Fuzzy transforms – a new bases for image fusion

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Problem specification

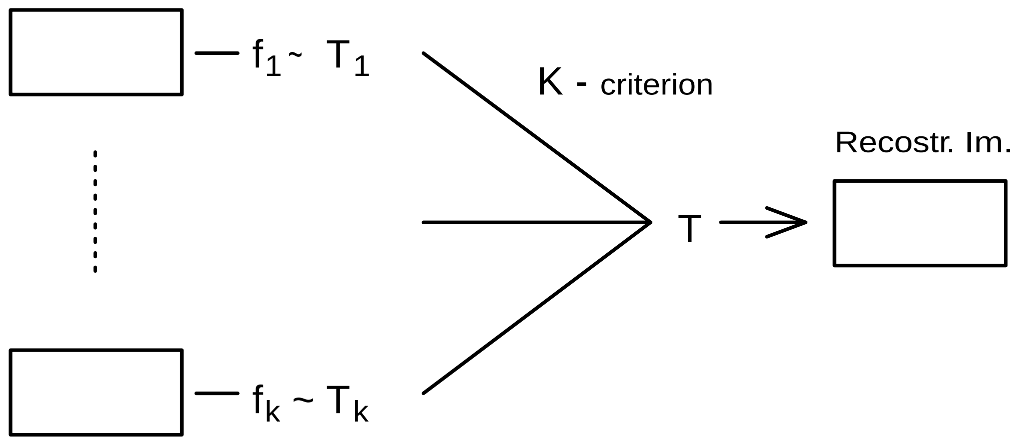
- What?

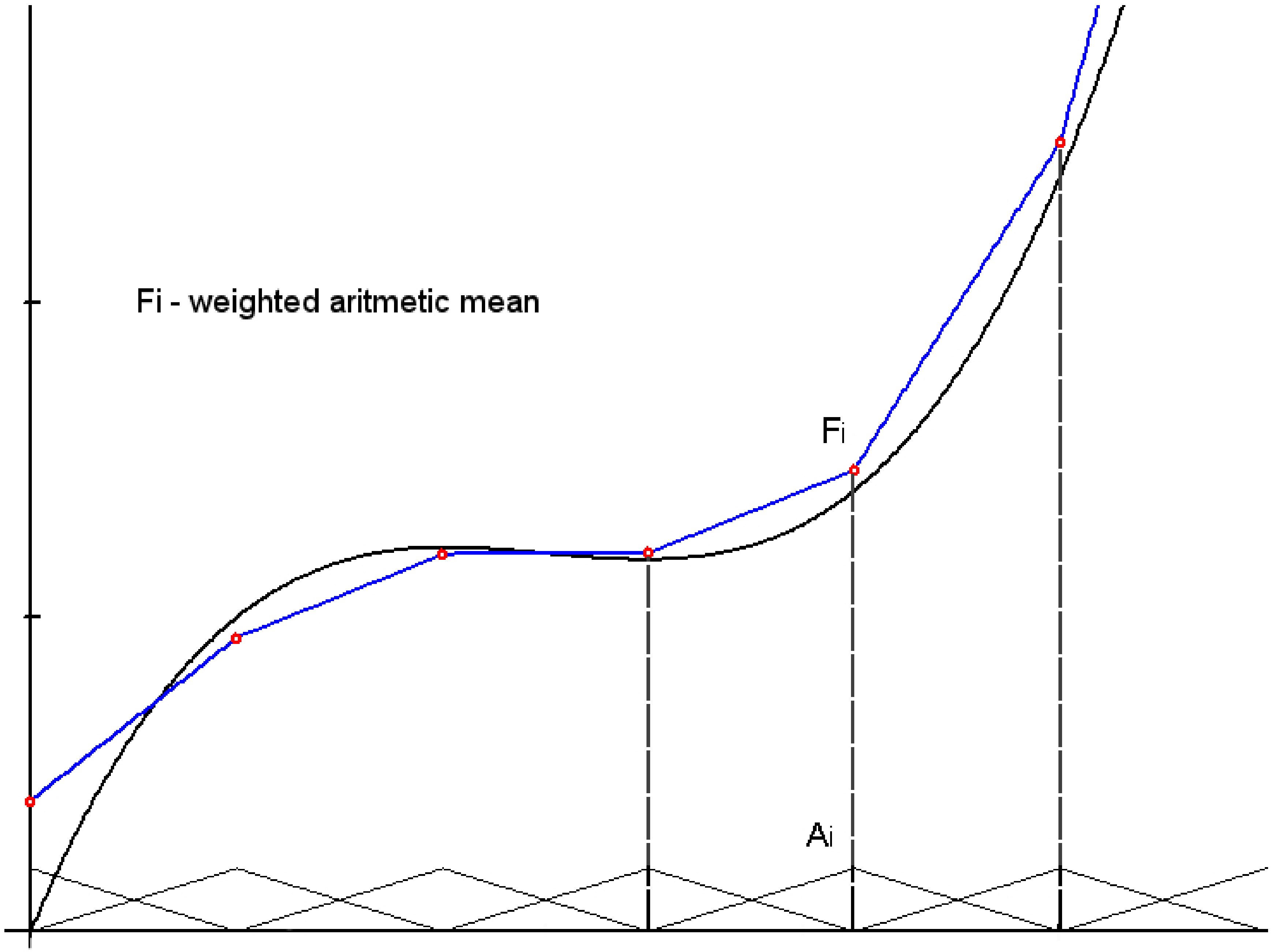
  Image fusion -

$$\text{"k" partially distorted images} \\ \Downarrow \\ \text{one "good" image}$$

- Tool?
  - Wavelet transform
  - Fuzzy transform

- How (Principle)?




Fi - weighted aritmetic mean
Fi
Ai

F-transform: Let $f(x)$ be given at nodes $x_1, \dots, x_l \in [a, b]$, and $\mathbb{A} = \{A_i(x)\}_{i=1}^n$ be Ruspini's partition. The vector of real numbers $[F_1, \dots, F_n]$ is a discrete F-transform of f if

$$F_i = \frac{\sum_{j=1}^{l} f(x_j) A_i(x_j)}{\sum_{j=1}^{l} A_i(x_j)},$$

where $1 \leq i \leq n$ and $n < l$.

Inverse F-transform: Let $[F_1, \dots, F_n]$ be the F-transform of $f(x)$ w.r.t. $\mathbb{A}$. Then

$$T_{f,n}(x) = \sum_{i=1}^{n} F_i A_i(x)$$

is called the inverse F-transform.

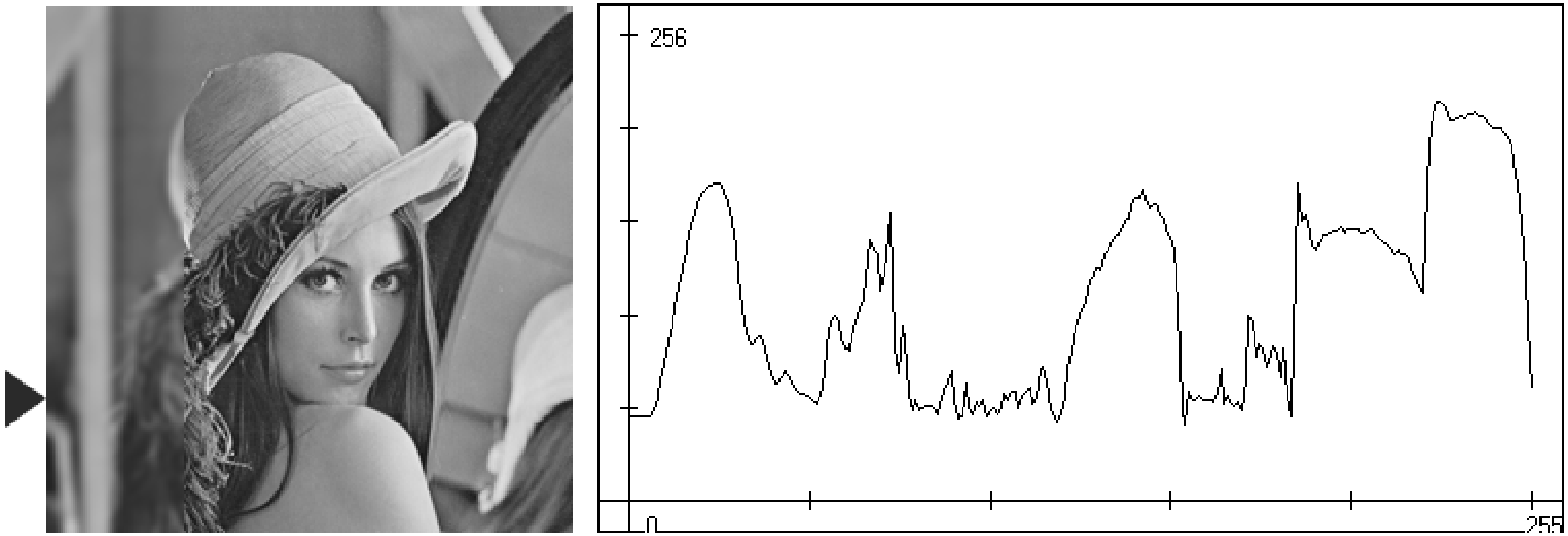
Removing noise by F-transform

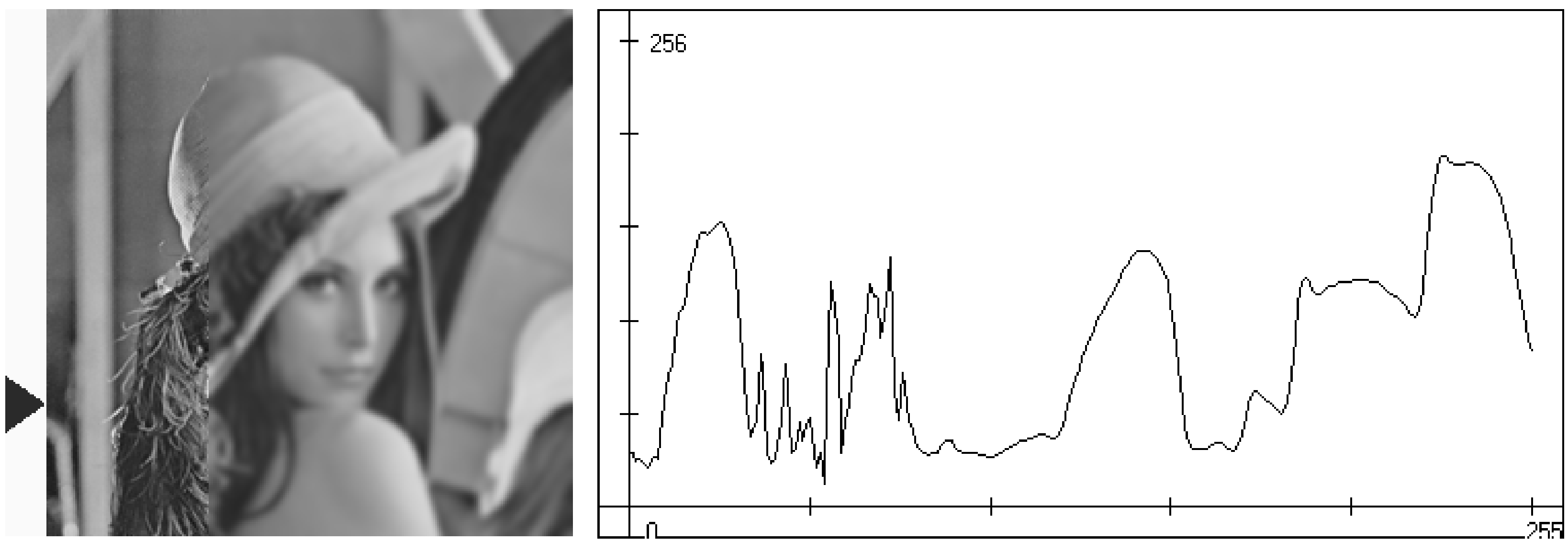
Lemma1:

$\mathbb{A}$ – a uniform fuzzy partition on $[a, b]$, $h = (b-a)/(n-1))$.
$s(x)$ – continuous periodical function on $[a, b]$ with period $2h$ such that

$$s(x_k - x) = -s(x_k + x), \quad x \in [x_{k-1}, x_{k+1}].$$

Then F_i are equal to zero and noise $s(x)$ is removable.


256
255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Description of fusion using fuzzy transform

- $f_{T,0}(x)$ stands for arithmetic mean of $f(x)$ and error function $e_0 = f(x) - f_{T,0}(x)$.

- For $i \geq 1$,

$$f_{T,i}(x) = T_{e_{i-1},2^i},$$

represents fuzzy transform of e_{i-1} in 2^i nodes and

$$e_i(x) = e_{i-1}(x) - f_{T,i}(x).$$

- Fusion function operates over the coefficients of fuzzy transforms of $f_{1_{T,i}}, \ldots, f_{p_{T,i}}$ in each level and it is defined by

$$\kappa(x,y) = \begin{cases} y, & |x| \leq |y| \\ x, & \text{otherwise.} \end{cases}$$

- Fused function is given by

$$F_T(x) = \bar{f}_{T,1}(x) + \bar{f}_{T,2}(x) + \bar{f}_{T,3}(x) + \ldots = \sum_{i=0}^{\infty} \bar{f}_{T,i}(x),$$

  where

$$\bar{f}_{T,i}(x) = \sum_{i \in I} F_i A_i(x)$$

  for each i and $F_1, \ldots, F_{2^i}$ are determined on the basis of coefficients of fixed fuzzy transformations $f_{1_{T,i}}, \ldots, f_{p_{T,i}}$ using κ.

Another F-transform

A residuated lattice on L is an algebra

$$\mathcal{L} = \langle L, \vee, \wedge, *, \rightarrow, 0, 1 \rangle \tag{1}$$

with four binary operations and two constants such that

- $\mathcal{L} = \langle L, \vee, \wedge, 0, 1 \rangle$ is a lattice with the largest element 1 and the least element 0 w.r.t. the lattice ordering $\leq$,

- $\mathcal{L} = \langle L, *, 1 \rangle$ is a commutative semigroup with the unit element 1, i.e. $*$ is commutative, associative, and $1 * x = x$ for all $x \in L$,

- $*$ and $\rightarrow$ form an adjoint pair, i.e.

$$z \leq (x \rightarrow y) \quad \text{iff} \quad x * z \leq y \quad \text{for all } x, y, z \in L. \tag{2}$$

F-transform on $\mathcal{L}$: Let $f(x) : [a, b] \to L$ be given at nodes $x_1, \ldots, x_l \in [a, b]$, and $\mathbb{A} = \{A_i(x)\}_{i=1}^n$ be partition on $[a, b]$ such that $\bigvee_i A_i(x) = 1$ for all $x \in [a, b]$. The vector $[F_1, \ldots, F_n] \in L^n$ is a discrete F-transform of f if

$$F_i = \bigwedge_{j=1}^{l} A_i(x_j) \to f(x_j),$$

where $1 \leq i \leq n$ and $n < l$.

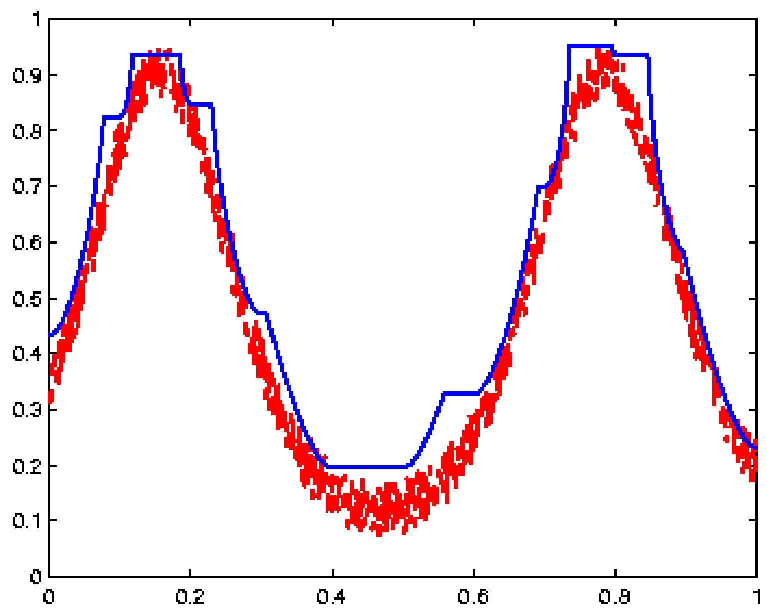
Inverse F-transform: Let $[F_1, \ldots, F_n]$ be the F-transform on $\mathcal{L}$ of $f(x)$ w.r.t. $\mathbb{A}$. Then

$$T_{f,n}(x) = \bigvee_{i=1}^{n} F_i * A_i(x)$$

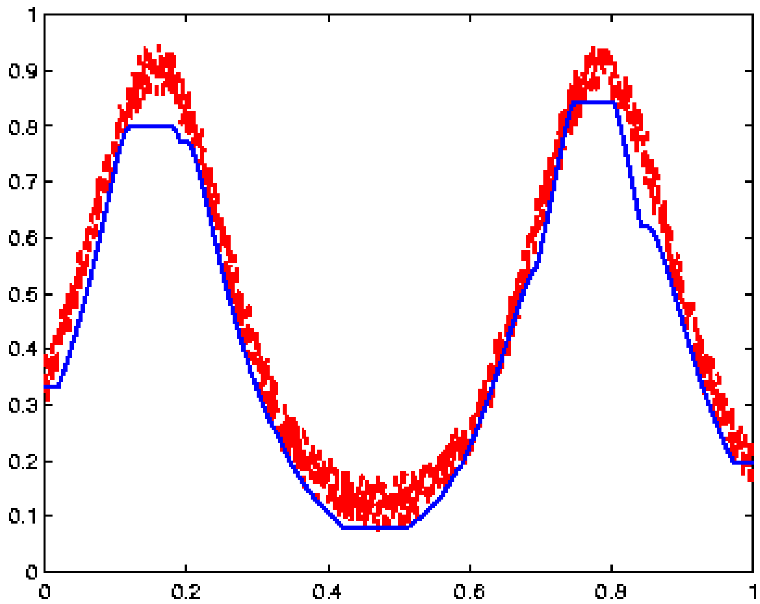
is called the inverse F-transform.

the coefficients carry a different information –

- weighted arithmetical means (fuzzy sets represents weights)
- minima on the respective fuzzy subset
- maxima on the respective subset



0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
0.2
0.4
0.6
0.8
1



0
0.1
0.2
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0.4
0.5
0.6
0.7
0.8
0.9
1
0
0.2
0.4
0.6
0.8
1

Thank you for the attention.