}_{if\ \ q=0} \leq P(B) \leq \underbrace{xy + (1-x)}_{if\ \ q=1}$$

Probabilistic MODUS PONENS

from $\boxed{P(A) = .7 \text{ and } P(B|A) = .9}$ infer $\boxed{P(B)}$

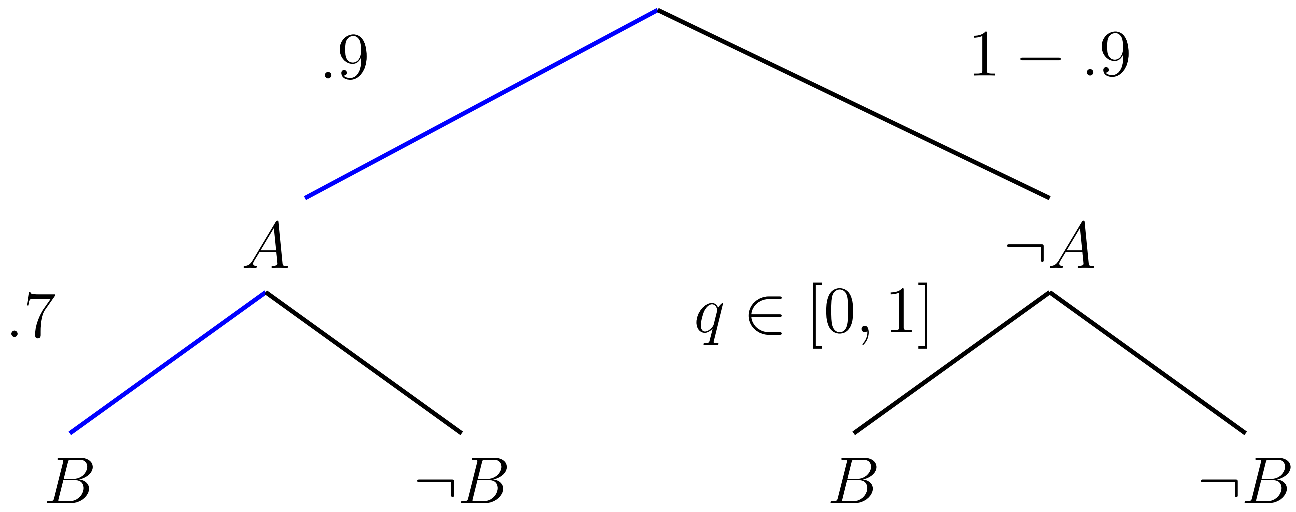


$$P(B) = \underbrace{P(A)}_{.7}\,\underbrace{P(B|A)}_{.9} + \underbrace{P(\neg A)}_{1-.7}\,\underbrace{P(B|\neg A)}_{q\in[0,1]}$$

$$\underbrace{.63}_{if\ \ q=0} \leq P(B) \leq \underbrace{.93}_{if\ \ q=1}$$

Probabilistic MODUS PONENS

from $\boxed{P(A) = .9 \text{ and } P(B|A) = .7}$ infer $\boxed{P(B)}$

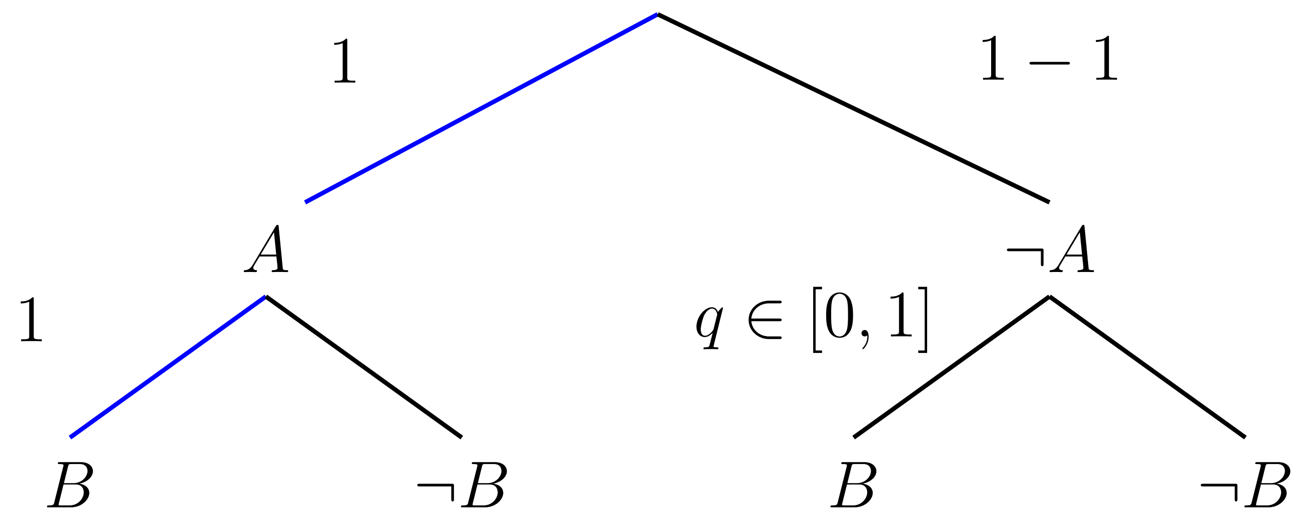


$$P(B) = \underbrace{P(A)}_{.9}\,\underbrace{P(B|A)}_{.7} + \underbrace{P(\neg A)}_{1-.9}\,\underbrace{P(B|\neg A)}_{q\in[0,1]}$$

$$\underbrace{.63}_{if\ \ q=0} \leq P(B) \leq \underbrace{\mathbf{.73}}_{if\ \ q=1}$$

Probabilistic MODUS PONENS

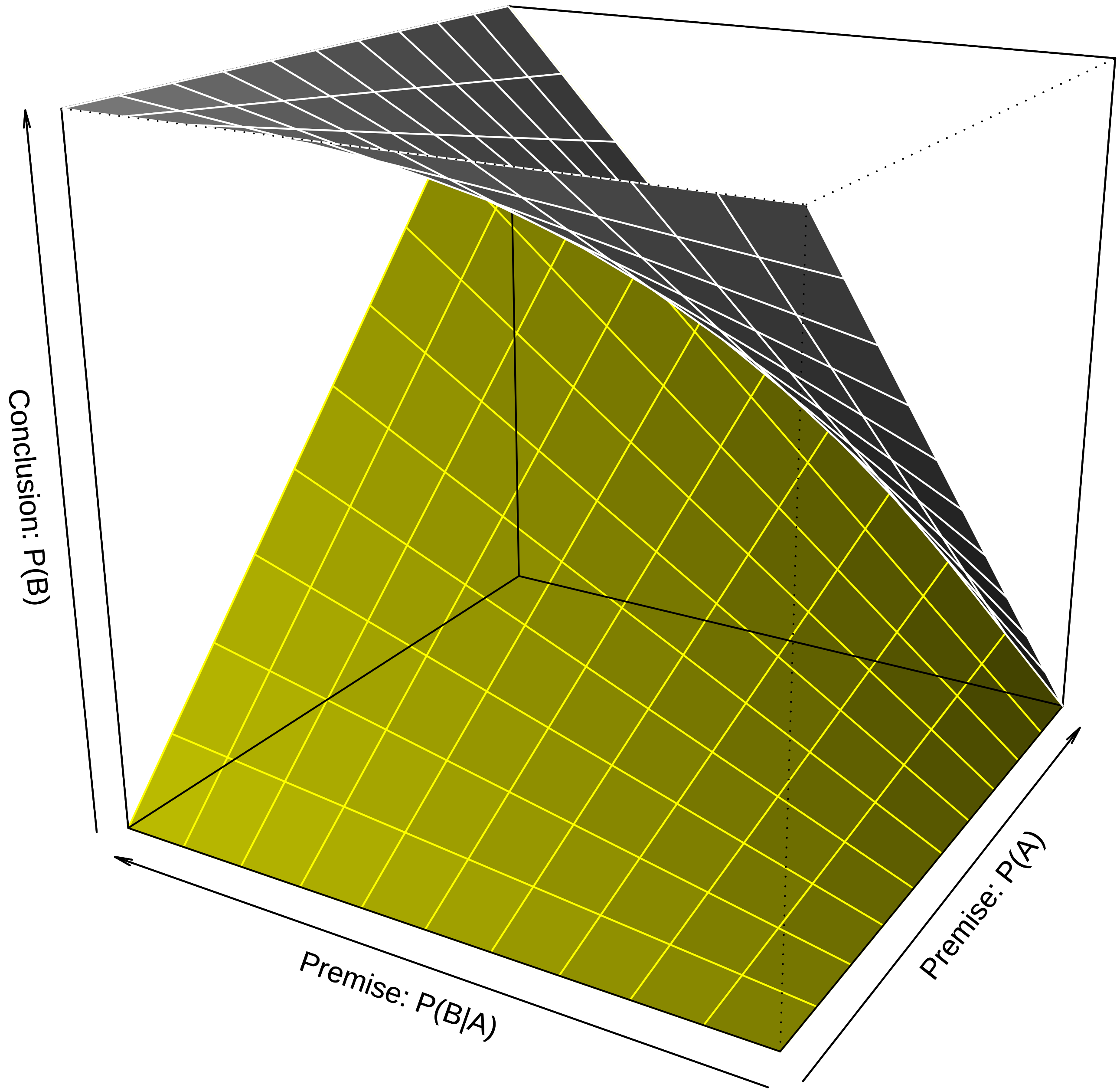
from $\boxed{P(A) = 1 \text{ and } P(B|A) = 1}$ infer $\boxed{P(B)}$



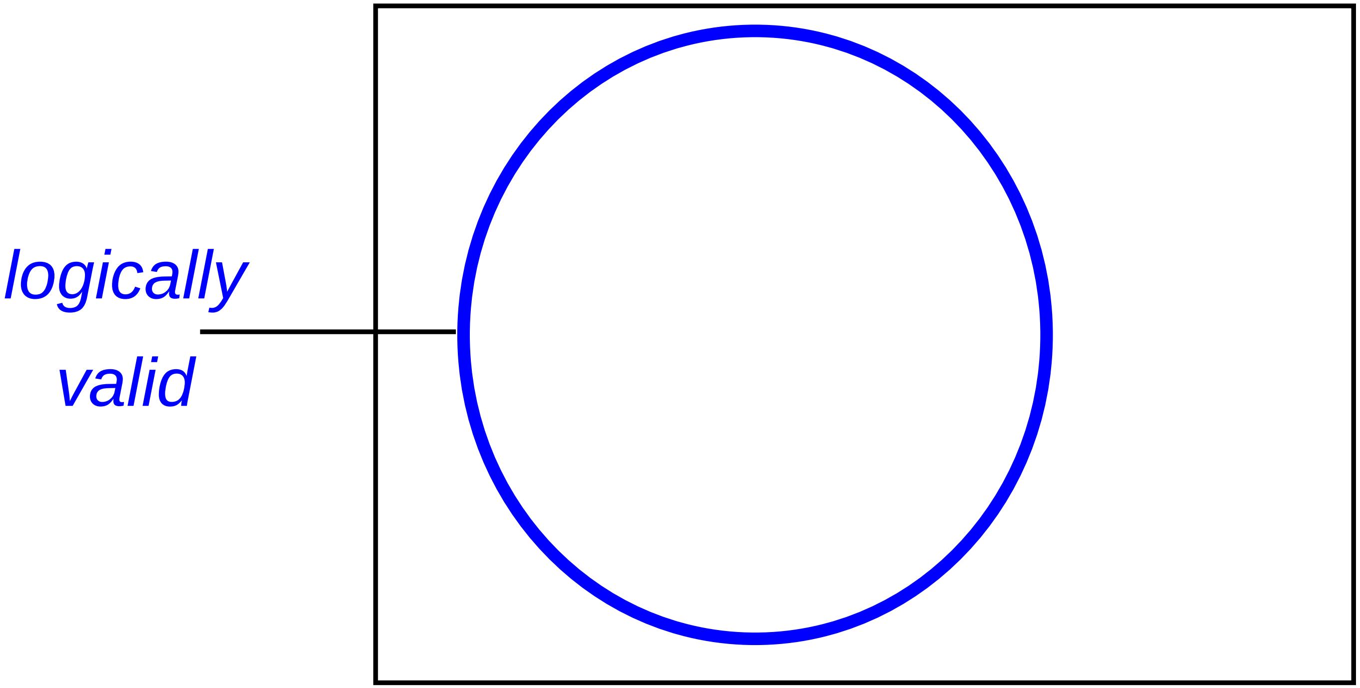
$$P(B) = \underbrace{P(A)}_{1}\,\underbrace{P(B|A)}_{1} + \underbrace{P(\neg A)}_{1-1}\,\underbrace{P(B|\neg A)}_{q\in[0,1]}$$

$$\underbrace{1}_{if\ \ q=0} = P(B) = \underbrace{1}_{if\ \ q=1}$$

Probabilistic MODUS PONENS



Logically valid–probabilistically informative



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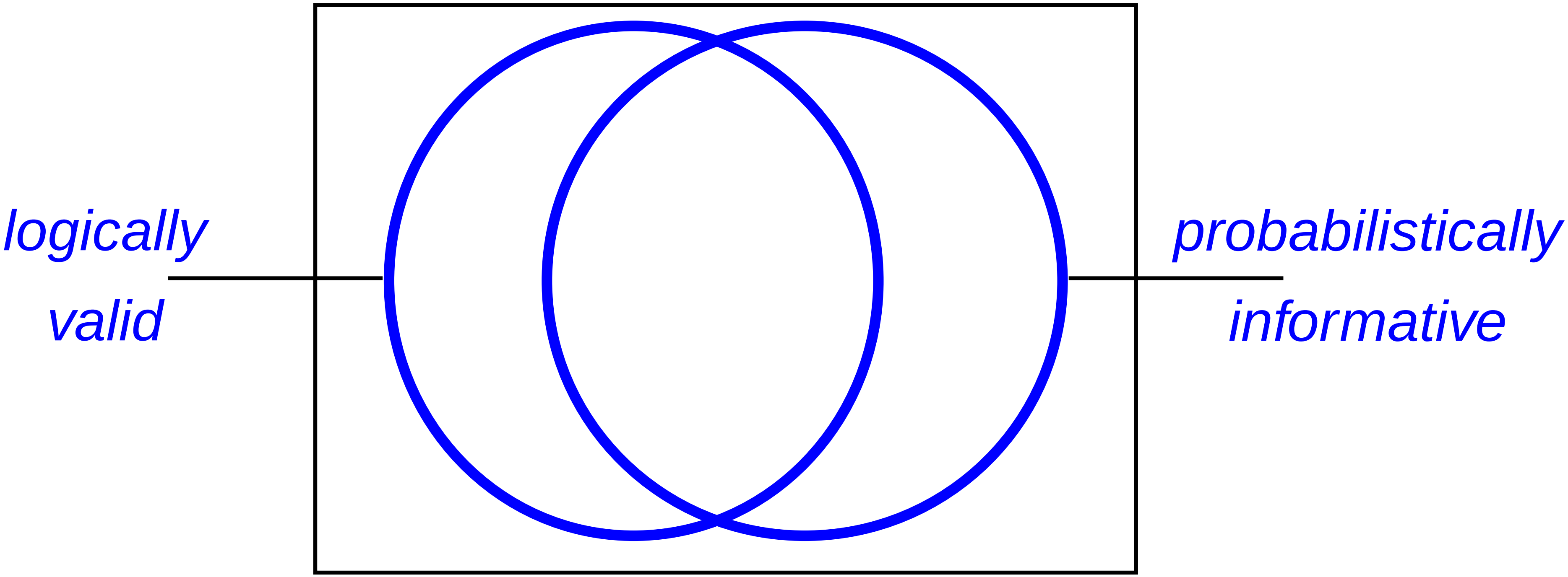

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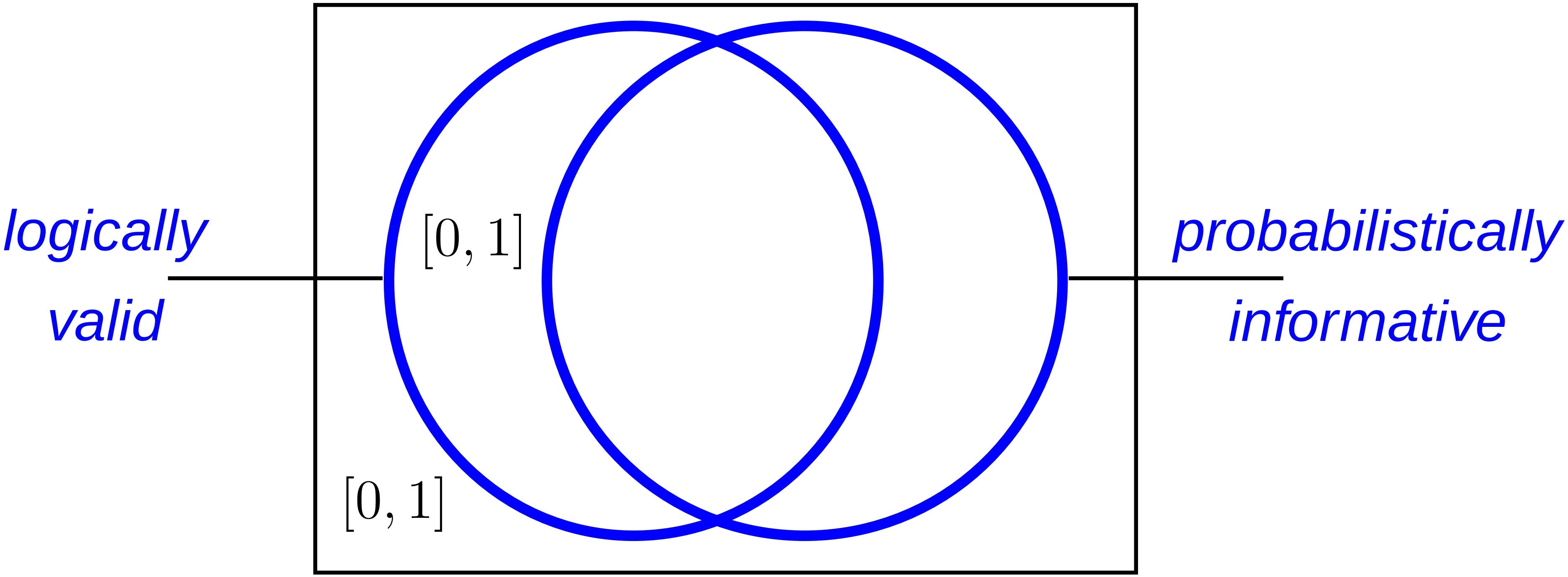

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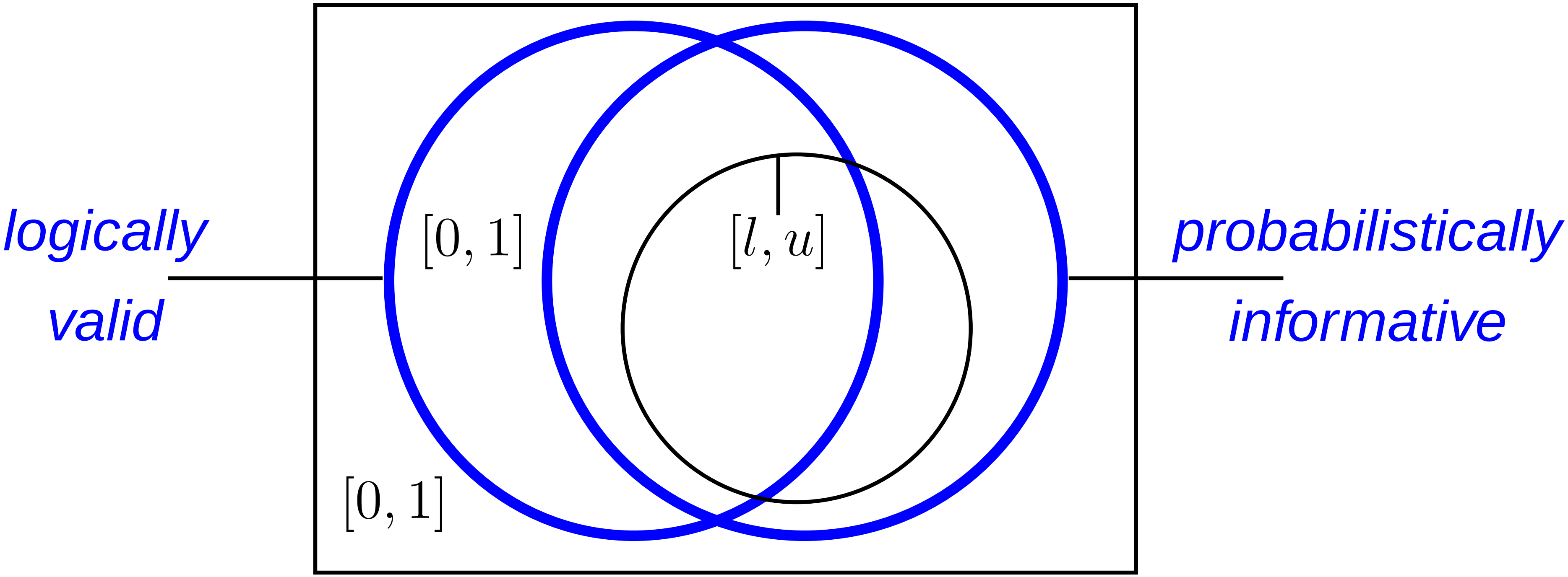

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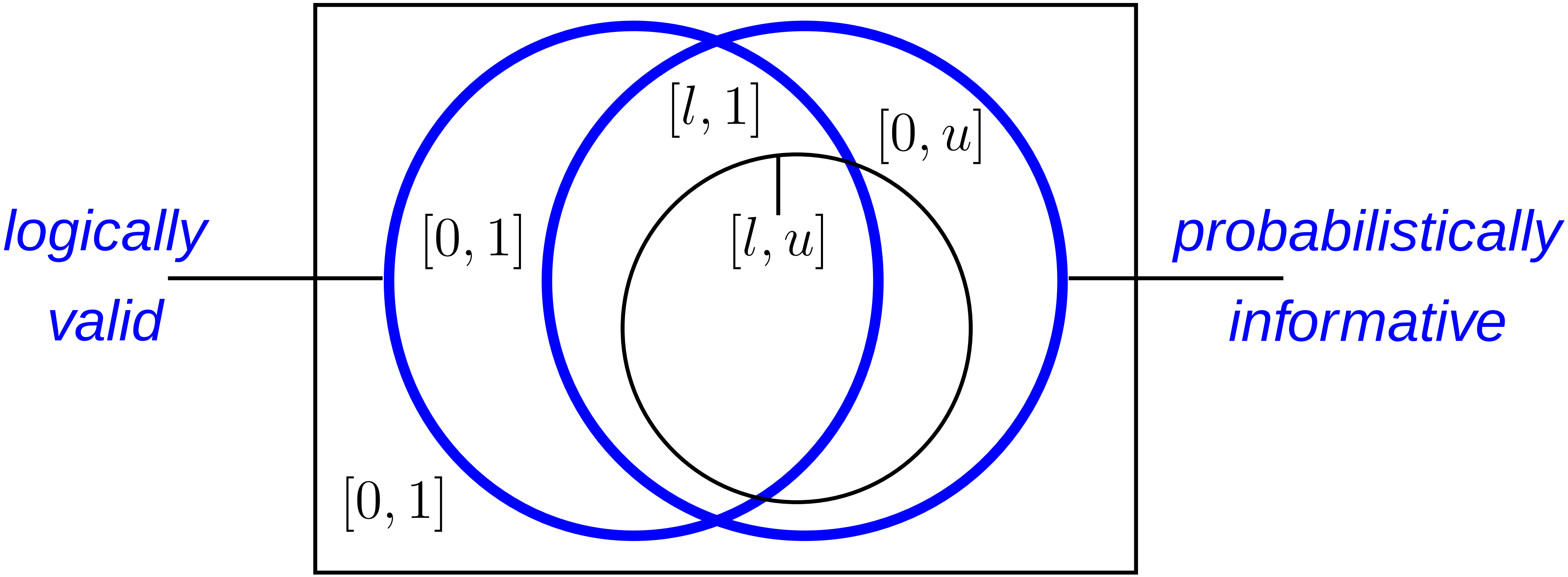

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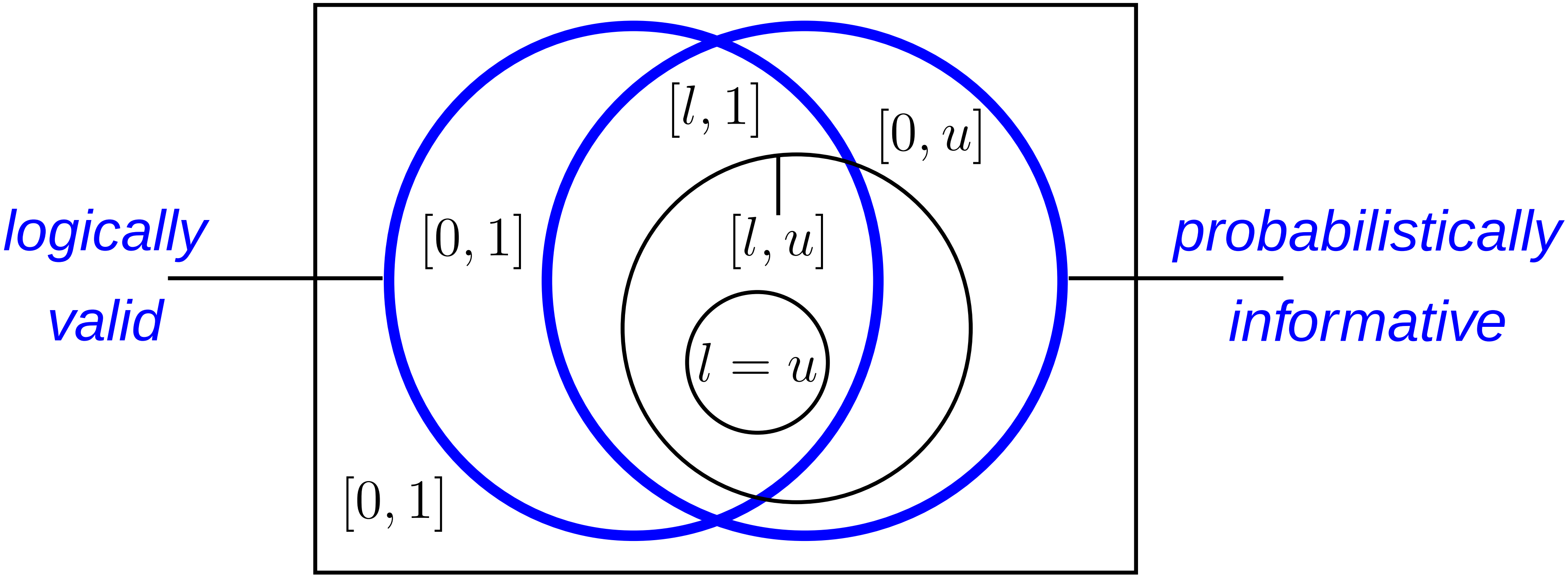

Example task: MODUS PONENS

Claudia works at the blood donation services. She investigates to which blood group the donated blood belongs and whether the donated blood is Rhesus-positive.

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If the donated blood belongs to the blood group 0,

then the donated blood is Rhesus-positive.

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How certain should Claudia be that a recent donated blood is Rhesus-positive?

Response Modality

The solution is either a point percentage or a percentage between two boundaries (from at least . . . to at most . . .):

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Claudia is at least . . . .% and at most . . . .% certain, that the donated blood is Rhesus-positive.

Within the bounds of:

0 25 50 75 100 %

Results

Premise 1	2	*coherent* LB.	UB.	*response* LB.	UB.	*coherent* LB.	UB.	*response* LB.	UB.
		MODUS PONENS				NEGATED MODUS PONENS			
1	1	1	1	1	1	.00	.00	.00	.00
.7	.9	.63	.73	.62	.69	.27	.37	.35	.42
.7	.5	.35	.85	.43	.55	.15	.65	.41	.54
		DENYING THE ANTECEDENT				NEGATED DENYING THE ANTECEDENT			
1	1	.00	1	.37	.85	.00	1	.01	.53
.7	.2	.20	.44	.19	.42	.56	.80	.52	.76
.7	.5	.15	.65	.25	.59	.35	.85	.33	.65

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“certain” MODUS PONENS tasks: all participants inferred correctly “1” or “0”

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“certain” DENYING THE ANTECEDENT tasks: most participants inferred intervals close to $[0, 1]$

Results

Premise 1	2	*coherent* LB.	UB.	*response* LB.	UB.	*coherent* LB.	UB.	*response* LB.	UB.
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good overall agreement between the normative bounds and the mean responses

Conjugacy

All participants inferred a probability (interval) of a conclusion $P(\mathfrak{C}) \in [z', z'']$ and the probability of the associated negated conclusion, $P(\neg\mathfrak{C})$.

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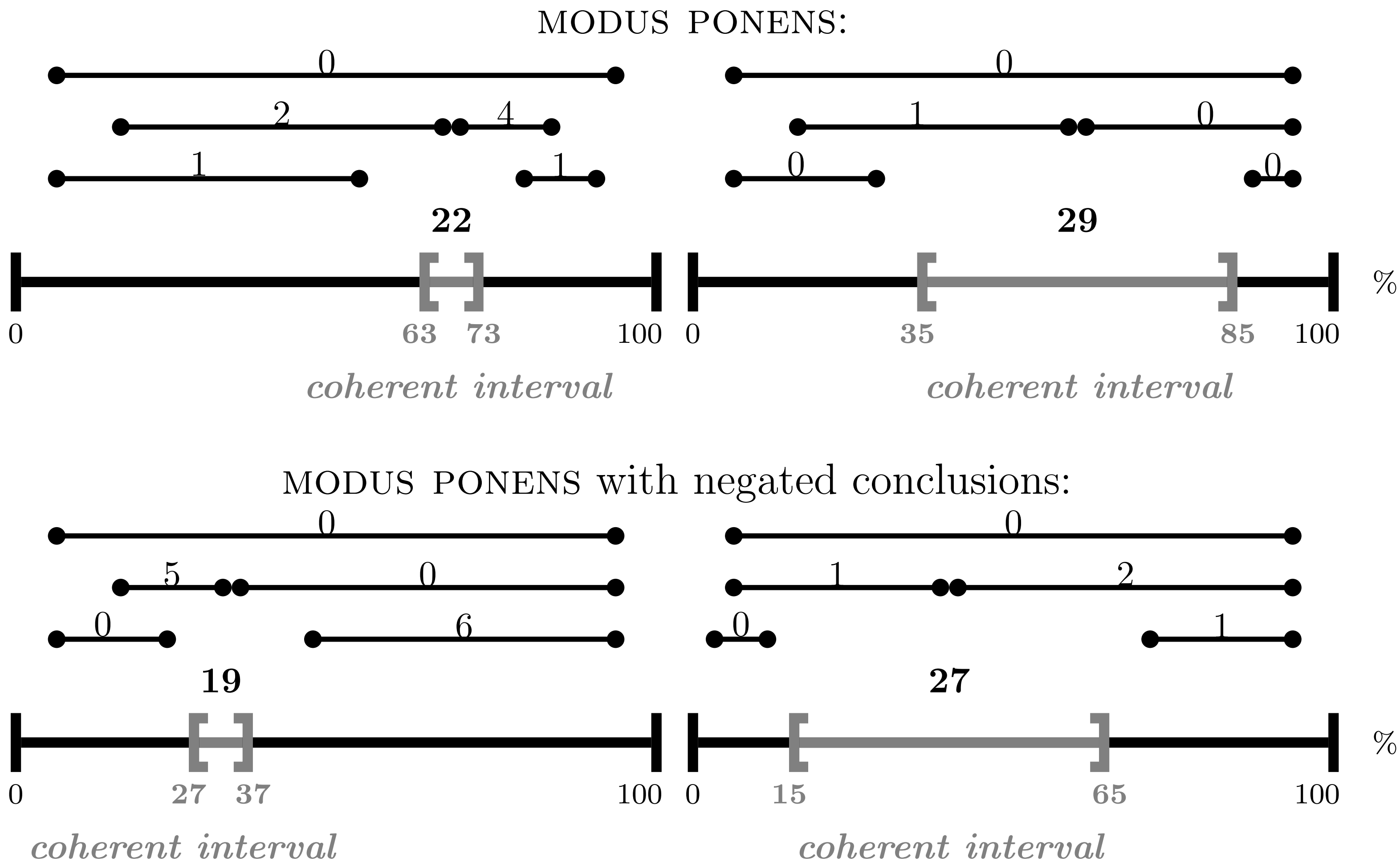
(*Premise 1, Premise 2*)	$(1, 1)$	$(.7, .9)$	$(.7, .5)$	$(.7, .2)$
MODUS PONENS	100%	53%	50%	
DENYING THE ANTECEDENT	67%		30%	0%

…percentages of participants satisfying both $z'_{\mathfrak{C}} + z''_{\neg\mathfrak{C}} = 1$ and $z'_{\neg\mathfrak{C}} + z''_{\mathfrak{C}} = 1$

Results: Interval Responses



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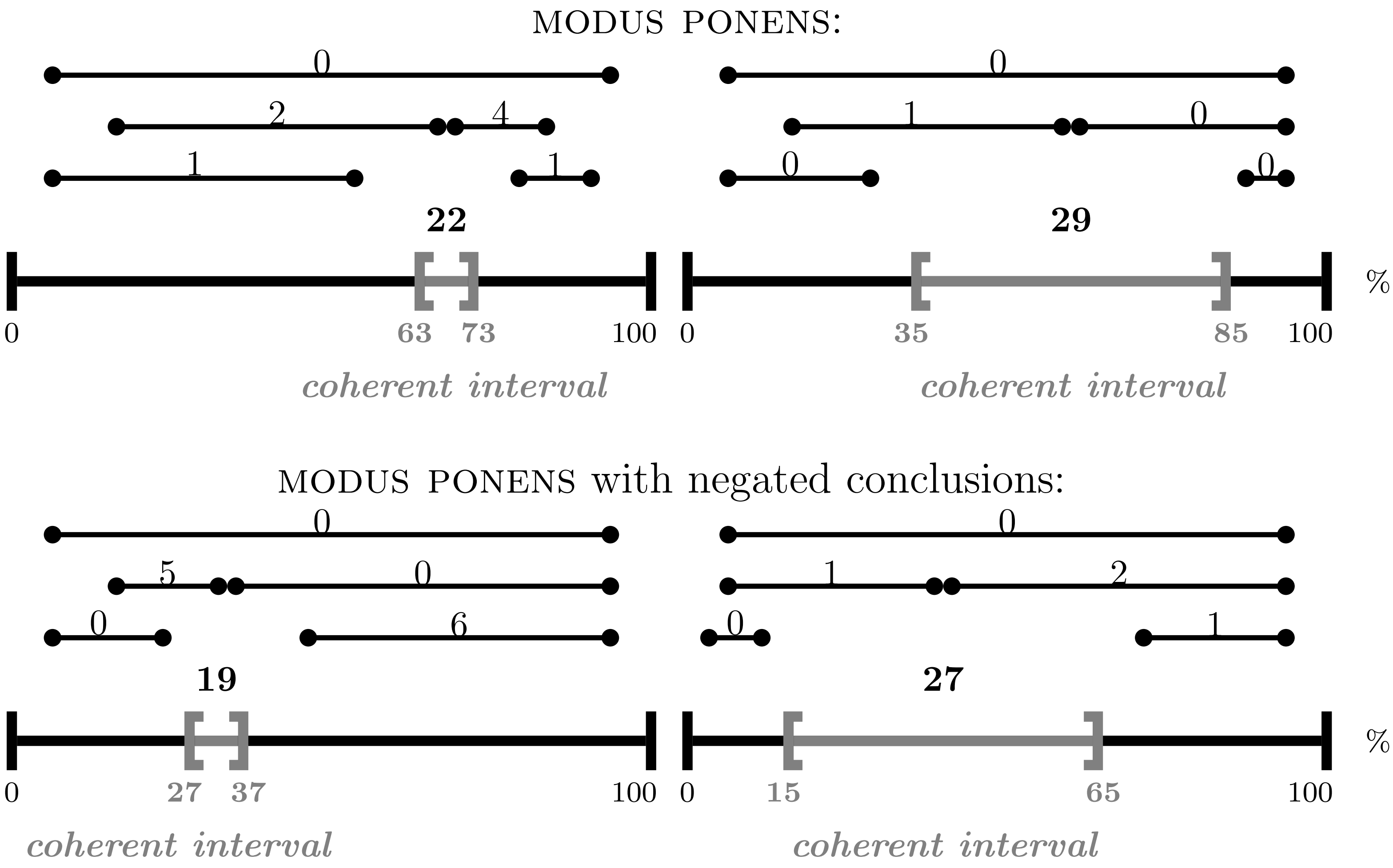

more observed responses are coherent than expected
(assuming a random interval generator)

Example 2: PREMISE STRENGTHENING

- In logic

  from $\boxed{A \supset B}$ infer $\boxed{(A \wedge C) \supset B}$

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Example 2: PREMISE STRENGTHENING

- In logic

  from $A \supset B$ infer $(A \wedge C) \supset B$

- In probability logic

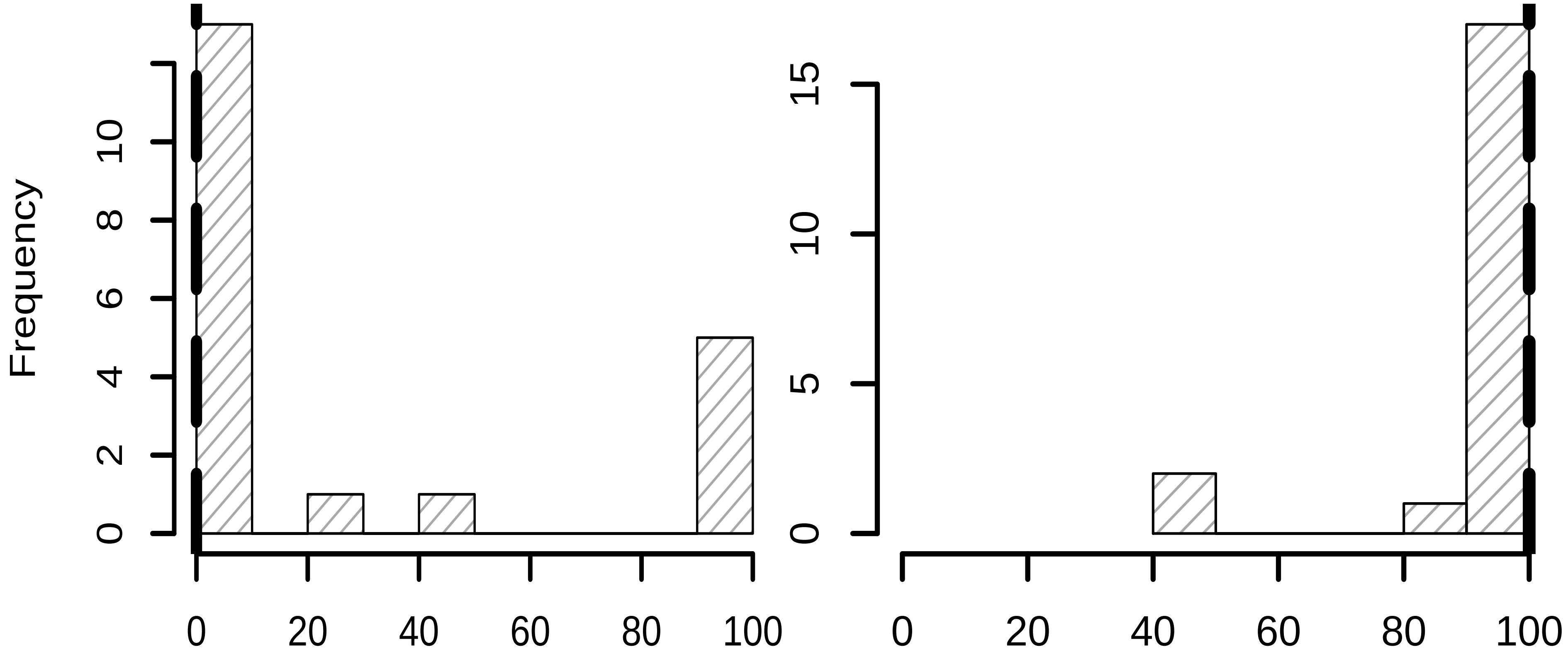
  from $P(B|A) = x$ infer $P(B|A \wedge C) \in [0, 1]$

- CAUTIOUS MONOTONICITY

  from $P(B|A) = x$ and $P(C|A) = y$

  infer $P(C|A \wedge B) \in [\max(0, (x + y - 1)/x), \min(y/x, 1)]$

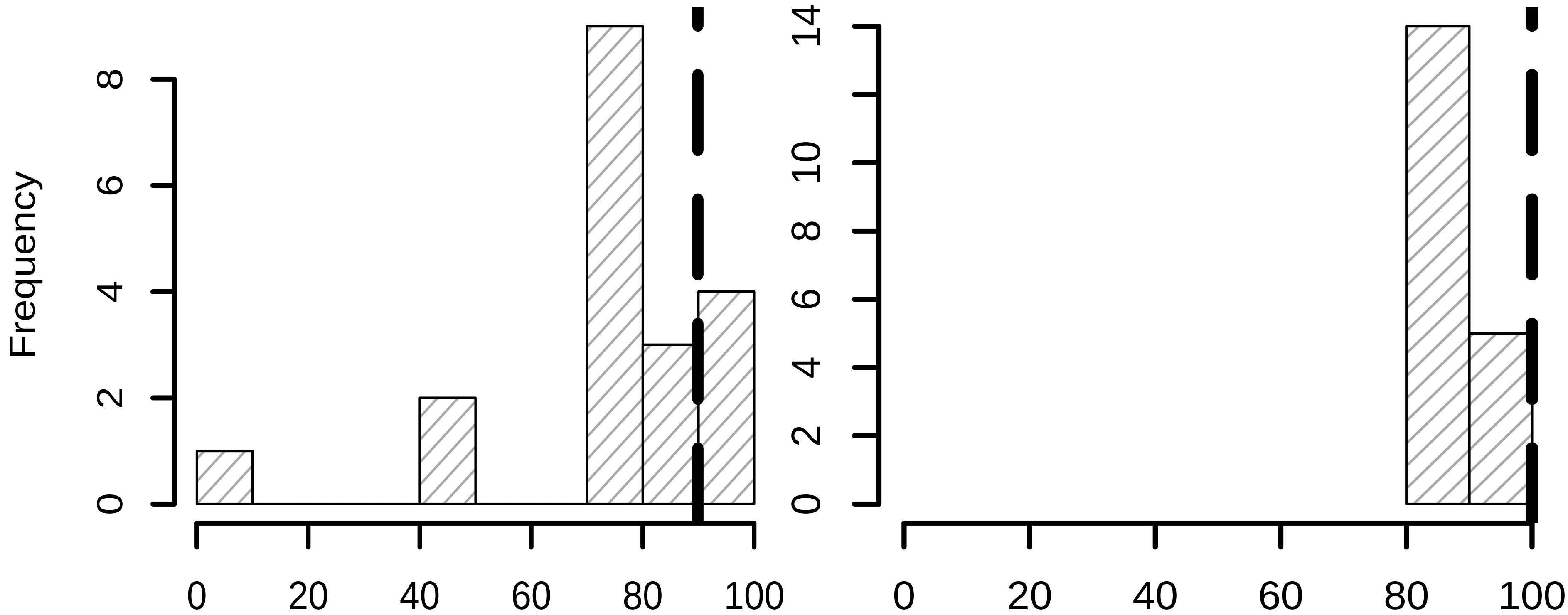
Results – PREMISE STRENGTHENING (Example Task 1)



lower bound responses upper bound responses

$$(n_1 = 20)$$

Results – CAUTIOUS MONOTONICITY (Example Task 1)



lower bound responses upper bound responses

$(n_2 = 19)$

Example 3: CONTRAPOSITION

- In logic

  from $\boxed{A \supset B}$ infer $\boxed{\neg B \supset \neg A}$

  from $\boxed{\neg B \supset \neg A}$ infer $\boxed{A \supset B}$

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- In logic

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- In probability logic

  from $\boxed{P(B|A) = x}$ infer $\boxed{P(\neg A|\neg B) \in [0,1]}$

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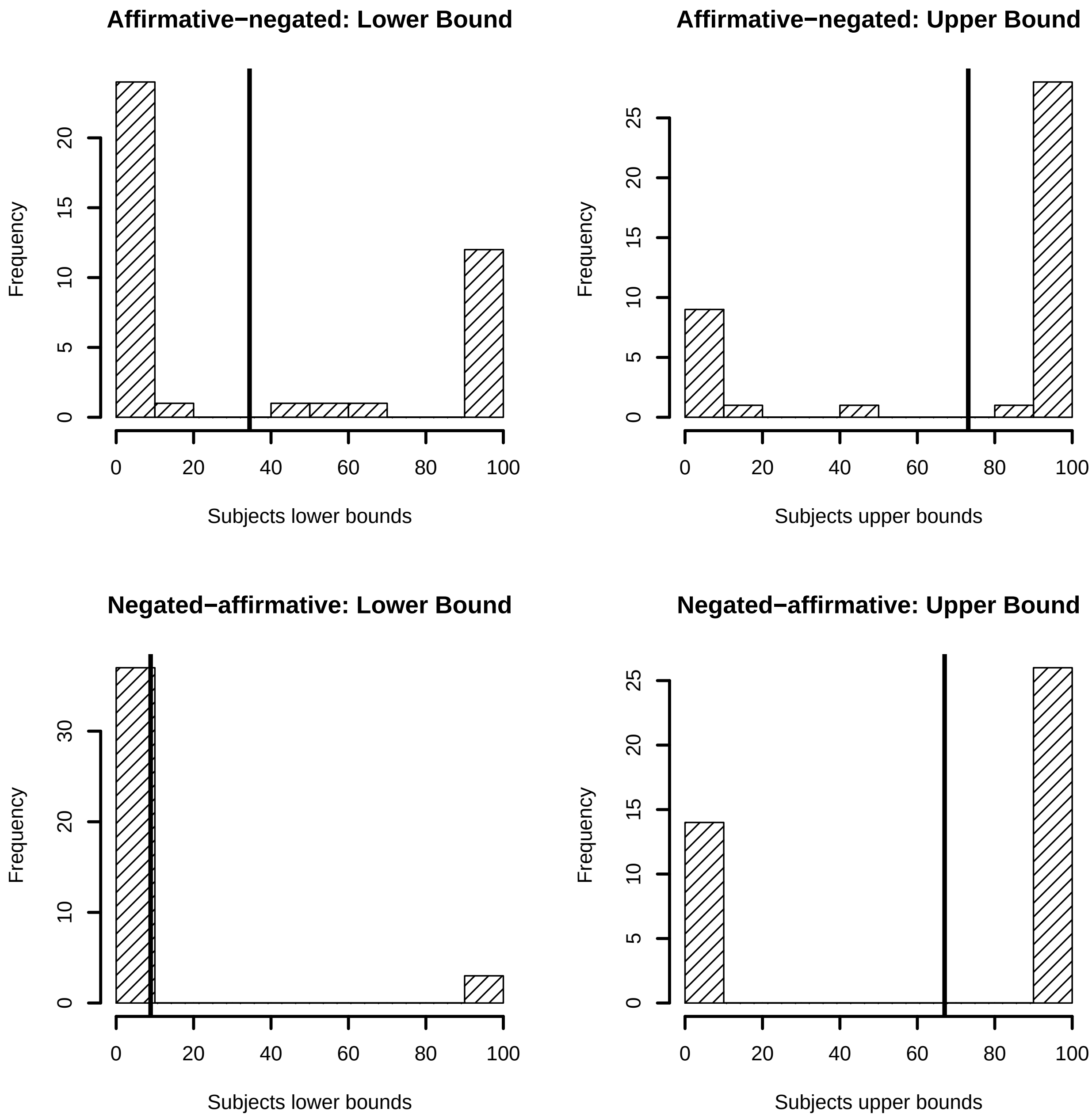
  from $\boxed{P(\neg A|\neg B) = x}$ infer $\boxed{P(B|A) \in [0,1]}$

- but

$$P(A \supset B) = P(\neg B \supset \neg A)$$

Results CONTRAPOSITION $(n_1 = 40, n_2 = 40)$



Example 4: HYPOTHETICAL SYLLOGISM

- In logic

  from $\boxed{A \supset B \text{ and } B \supset C}$ infer $\boxed{A \supset C}$

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Example 4: HYPOTHETICAL SYLLOGISM

- In logic

  from $A \supset B$ and $B \supset C$ infer $A \supset C$

- In probability logic

  from $P(B|A) = x$ and $P(C|B) = y$ infer $P(C|A) \in [0, 1]$

- cut

  from $P(B|A) = x$ and $P(C|A \wedge B) = y$

  infer $P(C|A) \in [xy, 1 - y + xy]$

Concluding remarks

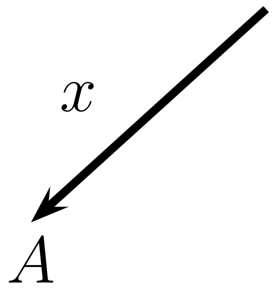
- Framing human inference by coherence based probability logic
  - investigating nonmonotonic conditionals in agrument forms
  - interpreting the if–then as high conditional probability
  - coherence based
  - competence theory (“Mental probability logic”)

Concluding remarks

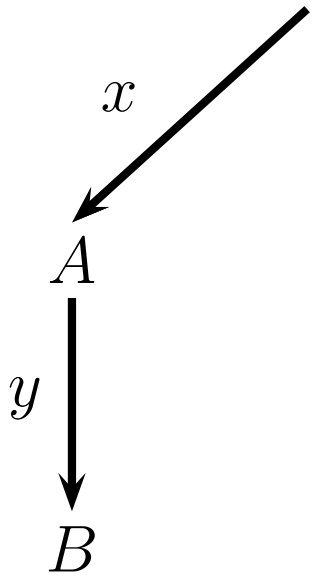
- Framing human inference by coherence based probability logic
  - investigating nonmonotonic conditionals in agrument forms
  - interpreting the if–then as high conditional probability
  - coherence based
  - competence theory ("Mental probability logic")
- Good overall agreement of human reasoning and basic predicitons
  - esp. MODUS PONENS, conjugacy, forward & affirmative
  - understanding of probabilistically non-informative PREMISE STRENGTHENING and CONTRAPOSITION
  - TRANSITIVITY converstationally implies CUT

Towards a process model of human conditional inference

Propositional graph: Notation



Propositional graph: Notation



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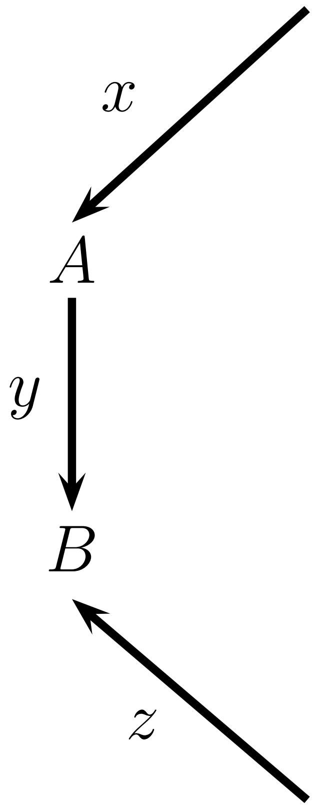

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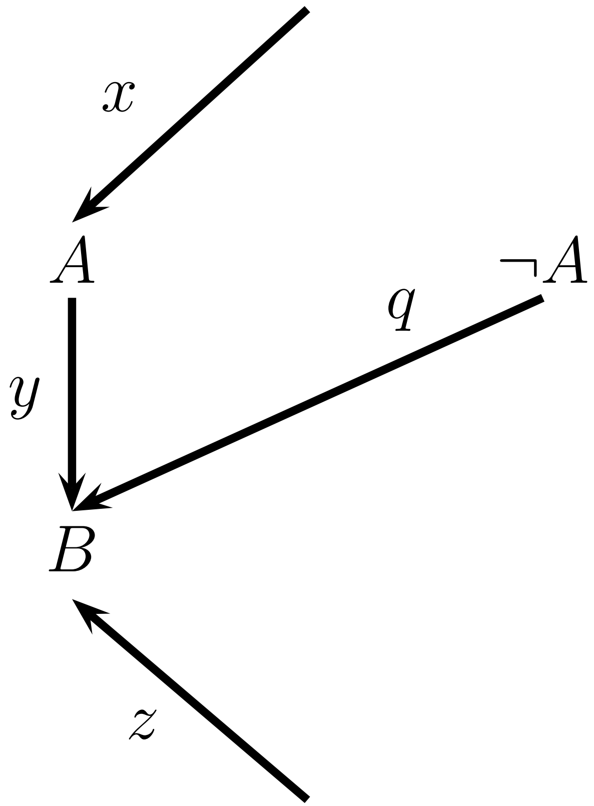

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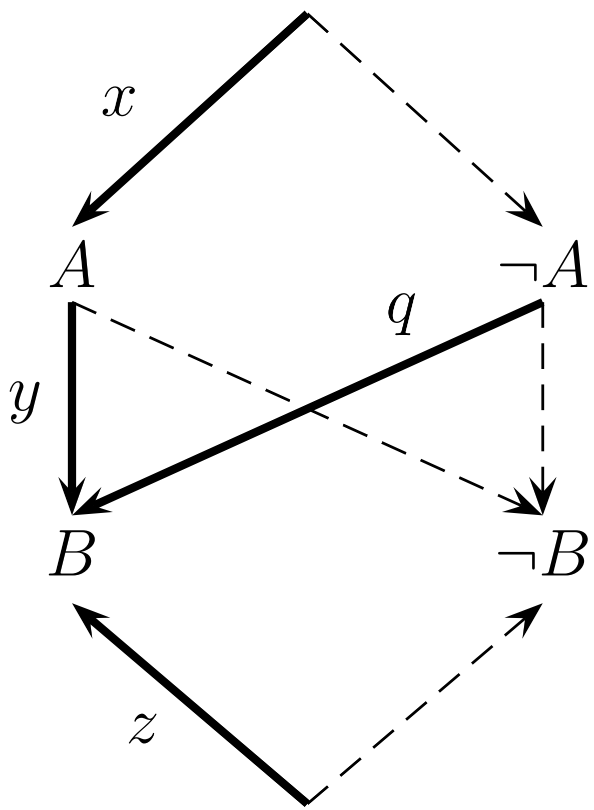
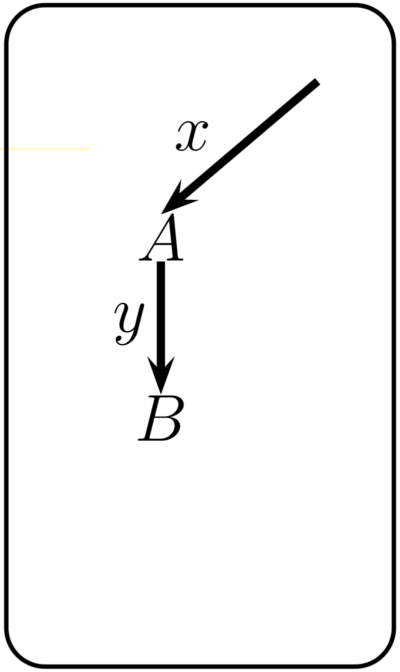
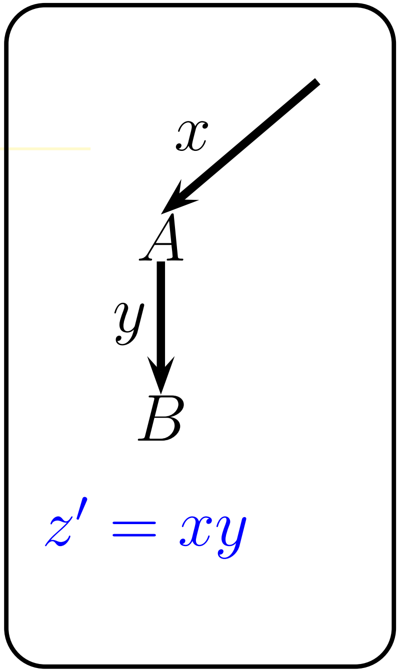




MODUS PONENS

$$P(B) = ?$$



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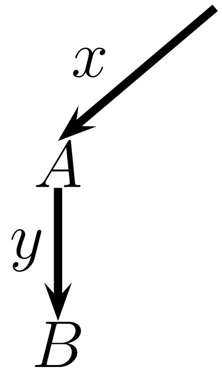
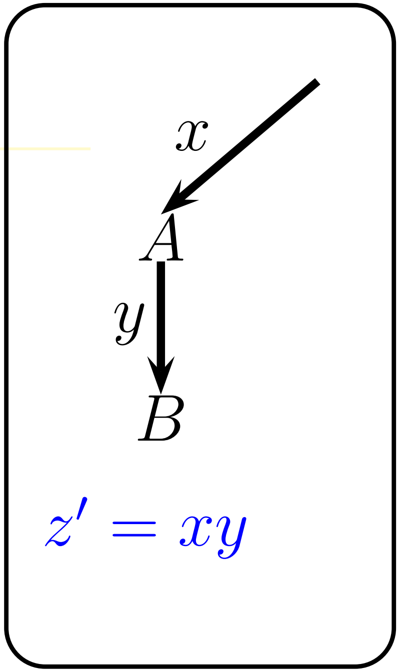


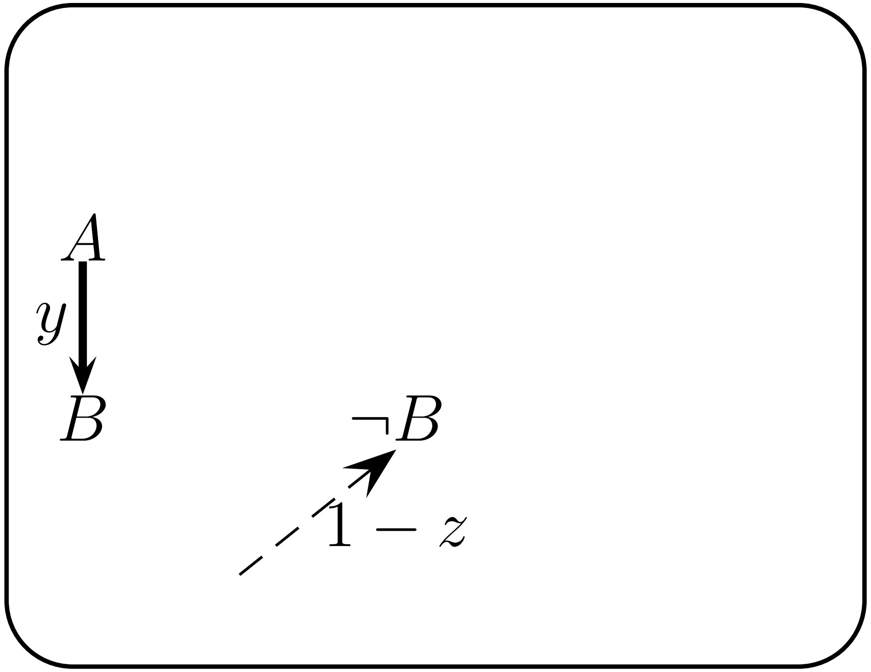
$z' = xy$

MODUS PONENS

$$P(B) = ?$$

forward
affirmative





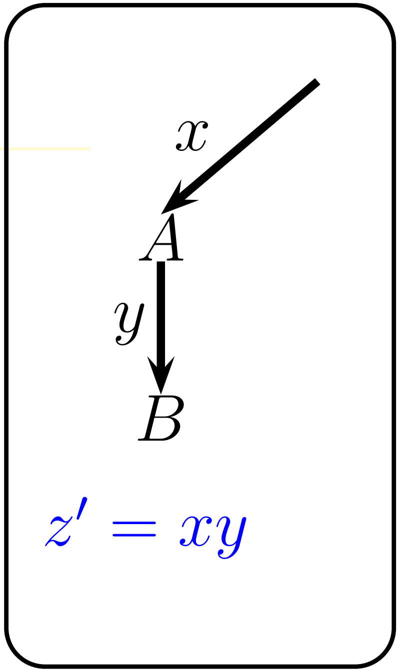
MODUS PONENS

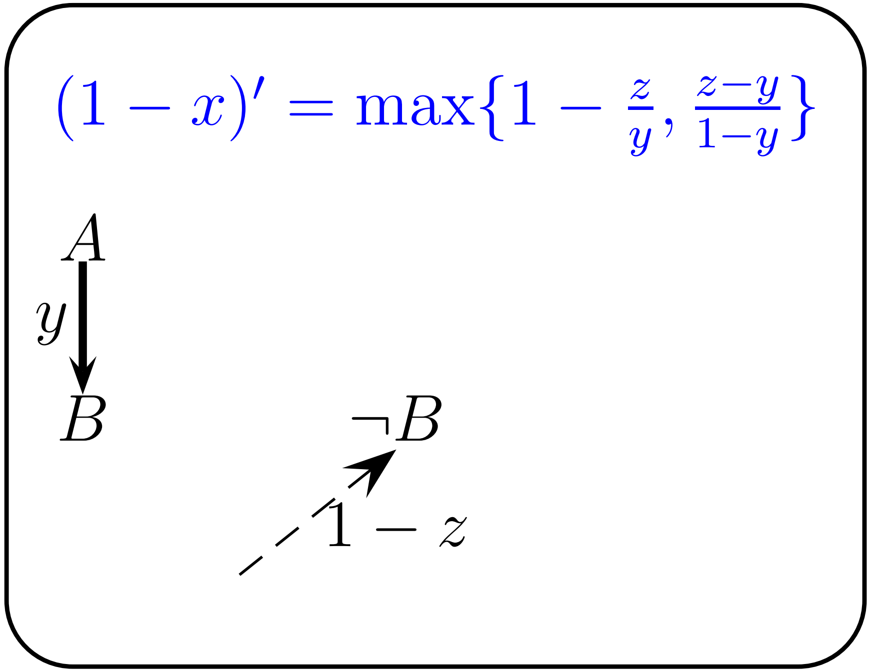
$$P(B) =?$$

forward
affirmative

MODUS TOLLENS

$$P(\neg A) =?$$





MODUS PONENS

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affirmative

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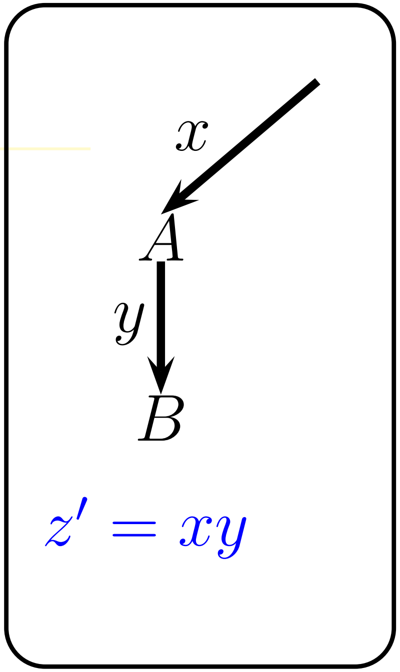
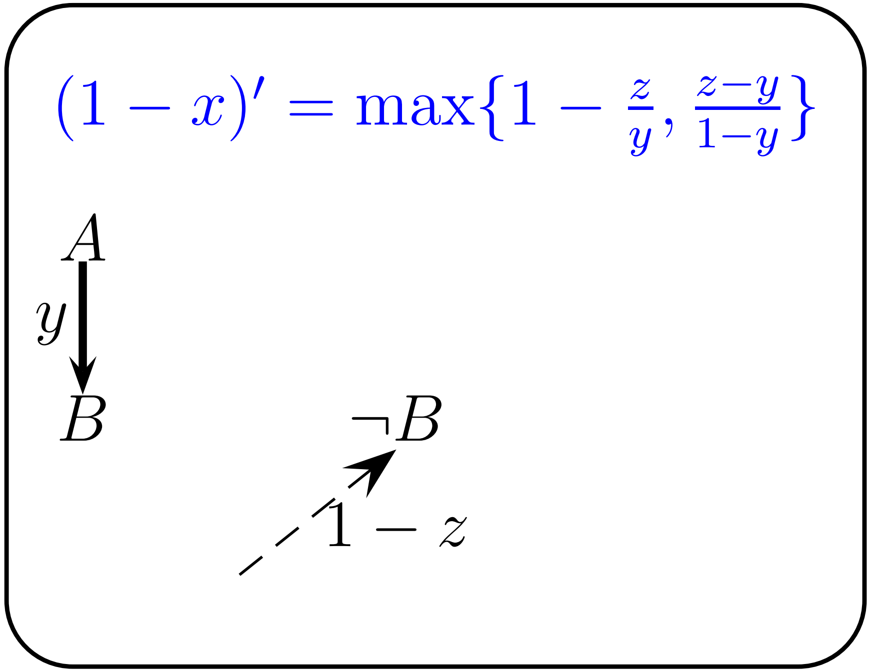




MODUS PONENS

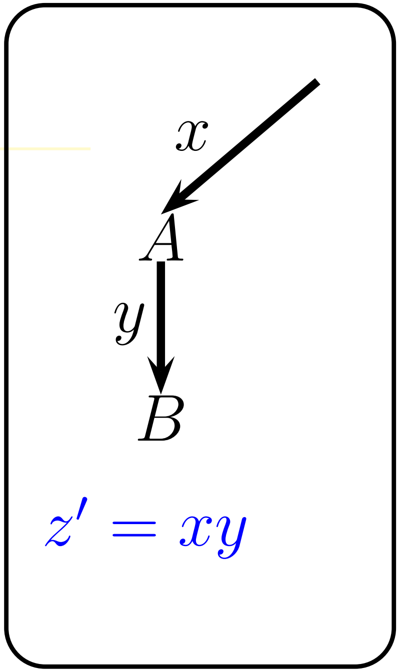
$P(B) = ?$

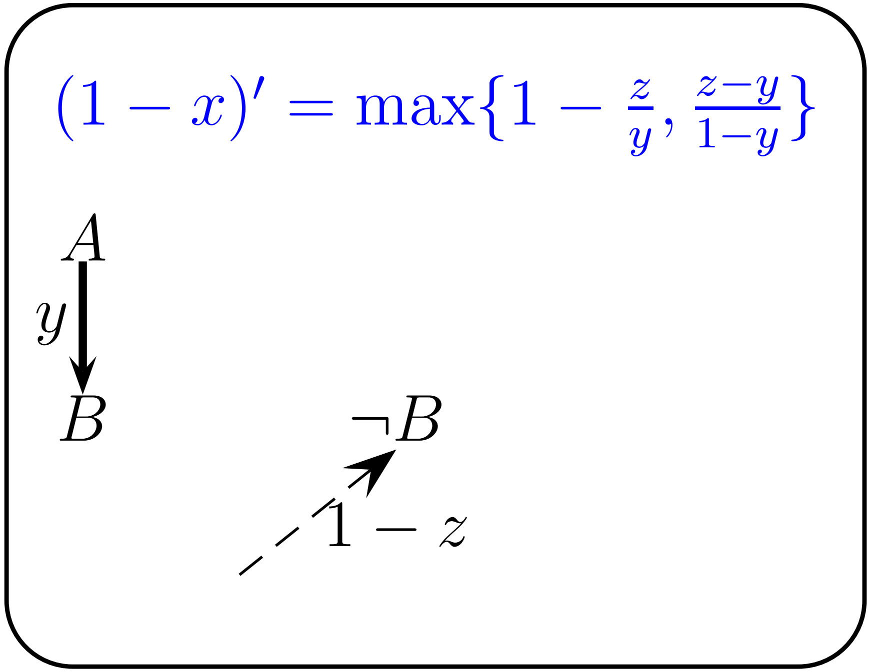
forward
affirmative

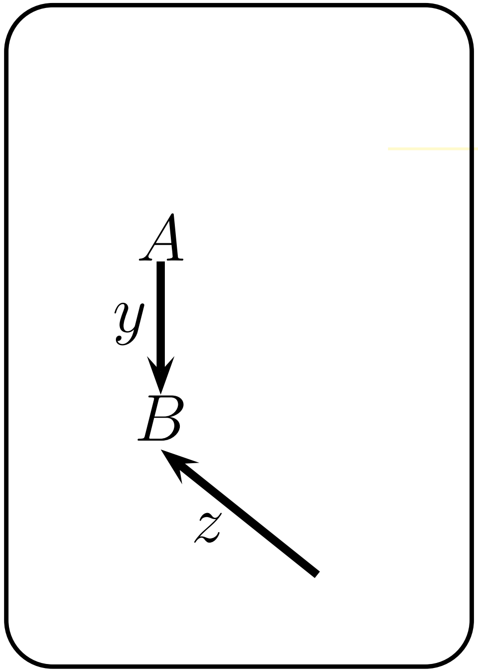
MODUS TOLLENS

$P(\neg A) = ?$

backward
negated







MODUS PONENS

$P(B) =?$

forward
affirmative

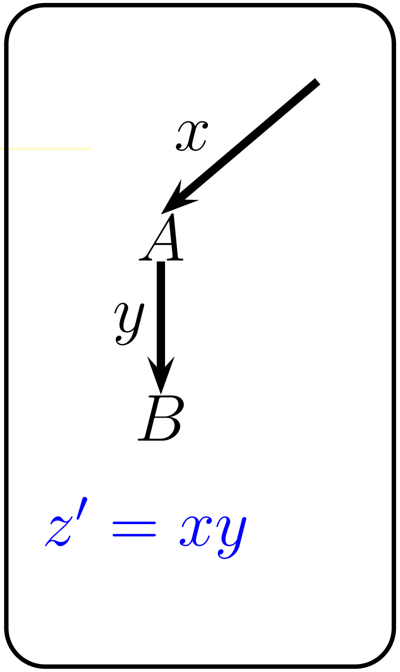
MODUS TOLLENS

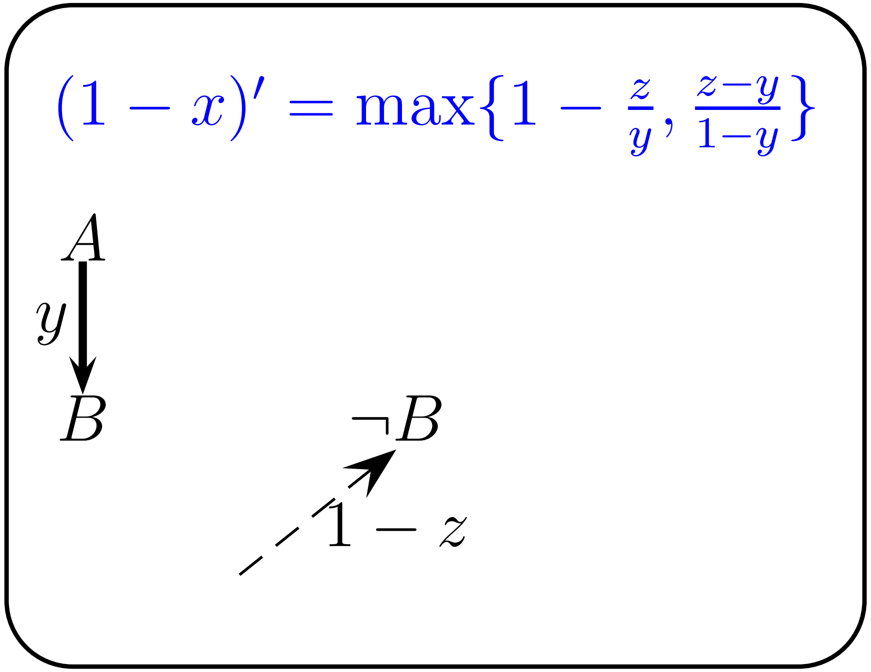
$P(\neg A) =?$

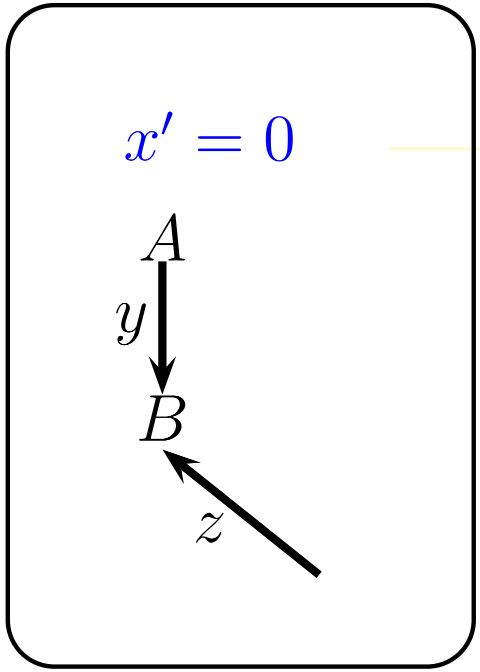
backward
negated

AFFIRMING THE CONSEQUENT

$P(A) =?$







MODUS PONENS

$$P(B) = ?$$

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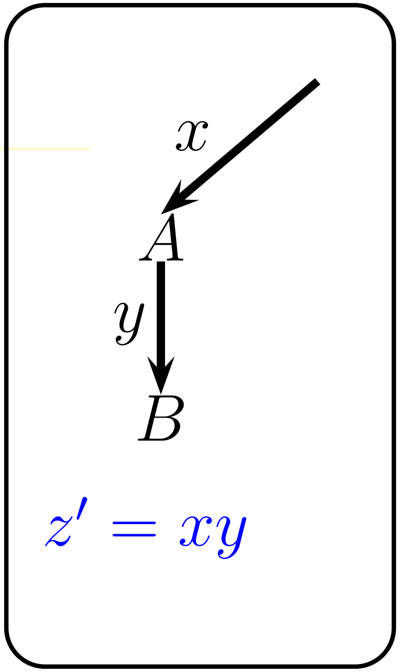
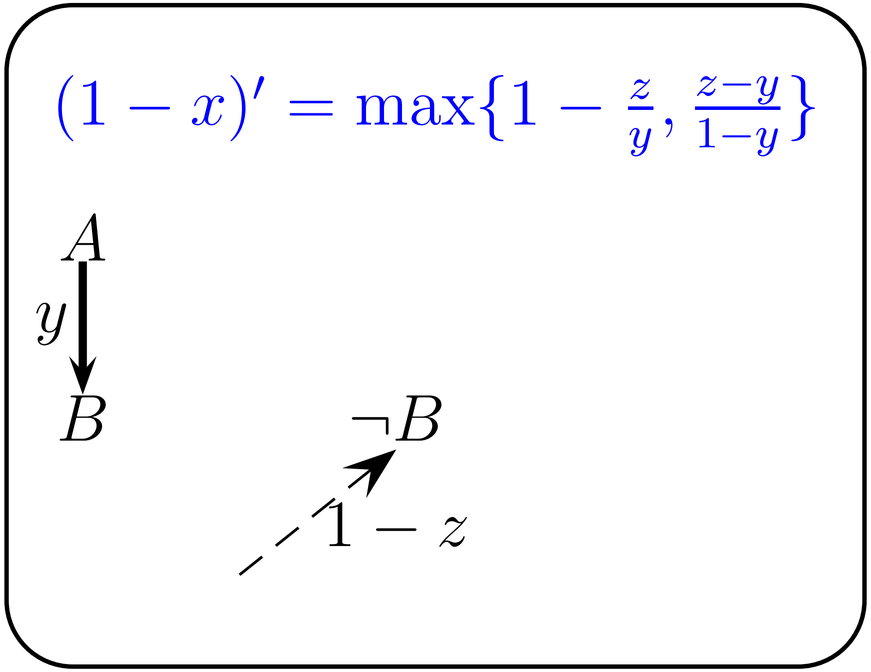


MODUS PONENS

$$P(B) = ?$$

forward
affirmative



MODUS TOLLENS

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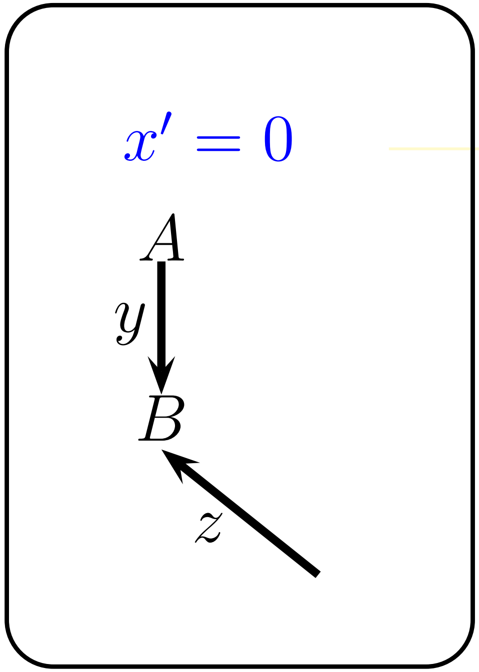
backward
negated



AFFIRMING THE CONSEQUENT

$$P(A) = ?$$

backward
affirmative

Logical validity vs. soundness

	MP
P_1:	$A \supset B$
P_2:	A
$\mathfrak{C}$:	B

Logical validity vs. soundness

	MP	NMP
P_1:	$A \supset B$	$A \supset B$
P_2:	A	A
$\mathfrak{C}$:	B	$\neg B$

Logical validity vs. soundness

	MP	NMP	DA	NDA
P_1:	$A \supset B$	$A \supset B$	$A \supset B$	$A \supset B$
P_2:	A	A	$\neg A$	$\neg A$
$\mathfrak{C}$:	B	$\neg B$	$\neg B$	B

Logical validity vs. soundness

	MP	NMP	DA	NDA
P_1:	$A \supset B$	$A \supset B$	$A \supset B$	$A \supset B$
P_2:	A	A	$\neg A$	$\neg A$
$\mathfrak{C}$:	B	$\neg B$	$\neg B$	B
L-valid:	yes	no	no	no

Logical validity vs. soundness

	MP	NMP	DA	NDA
P_1:	$A \supset B$	$A \supset B$	$A \supset B$	$A \supset B$
P_2:	A	A	$\neg A$	$\neg A$
$\mathfrak{C}$:	B	$\neg B$	$\neg B$	B
L-valid:	yes	no	no	no
$V(\mathfrak{C})$	t	f	?	?

$V(\mathfrak{C})$ denotes the truth value of the conclusion $\mathfrak{C}$ under the assumption that the valuation-function V assigns t to each premise.

Probabilistic argument forms

	Probabilistic versions of the			
	MP	NMP	DA	NDA
P_1:	$P(B\|A) = x$	$P(B\|A) = x$	$P(B\|A) = x$	$P(B\|A) = x$
P_2:	$P(A) = y$	$P(A) = y$	$P(\neg A) = y$	$P(\neg A) = y$
$\mathfrak{C}$:	$P(B) = z$	$P(\neg B) = z$	$P(\neg B) = z$	$P(B) = z$

The "IF A, THEN B" is interpreted as a conditional probability, $P(B|A)$.

Probabilistic argument forms

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	MP	NMP	DA	NDA
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$\mathfrak{C}$:	$P(B) = z$	$P(\neg B) = z$	$P(\neg B) = z$	$P(B) = z$
z'	xy		$(1-x)(1-y)$	
z''	$1-(y-xy)$		$1-x(1-y)$	

$z = f(x, y)$ and $z \in [z', z'']$

Probabilistic argument forms

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z'	xy	$y - xy$	$(1-x)(1-y)$	$x(1-y)$
z''	$1-(y-xy)$	$1-xy$	$1-x(1-y)$	$1-(1-x)(1-y)$

…by conjugacy: $P(\neg\mathfrak{C}) = 1 - P(\mathfrak{C})$

Probabilistic argument forms

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Chater, Oaksford, et. al: Subjects' endorsement rate depends only on the conditional probability of the conclusion given the categorical premise, $P(\mathfrak{C}|P_2)$

- the conditional premise is ignored
- the relation between the premise(s) and the conclusion is uncertain

Probabilistic argument forms

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Mental probability logic: most subjects infer coherent probabilities from the premises

- the conditional premise is not ignored
- the relation between the premise(s) and the conclusion is deductive

Results—Certain Premises (Pfeifer & Kleiter, 2003*, 2005a**, 2006)

Condition	lower bound		upper bound		
(Task B7)	M	SD	M	SD	n_i
CUT1	95.05	22.14	100	0.00	20
CUT2	93.75	25.00	93.75	25.00	16
RW	95.00	22.36	100	0.00	20
OR	99.63	1.83	99.97	0.18	30
CM*	100	0.00	100	0.00	19
AND**	75.30	43.35	90.25	29.66	40
M*	41.25	46.63	92.10	19.31	20
TRANS1	95.00	22.36	100	0.00	20
TRANS2	95.00	22.36	100	0.00	20
TRANS3	77.95	37.98	94.74	15.77	19

Inference from imprecise premises – “Silent bounds”

"Silent" bounds

A probability bound b of a premise is silent iff b is irrelevant for the probability propagation from the premise(s) to the conclusion.

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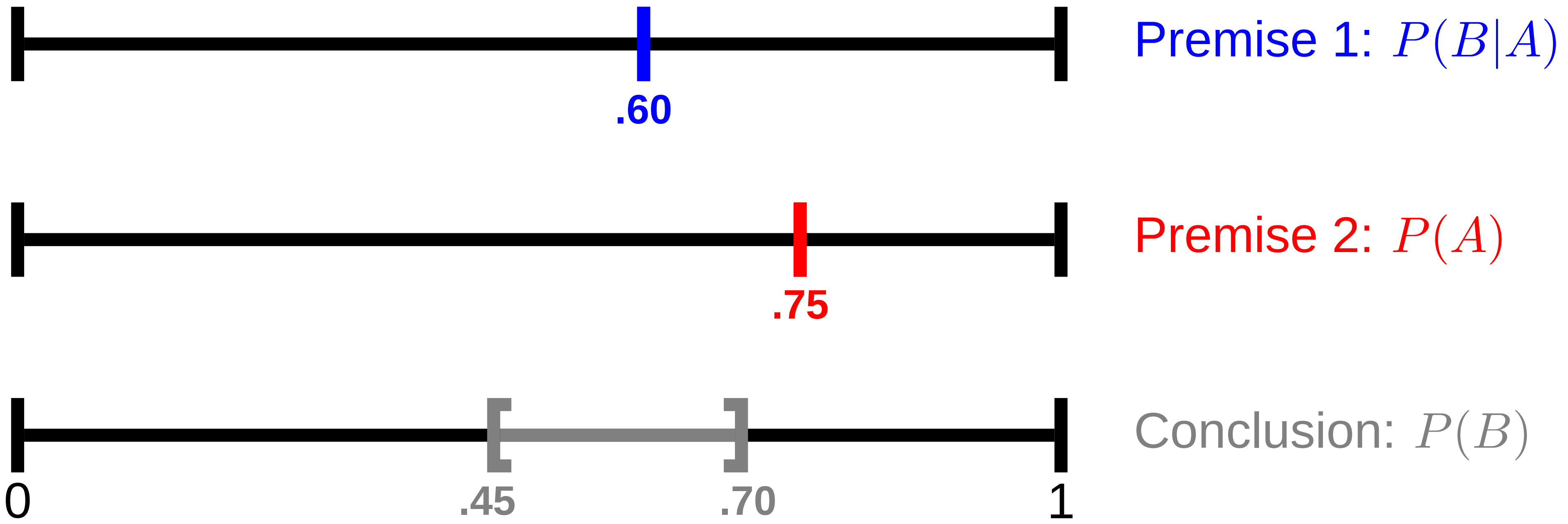

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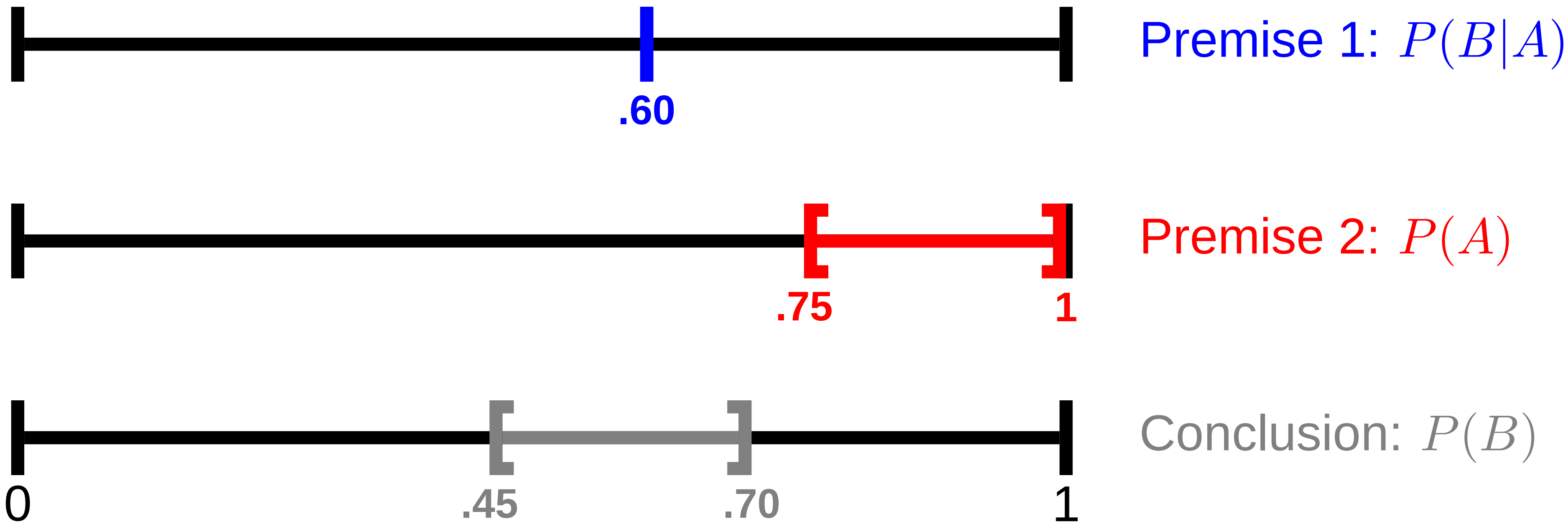

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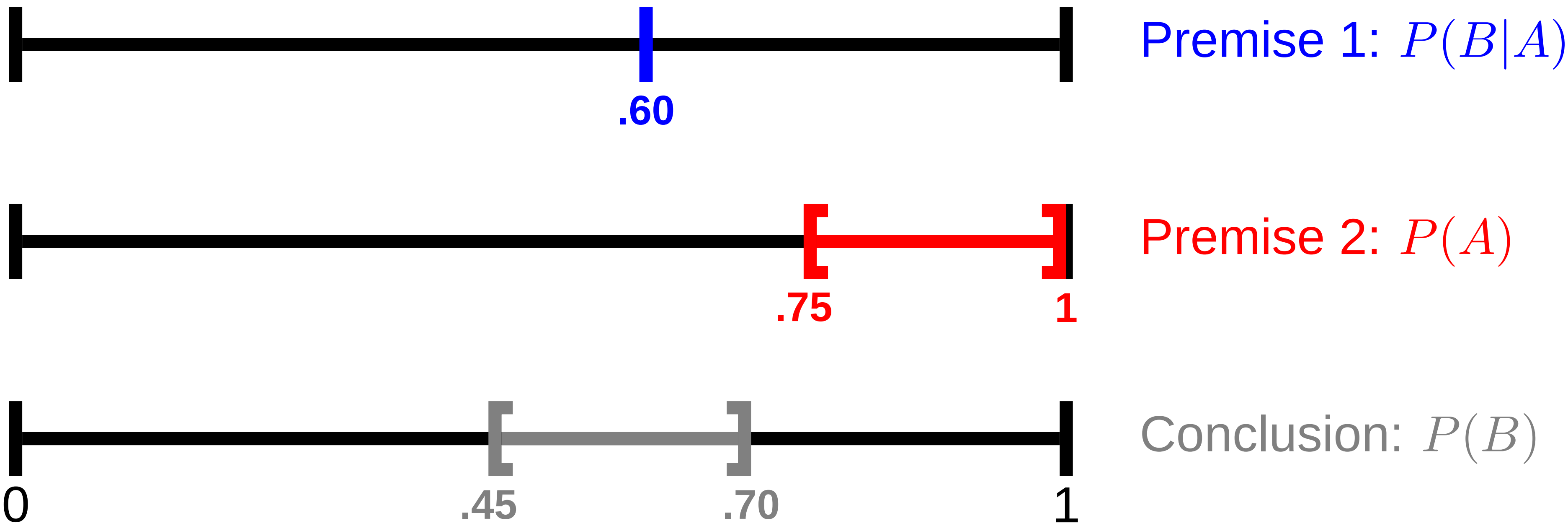

$$P(B|A) \in [x', x''], \; P(A) \in [y', \; y''] \; \therefore \; P(B) \in [x'y', 1 - y' + x''y']$$

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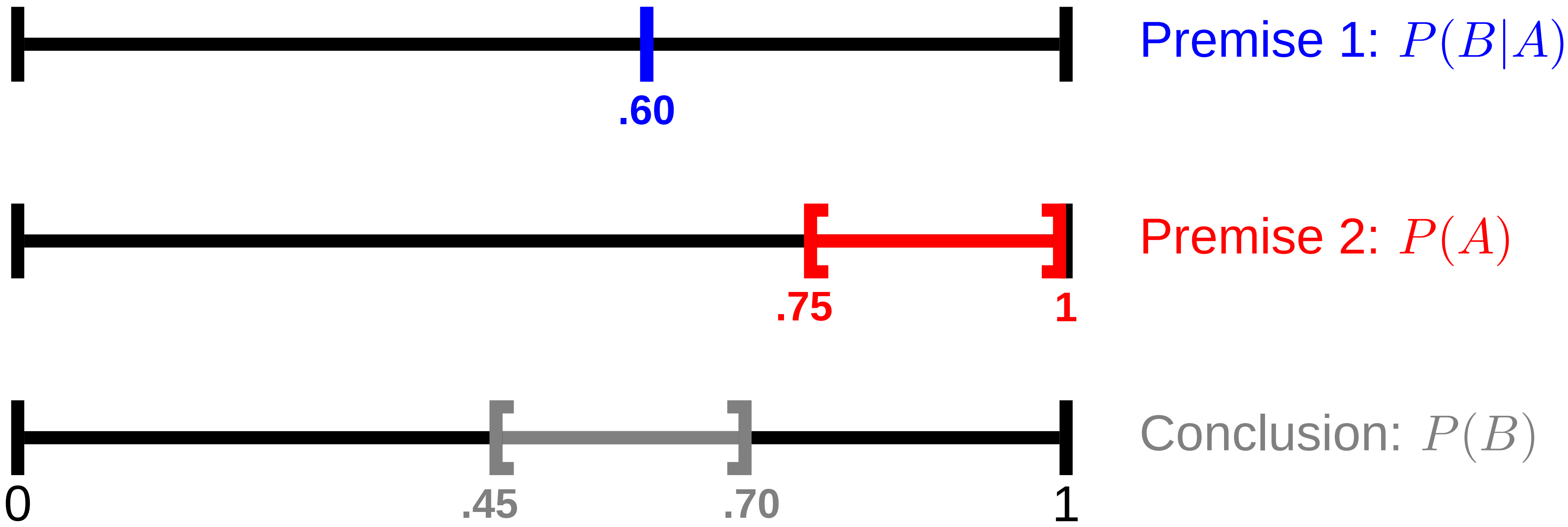

$$P(B|A) \in [x', x''],\ P(A) \in [y', \overbrace{y''}^{silent}]\ \therefore\ P(B) \in [x'y', 1 - y' + x''y']$$

MODUS PONENS **task with silent bound** (Bauerecker, 2006)

Claudia works at blood donation services. She investigates to which blood group the donated blood belongs and whether the donated blood is Rhesus-positive.

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How certain should Claudia be that a recent donated blood is Rhesus-positive?

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Results: Mean Responses (Bauerecker, 2006)

Task	*Premise*		*Coherent*		*Response*	
	1	*2*	*LB*	*UB*	*LB*	*UB*
MP	.60	.75-1^*	.45	.70	.45	.72
	.60	.75	.45	.70	.47	.60
NMP	.60	.75-1^*	.30	.55	.17	.46
	.60	.75	.30	.55	.23	.42

- Participants inferred higher intervals in the MP tasks: participants are sensitive to the complement

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- Participants inferred higher intervals in the MP tasks: participants are sensitive to the complement
- Participants inferred wider intervals in the tasks with the silent bound, 1^*: they are sensitive to silent bounds (i.e., they neglect the irrelevance of 1^*)
- More than half of the participants inferred coherent intervals

Frege's 1879 axioms for the propositional calculus

- $X \supset (Y \supset X)$
- $[X \supset (Y \supset Z)] \supset [(X \supset Y) \supset (X \supset Z)]$
- $[X \supset (Y \supset Z)] \supset [Y \supset (X \supset Z)]$
- $(X \supset Y) \supset (\neg Y \supset \neg X)$
- $\neg\neg X \supset X$
- $X \supset \neg\neg X$

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