FULLY PROBABILISTIC DECISION MAKING

at e-Democracy Service

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Abstract E-democracy has many facets addressed by respective experts. The paper contributes to this domain by putting its problems into the general framework of a suitable probabilistic decision making (DM). The approach: i) provides the methodology of fully probabilistic design of decision strategies that, among others, allows efficient handling of multi-criterion problems; ii) presents tools for cooperation of participants involved; iii) characterizes respective design elements and outlines ways how to construct them; iv) points to consequences of limiting perceiving and processing capabilities of DM participants.

Keywords: Bayesian decision making, randomized strategies, multi-criterion decision making, multi-participant decision making

1. Introduction

Technically, e-democracy can be seen as complex dynamic decision making (DM) with heterogenous participants. Each of them has its multiple aims and limited communication abilities. The multiplicity and limitations influence DM process significantly. Supporters and developers of e-democracy framework are aware of this fact but relative infancy of the field make them to orient themselves to other aspects. This state brings, however, problems, which could be counteracted when respecting the mentioned multiplicity and limitations. For instance, treatment of multiple aims has been addressed repeatedly, see e.g. the classical book [Keeny and Raiffa, 1978], but the methodology can hardly be applied at large scale inherent to the domain of e-democracy. Similarly, politicians as well as researchers are repeatedly disappointed by a weak response of citizens: they seem to resign on the participation offered to them. It is intuitively obvious that it is caused by citizens' information overload, which is, however, studied only commercial sphere, see [Malhorta, 1982] and reference in it, and in market modeling [Sims, 2002].

The paper touches the above aspects and provides a brief overview of the fully probabilistic design of decision-making strategies, Section 2. It includes also description of tools supporting cooperation of participants. The application of this theory to a prototype problem in participatory democracy forms the paper core, Section 3. Concluding remarks, Section 4, close the paper.

2. Prescriptive fully probabilistic DM

This section summarizes *formulation* of a prescriptive version of multiple participants' DM labeled as fully probabilistic DM. Details and solutions of respective subtasks can be found in [Guy et al., 2004, Kárný and Guy, 2004, Kárný et al., 2005, Kárný and Guy, 2006]. The formulation prepares the discussion of the e-democracy as a specific instance of such DM.

2.1 Single-participant DM

The participant is a decision maker, which has a freedom to select an *decisions* A from a non-empty set A^*. The decisions influence participant's environment, a part of the world. The prescriptive DM theory provides the participant with the methodology of constructing decisions so that the participant's aims are achieved in the best possible way under given circumstances.

DM always supposes a kind of uncertainty, i.e., some unknown internal quantities should be considered. The DM exploits combination of available *observations* Y and unavailable *internals* X for expressing the participant's aims with respect to the closed-loop formed of participant and its environment.

The design selects DM strategy $(\mathsf{R} : D^* \equiv (A, Y)^* \rightarrow A^*) \equiv$ (mapping: already observed *data* $\rightarrow$ decisions). Under rather general conditions, [Savage, 1954, DeGroot, 1970, Berger, 1985], the best strategy is to be defined as minimizer of the expectation of the loss function $\mathsf{L}(D, X)$. The *joint distribution* (probability density function, pdf) $f(D, X)$ of observations and internals is the needed model of the closed-loop "participant-environment". The adopted *fully probabilistic design* (FPD) selects the loss function in the form

$$\mathsf{L}(D, X) = \ln\left(\frac{f(D, X)}{{}^{I}f(D, X)}\right), \tag{1}$$

where ${}^{I}f(D, X)$ is an *ideal pdf* expressing the participant's aims as the desired distribution the closed-loop *behavior* $\mathcal{Q} \equiv (D, X)$. The corresponding expected loss becomes the *Kullback-Leibler divergence* (KLD) of the behavior model $f(D, X) = f(\mathcal{Q})$ on its ideal version ${}^{I}f(D, X) = {}^{I}f(\mathcal{Q})$

$$\mathsf{D}\left(f || {}^{I}f\right) \equiv \int f(\mathcal{Q}) \ln\left(\frac{f(\mathcal{Q})}{{}^{I}f(\mathcal{Q})}\right) d\mathcal{Q}. \tag{2}$$

The KLD [Kullback and Leibler, 1951] is a good proximity measure of a pair of distributions, which among other, reaches the smallest zero value for $f = {}^{I}f$. Moreover, the KLD is infinite if the support of ${}^{I}f$ is not included in that of f. Thus, the loss (1) represents also hard constraints on the desirable behavior.

The joint pdf describing behavior can be factorized to the *model of the environment* $f\left(Y, X \middle| A\right)$, describing response of the environment on decisions A, and the pdf $f\left(A\right)$, describing the distribution of decisions, i.e., *DM strategy*. The former one results from *modeling and learning*, the latter one results from the *design*. Thus, among other advantages, the FPD treats the model, aims and the constructed strategy in the same probabilistic way and the optimal DM strategy is searched via the formally simple optimization:

$$\text{optimal strategy} \equiv {}^{O}f\left(A\right) \in \arg\min_{f(A)^{*}} \mathsf{D}\left(f \middle|\middle| {}^{I}f\right). \tag{3}$$

Remark. The complexity of (3) is given by the fact that overall *decision making process* of one participant supposes a number of different DM tasks solved “simultaneously". Moreover, at any specific *instance of the real continuous time* (say, absolute time), the participant solves *only* one *DM task*.

As the solution of any DM task is not one-shot act and requires a time period, “simultaneous" implies *consecutive switching* of participant’s attention over its DM tasks. While switching, the participant interrupts solving one DM task and deals with another DM task, possibly, already partially solved. Then, after some time period, it returns and continues in solving the original task.

To describe the return point to the original DM task, a *relative time* indicating how much time passed within the period needed to solve this task is stored. The set of these relative times as well as switching sequence are “naturally" ordered by the continuous-time flow and participant’s schedule. The decisions, observations and internals involved in the *overall* DM process are thus various sequences of multivariate variables.

The model of behavior $\mathcal{Q}$ within a single DM task can be factorized:

$$f\left(\mathcal{Q}\right) = \prod_{t \in t^{*}} \overbrace{\overbrace{f\left(y_t \middle| a_t, d^{1:t-1}, x^{1:t}\right)}^{observation\ model} \overbrace{f\left(x_t \middle| a_t, d^{1:t-1}, x^{1:t-1}\right)}^{model\ of\ internals} \overbrace{f\left(a_t \middle| d^{1:t-1}\right)}^{DM\ strategy}}^{design\ elements\ (including\ domains\ of\ pdfs)}, \tag{4}$$

where $A \equiv a^{1:\mathring{t}} \equiv (a_1, \ldots, a_{\mathring{t}})$, $Y \equiv y^{1:\mathring{t}}$, $X \equiv x^{1:\mathring{t}}$. The omission of $x^{1:t-1}$ in the strategy reflects the fact that, by definition, the internals are unobserved and thus unavailable for the choice of decisions. This assumption on conditional independence is one of the several needed during the construction of the observation model and the model of internals. Otherwise learning [Peterka, 1981], i.e., formally simple operations of inserting data and evaluation of the pdfs derived from the joint pdf (4), crosses quickly capabilities of any computer.

The ideal pdf ${}^{I}f\left(\mathcal{Q}\right)$ can be structured in a similar way as (4) and faces the same complexity barrier when trying to express, typically multi-modal, participant's aims and complex constraints.

In spite of a well-developed art of modeling and approximate learning, which balances feasibility and expressiveness of the involved pdfs, the complexity barrier of DM is reached very soon. It becomes even more obvious if the practical realization of DM is respected. Hence, a *real* participant can deal with DM problems of very limited complexity only. Naturally, the single-participant DM cannot cope with the growing complexity of modern DM tasks increased by the complexity of particular application domains. And it is not surprising that a multi-participants DM is more rule than exception.

2.2 Multi-participants DM

In multi-participants' setting, participant's environment includes other participants solving their own DM tasks. Participants exhibit some direct interaction when solving theirs DM problems are here called *neighbors*. The complexity barrier is not crossed when the participant just models other participants and counteract their undesirable influences and exploits desirable ones. An improvement can be hoped for only when a sort of *cooperation* between participants takes place.

Within the adopted framework, the cooperation means (possibly partial) sharing one or several design elements. The design elements concerns *models*, *ideals* as well as observation and decision *domains*. Sharing any design element can be compulsory or optional. The optional sharing is restricted only by abilities of cooperators, determining, for instance, the degree of behavior overlapping. The compulsory sharing establishes a cooperation hierarchy. It determines the participant's operation domain and the type of information shared in accordance with the participant's position within the cooperation structure.

Three types of cooperation scenarios differing in the strength of the mutual influence can be distinguished [Kárný and Guy, 2004].

The weakest *selfish scenario* supposes that the participant shares actively with another participant design elements. The receiving participant may behave in a selfish way and respect the received information only partially. In the extreme case, the receiving participant does not change own design elements. No common model and ideal have to arise even when the cooperating participants modify their design elements.

Even the selfish scenario requires some cooperation as the participants provide their design elements at least partially. Providing the correct models or the true ideal is not, however, compulsory, i.e., the participant may cheat.

In contrast to the selfish scenario, the *cooperative scenario* assumes that the mutual exchange of the design elements is followed by their *obligatory* updating to *common models and ideal* concerning to their common part of behaviors. They result from a, usually iterative, negotiation.

Potentially the most powerful *hierarchical scenario* assumes existence of a participant coordinating a group of participants. Exceptional role of the coordinator lies in its power to influence members of the group by enforcing the design elements. The potential of this scenario is restricted by the complexity of the DM faced by the coordinator and by always limited possibility to enforce anything completely: the transfer of the elements between hierarchical levels is always corrupted by a sort of noise.

2.3 Sharing of design elements

Sharing of the design elements is the key specificity of the multi-participants' DM comparing to its single-participant version. Processing of this information *must not require a substantial increase of capabilities of participants involved.* Otherwise, they would be optimally asked to care about the DM in its full extent and the troubles of the single-participant DM would be re-established.

Concerning the observations and internals, different participants use generally different types of models even on common parts of their domains. Thus, they can process only their specific derivatives they understand. A plausible solution has been developed for time-invariant internals, with

$$f\left(x_t \middle| a_t, d^{1:t-1}, x^{1:t-1}\right) = \delta(x_t - \Theta) \equiv \text{Dirac function}, \ \Theta \in \Theta^*, \forall\, t \in t^*,$$

see (4), where Θ is called *parameter* and learning reduces to its estimation

$$f\left(\Theta \middle| d^{1:t}\right) \underbrace{\propto}_{proportionality} f\left(y_t \middle| a_t, d^{1:t-1}, \Theta\right) f\left(\Theta \middle| d^{1:t-1}\right). \tag{5}$$

Let us consider that the cooperation takes place at time τ when the participant starts the learning with the prior pdf $f\left(\Theta \middle| d^{1:\tau-1}\right)$. During the sharing, this pdf should be modified by the *predictor* – data-describing pdf – ${}^{n}f\left(d^{1:\tau}\right)$ provided by a neighbor n as follows

$$f\left(\Theta \middle| d^{1:\tau-1}\right) \exp\left[\nu \int {}^{n}f\left(d^{1:\tau}\right) \ln\left(f\left(y_\tau \middle| a_\tau, d^{1:\tau-1}, \Theta\right)\right)\right] dd^{1:\tau} \tag{6}$$

with $\nu \geq 0$ selected by the receiving participant. The procedure is discussed at length in [Kárný et al., 2006]. The following remarks are relevant to this text.

- The formula (6) reduces to the standard Bayesian estimation when the supplied predictor is the sample pdf representing some observed data.
- Typically, the overlap of behaviors is only partial. Then, the formula (6) has to be applied to appropriate marginal pdfs.
- The weight ν can be interpreted as a degree of confidence into the shared information. In the selfish scenario, the participant selects it freely. The information is discarded by setting $\nu = 0$. In the cooperation scenario, the negotiation just harmonizes the weights ν assigned by the cooperating participants. In the hierarchic scenario, the top level participant enforces extremely high weigh to the predictor.
- The justification of (6) seems to be extendable to the general internals.

The participant can be informed about its neighbor decisions, as their observable part belongs to participant's observations. Even in this situation, it is reasonable to share and possibly modify the ideal pdfs.

Naturally, the participants want to stick to their ideal pdfs. At the same time, it is clear if the participant's aims contradict to the aims of a neighbor then the achievement of its aims is endangered. Thus, kth participant searches for an ideal pdf ${}^{kI}\hat{f}(Q)$, which approximates the original one ${}^{kI}f\left(\mathcal{Q}\right)$ and achieves the necessary compromise with its neighbors.

When staying within the adopted fully probabilistic approach, it can be shown [Bernardo, 1979] that ${}^{kI}\hat{f}(Q)$ should minimize the KLD of ${}^{kI}f\left(\mathcal{Q}\right)$ on ${}^{kI}\hat{f}(Q)$. The other participants have the same wishes. Thus, a commonly acceptable compromise has to be searched on Pareto frontier. This determines

$$\text{optimal compromise ideal } \equiv {}^{kI}\hat{f}(Q) \in \mathrm{Arg} \min_{{}^{I}f(\mathcal{Q})} \sum_{p \in {}^{k}p^{*}} \alpha_p \mathsf{D}\left({}^{pI}f \middle|\middle| {}^{I}f \right), \tag{7}$$

where α_p are optional probabilistic weights and summation runs over set of neighbors ${}^{k}p^{*}$ of the kth participant. This composition rule, proposed in [Kracík, 2004], complements the tool set needed for the cooperation. Its use in various scenarios is controlled via weights α_p. The specific option follows the pattern of choosing the weight ν.

3. DM in e-democracy: problem peculiarities

Here we try to map the theory sketched in previous sections on the domain of *participatory democracy* that aims at maximal engagement of citizens in governance processes. *E-democracy* is its version that exploits modern communications tools to the same purpose.

In society a multitude of DM tasks is addressed. A democratic organization of society is inevitable, when the complexity of DM crosses capabilities of the fully centralized solution. This is caused by the following reasons, which to be respected by any practically applicable methodology of a societal DM.

- There are the strict *limits on information load* of citizens, [Malhorta, 1982]. People are unable to receive and analyze a large number of data, especially of the different nature [Jacoby, 1984].
- There are the strict *limits on ability to make decisions in the multiple-aims setting*.
- The citizens solve a range of their personal DM problems so that the *effort they can spent on common problems is strictly limited*. Among others, this implies they use simple models on predicting consequences of decisions. Also, the citizens focus on predicting the consequences directly related to their personal aims.

The subsequent discussion, illustrating the use and consequences of the outlined theory, deals with a particular but typical problem of participatory democracy, namely, planning of a municipal budget. It can be directly translated to deciding on any significant change related to the city and its inhabitants. The problem concerns the following stake holders.

A *"global"decision maker* (GDM) is a participant who's decisions or their observable consequences influence, directly or indirectly, a large group of other participants, called here *"local" decision makers* (LDMs). A mayor of city, or city hall as an institution, serves as an example of the global decision maker, while city inhabitants represent local decision makers.

The legal framework to the budget planning is enforced externally. The GDM has, however, a freedom to decide about a part of this budget. The GDM usually uses this freedom to reach its own aims, typically: increasing his popularity, further re-election, personal profit, etc. For this, the GDM may involve into the budget planning LDMs or their subgroups, for instance, members of GDM's party. In this context, an additional (to many existing ones) task for LDMs arises: the attempt to influence the budget, via influencing the GDM.

As the roles of the GDM and LDMs differ, it is useful to discuss the FPD they should address separately.

3.1 GDM perspective

The construction of the specific design elements is outlined below in a static way. In reality, it is an iterative process. At least after creating all elements and proposing the FPD-based strategy, the prediction of its consequences should be inspected, ideally, with LDMs involved in the correction processes. The number of iterations bothering the overloaded LDMs has to be extremely small. Thus, trial iterations and revisions have to modify the GDMs inputs to the GDM satisfaction before the results are presented to the LDMs' final revision.

3.1.1 Quantities forming the behavior. In connection with the *externally given aims of the GDM* (the budget planning), the decision consists of allocation of the free budget part to specific purposes. The choice of the set of competitive alternatives is the decision induced. Both these categories imply a collection of observable consequences. In addition to it, they influence the problem internals. The latter ones are important in connection with the *internally motivated aims of the GDM* (be popular or feared and thus re-elected).

Note that, irrespectively of the motivation, the internal aims are inherent to democratic governing and heavily influence the design of the GDM's strategy. Obviously, the wish to meet the internal aims calls for additional decisions like:

- The decision on the extent to which LDMs will be allowed to influence the budget. For instance, LDMs can be given the possibility to modify the set of alternative spending ways.
- The choice of ways and means for obtaining information about LDMs wishes and preferences (public opinion investigation) as well as the ways of influencing them (meeting, leaflets, passive or active communication via Internet). The form of this investigation should be paid special attention, as it has to respect limited perceiving capabilities of LDMs. Typically, questions on ranges of commonly known quantities can be possed with the freedom to answer only their subset. The information gathered to alternative purposes is to be used. For instance, investigation made by State Statistical Office should give a background picture about the population at large. If it is felt to be useful, it implies that the questions should as much as possible follow standards, for instance, Classification of Individual Consumption by Purpose.
- The gathering of the specific information from LDM and influencing them need additional resources. The gaining of these resources may call for additional decisions (use of a part of the free budget, search for sponsors, etc.).

Generally, any e-democracy DM has the following features:

- Single externally supplied decision problem implies a whole chain of mutually interconnected DM tasks.
- The relevant behavior is indeed a mixture of observations and internals of a quite different physical nature. Some of them have no quantitative value at all (for example, satisfaction with this or that decision). This makes the common probabilistic description invaluable as it makes this "mess" comparable.
- The structured formulation allows the GDM to think about problem data and leaves the choice of the final strategy to an optimizing mechanism.

- The extent and form of the interaction with LDMs have to respect their overload caused by switching between their multiple DM tasks, see Remark, Section 2.1. A sort of experiment design is needed to this purpose [Zarrop, 1979, Curtice and Sparrow, 1997].

3.1.2 Models. In the societal domain, the models needed for expressing relationships of decisions to consequences (4) have to be chosen from a universally approximating class [Haykin, 1994]. In the probabilistic framework, mixture models [Titterington et al., 1985] and Hidden Markov models [Elliot et al., 1995] are relevant general candidate. Parameters of these models extend the set of the problem internals. In their construction, the GDM can offer the expertise on global properties of modeled relationships, typically:

- *Dependence structure of modeled relationships* For instance, "satisfaction" of citizens with governing depends on the crime rate, while does not depend on the wages of the city mayor.

  These knowledge pieces restrict the structures of the model inspected. Without it, learning of a good model is almost hopeless as the number of compared alternatives is too large.

- *(Semi)quantitative statements about modeled relationships* For instance, if the local tax increases between 5 to 10 % the city income will increase in the range 2 and 3 %.

  These knowledge pieces are essentially predictors and thus they can influence significantly the prior distribution of internals through the formula (6) or its foreseen extensions. We claim that this is to be the main tool for translating the domain-specific prior knowledge into pdfs.

The specific peculiarities of e-democracy with respect to GDM are:

- "Translation" of the domain knowledge into pdfs, i.e., automatic knowledge elicitation [Garthwaite et al., 2005], is inevitable and still poorly supported. Obviously, the majority of GDMs cannot afford to leave this translation to highly qualified and thus expensive "translating experts" (statisticians or analyst of various type). The automatic translation could be feasible as various GDMs solving various DM tasks provide structurally similar information pieces. In this sense, the similar situation is met in technical domains [Kárný et al., 2001].

- The need for combination of various pieces of prior knowledge arises.

- Observed quantities – for example, number of votes – are always combined with qualitative internals, say, satisfaction. This urgently calls for improving the support of models with mixed-type data.

- The observation model has to respect that the processed data are always incomplete and noisy. The model of internals has to respect their stochastic nature. For instance, the relationship of satisfaction and wealth is far from being deterministic).
- Models should respect that possibilities of experimental design in society are strongly limited and thus the processed data will be poorly informative. Thus, robustness analysis [Rios-Insua and Rugerri, 2000] is an indispensable part of modeling.

3.1.3 Learning phase. Formally, this DM part consists of collecting data (mostly from LDMs) and inserting them into prepared models. Practically, it demonstrates all known troubles justifying the statement that informative data are expensive. In e-democracy, the most pronounced problems are:

- *LDMs are not responding*. The overload of LDMs is the main cause of this fact. The GDM can counteract it in the previous steps of the design by: i) sending the information at time moments when LDMs are ready to receive messages concerning the GDM task (viz. fight for the prime time in broadcasting), or ii) connect the message passing with regular activities of LDMs containing a substantial portion of idle time (viz. distribution of free newspapers to passengers in public transport).
- *Obtained data are poor*. They are incomplete, of a mixed type (qualitative and quantitative), irregularly sampled (in the real continuous time) and typically under-sampled, etc.

  Many of those features are covered by statistics, signal processing and other research domains, but a systematic tailoring to the participatory democracy waits for its matured realization.

3.1.4 Ideal Pdfs. Construction of the ideal pdf, see Section 2.1, expressing the externally supplied aim, is relatively straightforward. Variants have to meet externally supplied constraints and, moreover, their consequences can be expressed in quantitative terms. Then, a conservative algorithmic modification of the model of the current behavior to the ideal model can be applied [Kárný, 2006].

Similarly, the internal aims can be expressed relatively well via some indirect countable or measurable quantities, say, number of votes instead of citizens “satisfaction" with the governing policy.

The combination of these ideals into a single one represents the real challenge. We conjecture, that the combination way described by (7) is adequate. Just the set ${}^{k}p^{*}$ is to be interpreted as pointers to external and internal ideals.

It can be shown that in simply structured cases the result has mixture character. Generally, however, the combination has much more complicated form, which is neither arithmetic nor geometric poll. In any case, the result is multi-modal one and the adopted FPD provides the compromise among both aims followed.

When using (7), the GDM can influence the compound ideal through the optional weights α by changing the stress on respective aims. It can be done purely subjectively. In a longer run, when a feedback from LDMs is available, the weights can be treated as additional decisions and optimized, too.

It is fair to say that the creation of the compound multi-modal ideal and its subsequent use in the design are computationally hard. The approximate solutions have to be constructed.

3.1.5 Knowledge sharing. The success of the communication of the GDM with LDMs is determined by the degree of feedback from LDMs. Due to the inevitable one-to-many communication structure, generic LDMs provide at most ranges of observable data or of intuitively interpreted notions like happiness or satisfaction. The formula (6) allows processing such data as they can be interpreted as predictors. The receiving GDM decides on the weights ν in conceptually same way as on the weights α when creating its ideal pdf. The (hopefully) large amount of the processed information pieces implies that the GDM can at most assign different weights to several group of answers: a sort of aggregation takes place.

3.1.6 Modification of ideal pdf. The sheer number of LDMs calls for an aggregation of LDM's ideals otherwise the choice of the weights α would be unmanageable. Moreover, the real LDMs can hardly be expected to provide detailed description of their ideals. Again, they provide at most desired ranges of variables in the overlap of behaviors with the GDM. Thus, it is "natural" to use these data and *estimate a collective ideal pdf*. In order to reach a fair compromise, multi-modal nature of preferences has to be respected. This calls for constructing such an ideal pdf as the mixture model. The solution in this vein was proposed in [Kárný and Kracík, 2004]. Its combination with the GDM ideal is then determined by (7) where p points to the GDM's ideal and the estimated collective ideal. The weight α is controlled by the GDM in the way described above.

Let us stress that the proposed "soft" treatment of multiple aims in multi-participants settings avoids the trap of the Arrow's impossibility theorem [Arrow, 1995]. Similar "soft" treatment was proposed in [Nurmi, 2001].

3.2 LDM perspective

The basic situation of a LDM is different as the discussed DM problem is the one of many the LDM faces. The degree of the LDM's participation depends strongly on the significance it assigns to the discussed DM problem. Its reactions may populate the range between the following extremes:

- The LDM ignores anything related to the problem. It may be a wise approach if the real consequences to its domain of interest is small enough.
- The LDM passively observes decisions of the GDM and expresses its attitude only at polls. This solution spares LDM's energy and may be wise when the usual four year period is sufficient for contributing to desirable changes of the governing style.
- The LDM responds to queries of the GDM and provides answers expressing its personal aims. This is based on recognition that the GDM is its almost permanent neighbor – i.e., their DM tasks interact almost permanently – and the GDM has a sufficiently strong influence on important LDM's aims.
- The LDM recognizes the importance of GDM's decisions to its life and find that individually is too weak to influence them. Then, it either join a sort of existing coalition that behaves according to its tastes or even actively tries to create such a coalition. The aim of the coalition is to increase the relative strength of its members with respect to the particular global DM task or, typically, a whole sequence of such tasks.

  Here, the LDM enters the hierarchical framework similar to the discussed one. It may act either as the LDM in a smaller and more homogenous coalition group or even as its GDM.

In all these variants, the discussed facets would re-appear with a different strength and complexity but the design elements and their handling remain conceptually the same.

4. Concluding remarks

We have outlined a mapping of problems e-democracy on a particular formal structure labeled as fully probabilistic design of decision making strategies within the multiple-participants' setting. The gains have been already commented within the text so that we can summarize them by slogans: i) common probabilistic language is proposed covering the DM completely; ii) inherent presence of internal aims within participatory democracy leads to multiple-aims DM; iii) multiple-aims can be address without the need for expensive experts and enumerative type analysis; iv) basic cooperation tools are available; iv) dynamic chaining of DM tasks is always present; v) it is observed that participants solve their multiple personal DM tasks, see Remark in Section 2.1; vi) consequently, the participants have to can be modeled and treated respecting their limited capabilities to pay attention and act with respect to public affairs. The items i), ii) and iii) bring the most important theoretical and algorithmic message of the paper. The items v) and vi) represent the most important practical message of the paper.

The mapping of the advocated methodology on a DM problem within participatory democracy does:

- indicate suitability of the proposed formulation,
- offer a help to researchers dealing with the e-democracy to harmonize solutions of treated subproblems with the overall problem,
- turn the attention of researchers to vital open problems specific to the participatory democracy,
- clarify what is behind the general notions of the FPD,
- open a way to a construction of a generic computer system supporting practitioners who deal with societal DM processes.

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