International Carpathian Control Conference

Zakopane, Polska 24-27 maj 2009

DATA FUSION IN ESTIMATION PROBLEMS

Jiří AJGL, Miroslav ŠIMANDL

Department of Cybernetics
Faculty of Applied Sciences
University of West Bohemia in Pilsen
Czech Republic

Outline:

- Introduction
- System description
- Estimation problems
- Data fusion
- Millman's formula
- Unification of problems
- Conclusion

There are many data sources. The data fusion is needed.

The goal of the paper:

To show basic principles of data fusion.

To unify several estimation problems
in the framework of data fusion.

Let the system be described by

$$x_{k+1} = F\,x_k + G\,w_k\,,$$
$$z_k^{(j)} = H^{(j)}\,x_k + v_k^{(j)}\,,$$

where $x_k \in \mathbb{R}^n$ is the system state, $z_k^{(j)} \in \mathbb{R}^{m_j}$ is j-th sensor measurement, $j = 1, \ldots, S$ is the sensor number, $k = 0, 1, \ldots, t$ is a time index, $\{w_k\}$, $\{v_k^{(j)}\}$ are zero mean white gaussian processes with covariances Q, $R^{(j)}$, $\mathrm{E}(w_k\, v_l^{(j)T}) = 0$ for all k, l, j.

Many estimation problems can be posed:

a) fusion of prior information and measurement data
b) multi-sensor filtering
c) decentralized fusion
d) smoothing
e) ...

Data fusion will be considered to be linear:

$$\hat{x} = \sum_{i=1}^{N} c_i \hat{x}_i, \quad \sum_{i=1}^{N} c_i = I_n$$

where $\hat{x}_i, \quad i = 1, 2, \ldots, N$, are the estimates to be fused and their covariance matrices are

$$P_{ij} = cov(x - \hat{x}_i, x - \hat{x}_j), \quad i, j = 1, 2, \ldots, N.$$

Having the MSE criterion

$$J(c_{1,}c_{2,}\ldots,c_N)=E\left(\left\|x-\sum_{i=1}^{N}c_i\hat{x}_i\right\|^2\right),$$

the optimal solution is obtained by the generalized Millman's formula

$$\sum_{i=1}^{N}c_i(P_{ij}-P_{iN})+c_N(P_{Nj}-P_{NN})=0,$$

$$j=1,2,\ldots,N-1,$$

$$\sum_{i=1}^{N}c_i=I_N.$$

The new view of the estimations problems:

The estimation problems could be unified by the (generalized) Millman's formula.

$$\Rightarrow$$

The data fusion can be understood as a tool for synthesis of different estimators.

For independent estimates, the solution to the generalized Millman's formula has a closed form:

$$\hat{x} = P \sum_{i=1}^{N} P_{ii}^{-1} \hat{x}_i ,$$

$$P = \left(\sum_{i=1}^{N} P_{ii}^{-1} \right)^{-1}$$

The application of that relation will be shown for the above mentioned estimation problems.

Prior information and measurement data

Inspecting the Kalman filter update equation, the filtering estimate could be understood as a fusion of two independent estimates, the prior information estimate $\hat{x}_{k|k} = E(x_k | z_0^{(j)k}) = E(x_k | z_0^{(j)}, \ldots, z_k^{(j)})$ and the maximum likelihood estimate $\hat{x}_{ML} = \arg\max_{x_k} p(z_k | x_k)$,

$$P_{k|k}^{-1} \hat{x}_{k|k} = P_{k|k-1}^{-1} \hat{x}_{k|k-1} + P_{ML}^{-1} \hat{x}_{ML},$$
$$P_{k|k}^{-1} = P_{k|k-1}^{-1} + P_{ML}^{-1},$$

$P_{k|k-1}, P_{ML}$ are corresponding covariance matrices.

Multisensor fusion with fusion center

The Millman's formula can be used to combine the local Kalman filter estimates $\hat{x}^{(j)}_{k|k} = E\left(x_k | z^{(j)k}_0\right)$ at the fusion center given by

$$\hat{x}_{k|k} = P_{k|k} \sum_{j=1}^{S} P^{(j)-1}_{k|k} \hat{x}^{(j)}_{k|k},$$

$$P^{-1}_{k|k} = \sum_{j=1}^{S} P^{(j)-1}_{k|k},$$

where $P^{(j)}_{k|k}$ are corresponding covariance matrices.

Decentralized fusion

If the local Kalman filters network is fully connected, it is algebraically equal to the case with fusion center and a feedback to the local filters.

$$P_{k|k}^{-1}\hat{x}_{k|k}=P_{k|k-1}^{-1}\hat{x}_{k|k-1}+\sum_{j=1}^{S}\left\{P_{k|k}^{(j)-1}\hat{x}_{k|k}^{(j)}-P_{k|k-1}^{(j)\;-1}\hat{x}_{k|k-1}^{(j)}\right\},$$

$$P_{k|k}^{-1}=P_{k|k-1}^{-1}+\sum_{j=1}^{S}\left\{P_{k|k}^{(j)-1}-P_{k|k-1}^{(j)\;-1}\right\}$$

The terms under the sums can be understood as a reverse use of the update equation in the prior information and measurement data fusion problem.

Smoothing

The Millman's formula is used to combine forward $\hat{x}_{k|k} = E(x_k | z_0^k)$ and backward $\hat{x}_{k|k+1} = E(x_k | z_{k+1}^t)$ state estimates. The fusion relations are

$$K_k = P_{k|k} S_{k|k+1} \left(I_n + P_{k|k} S_{k|k+1} \right)^{-1},$$

$$P_k = (I_n - K_k) P_{k|k},$$

$$\hat{x}_k = (I_n - K_k) \hat{x}_{k|k} + P_k \hat{y}_{k|k+1},$$

where

$$\hat{y}_{k|k+1} = S_{k|k+1} \hat{x}_{k|k+1}, \quad S_{k|k+1} = P_{k|k+1}^{-1}$$

is the information form of the backward estimate.

Smoothing cont'

$P_{k|k}, P_{k|k+1}$ are corresponding covariance matrices,
$P_{k|k} = cov(x_k | z_0^k), P_{k|k+1} = cov(x_k | z_{k+1}^t)$.

The relations can be expressed in an alternative form

$$P_k = (P_{k|k}^{-1} + P_{k|k+1}^{-1})^{-1},$$
$$\hat{x}_k = P_k P_{k|k}^{-1} \hat{x}_{k|k} + P_k P_{k|k+1}^{-1} \hat{x}_{k|k+1}$$

which better shows the use of the Millman's formula.

Conclusion:

Various estimation problems were introduced
by using the Millman's formulas.

$\Rightarrow$

These estimation problems can be expressed
in a unique framework.

$\Rightarrow$

The data fusion can serve
as a tool for synthesis of new estimators.