Applications of Fuzzy Modelling

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Outline

Geological application - coral reef growth

Bosser and Schlager (1992):

$$\frac{dh(t)}{dt} = G_m \tanh\left(\frac{I_0}{I_k} exp\left(-k[(h_0 + h(t)) - (s_0 + s(t))]\right)\right)$$

where

$h(t)$ - the growth increment

G_m - the maximal growth rate

I_0 - the surface light intensity

k - the extinction coefficient

I_k - the saturating light intensity

h_0 - the initial height

s_0 - the initial sea level position

$s(t)$ - the sea level variation

Fuzzy Partition

Partition of $[a, b]$

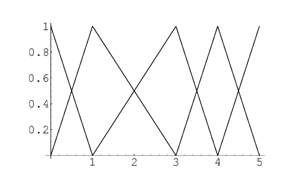
- $a = x_1 < x_2 < \cdots < x_n = b$
- $h(n) = \max_{k=1,\cdots,n-1} (x_{k+1} - x_k)$

Fuzzy Partition of $[a, b]$

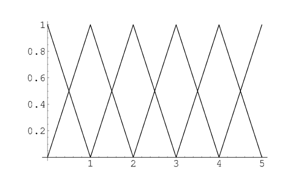
- $A_1(x), \ldots, A_n(x)$ - basis functions
- $A_k : [a, b] \to [0, 1],\ A_k(x_k) = 1$
- $A_k(x) = 0$ if $x \notin (x_{k-1}, x_{k+1})$ where $x_0 = a$ and $x_{n+1} = b$
- A_k is continuous
- $A_k(x)$ increases on $[x_{k-1}, x_k]$ and decreases on $[x_k, x_{k+1}]$
- $\sum_{k=1}^{n} A_k(x) = 1 \quad \forall x \in [a, b]$

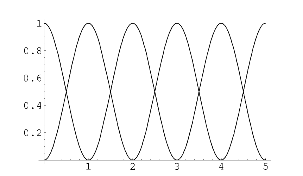
Examples of fuzzy partitions

General fuzzy partition



Uniform fuzzy partitions





Generalized Euler method

Cauchy problem

$$y'(x) = f(x, y) \qquad y(x_1) = y_1$$

Direct F-transform

$$Y_1 = y_1$$
$$Y_{k+1} = Y_k + h\hat{F}_k \qquad k = 1, \dots, n-1.$$
$$\hat{F}_k = \frac{\int_a^b f(x, Y_k) A_k(x) dx}{\int_a^b A_k(x) dx}$$

Inverse F-transform

$$y_{Y,n} = \sum_{k=1}^{n} Y_k A_k(x)$$

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Direct F-transform

$$\begin{aligned} Y_1 &= y_1 \\ Y_{k+1} &= Y_k + h\hat{F}_k \qquad\qquad k = 1, ..., n-1. \end{aligned}$$

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Inverse F-transform

$$y_{Y,n} = \sum_{k=1}^{n} Y_k A_k(x)$$

Reef near the island Belize

Bosser and Schlager (1992):

$$\frac{dh(t)}{dt} = G_m \tanh\left(\frac{I_0}{I_k} exp\left(-k[(h_0 + h(t)) - (s_0 + s(t))]\right)\right)$$

where

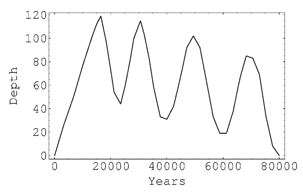
$G_m = 0,005\, mm\, yr^{-1}$ $\quad k = 0,05\, m^{-1}$

$I_0 = 2000\, \mu\, E\, m^{-2}\, s^{-1}$ $\quad I_k = 250\, \mu\, E\, m^{-2}\, s^{-1}$

$h_0 = 0\, m$ $\quad s_0 = 0\, m$

$s(t)$= sea level

Sea level

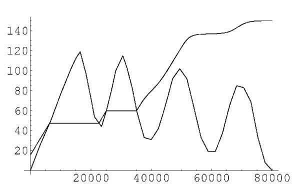


Choice of initial depth

$h(80000) = 0 - 200\ m$

Example

$h(80000) = 150\ m$



Original slope of reef

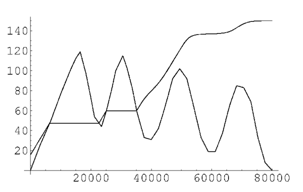
- 50°

Animation

- evolution of coral reef

Example

$h(80000) = 150\ m$



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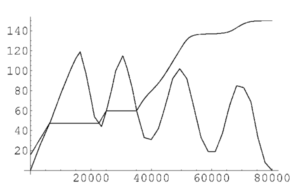

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Outline

Stabilization of water level in reservoir

Solid - cylinder

- height of cylinder: $h_{MAX} = 5\,m$
- radius of bottom: $r = 1\,m$

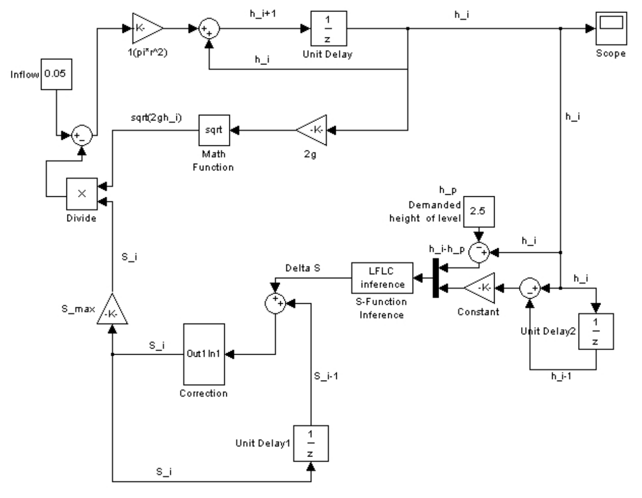
Other parameters

- constant inflow: $P = 0,05\,ms^{-1}$
- maximal outflow section: $S_{MAX} = 0,1\,m$
- demanded height of water level: $h_p = 2,5\,m$
- original height of water level: h_0

Equation

$$\pi r^2 \dot{h}(t) = P - \sqrt{2gh(t)}S(t)$$

Scheme



h_i+1
h_i
Inflow
0.05
-K-
1(pi*r^2)
h_i
1
z
Unit Delay
Scope
sqrt(2gh_i)
sqrt
Math Function
-K-
2g
h_i
×
Divide
h_p
Demanded height of level
2.5
h_i-h_p
h_i
S_i
Delta S
LFLC inference
S-Function Inference
-K-
Constant
h_i
S_max
-K-
Unit Delay2
S_i
Out1 In1
Correction
S_i-1
h_i-1
Unit Delay1
S_i

Fuzzy control

Fuzzy control

LFLC 2000 (Linguistic Fuzzy Logic Controller) - Institute for Research and Applications of Fuzzy Modeling

Inference method

Logical deduction

Defuzzification

Defuzzification of Linguistic Expression

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Input variables

- $(h_i - h_p) \in < -2,5; 2,5 >$
- $(h_i - h_{i-1}) \in < -10, 10 >$

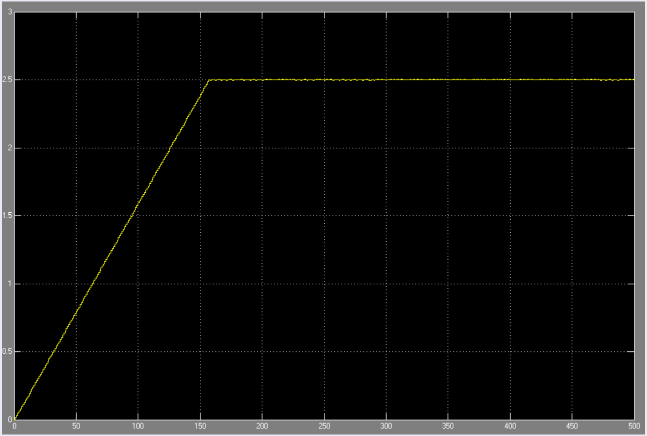
Output variable

- $\Delta S_i \in < -1; 1 >$

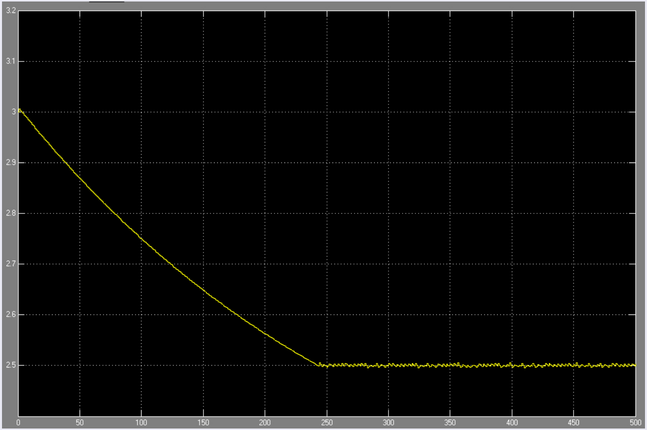
Rules

- Number - 13
- Examples:
  1. IF $h_i - h_p$ is *+ big* AND $h_i - h_{i-1}$ is *+ big* THEN $\triangle S_i$ is *+ extremely big*
  2. IF $h_i - h_p$ is *- zero* AND $h_i - h_{i-1}$ is *- small* THEN $\triangle S_i$ is *- small*

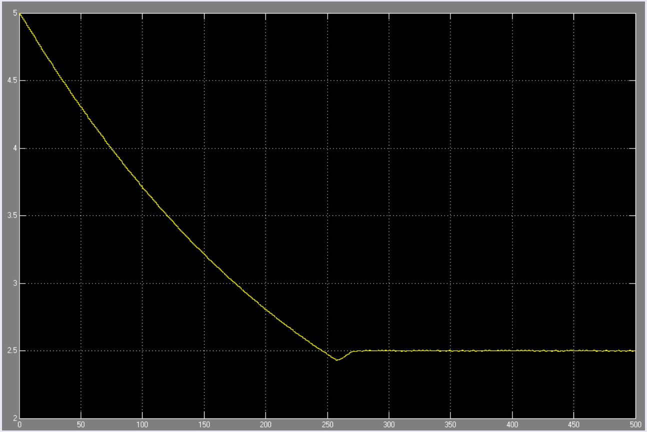
Original height of water level: $h_0 = 0\ m$



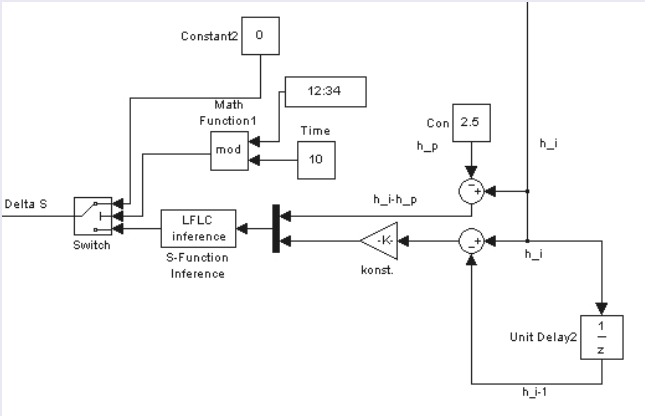
Original height of water level: $h_0 = 3\ m$



Original height of water level: $h_0 = 5\,m$

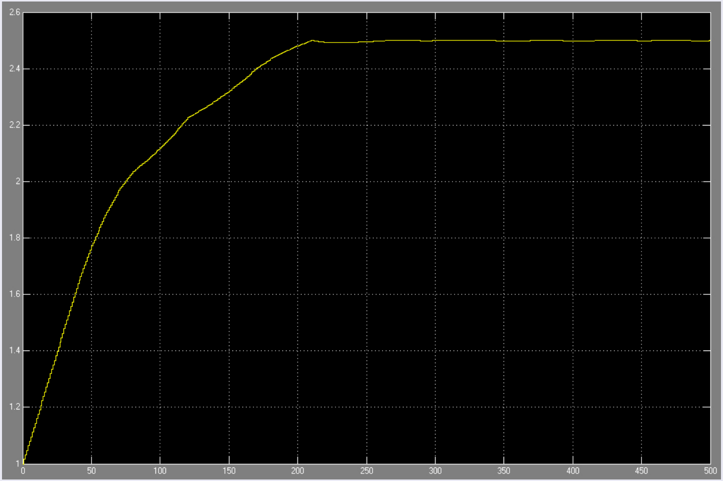


Simulation: different sample time for control LFLC and solver ODE

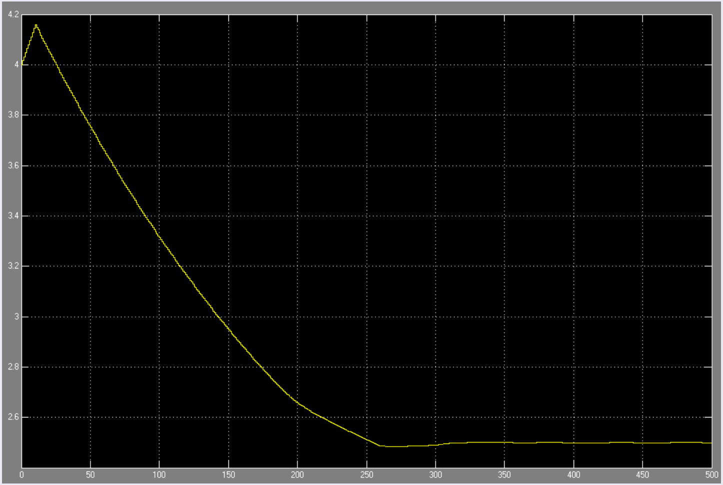


Number of rules: 16

Original height of water level: $h_0 = 1\ m$



Original height of water level: $h_0 = 4\ m$



Outline

Future work

Control of reservoir of Těrlicko

- control of water discharge from reservoir during flood passage
- Goal: elimination of spring flood
- Problem: keep required outflow
- Solving: PI or P controller

Thank you for your attention