Multisensor Constrained Estimation with Unscented Transformation

Jiří AJGL Miroslav ŠIMANDL

Department of Cyberetics
Faculty of Applied Sciences
University of West Bohemia
Pilsen, Czech Republic

Introduction

classical state estimation

system described by a stochastic model exactly

state estimation with constraints

system only appoximated by a stochastic model

BUT

some extra knowledge available, for example

- train follows railway
- ship cruises the sea
- mass is non-negative

i.e. some unmodelled physical constraints exist

Problem Formulation

a linear multisensor system is *described* by

$$\begin{aligned} \mathbf{x}_{k+1} &= \mathbf{F}_k \mathbf{x}_k + \mathbf{G}_k \mathbf{w}_k, \\ \mathbf{z}_k^{(j)} &= \mathbf{H}_k^{(j)} \mathbf{x}_k + \mathbf{v}_k^{(j)}, \quad j = 1, \ldots, N, \end{aligned}$$

where $\mathbf{x}_k \in \mathrm{R}^{n_x}$ is the state, $\mathbf{z}_k^{(j)} \in \mathrm{R}^{n_z^{(j)}}$ are local measurement coming from j-th sensor, the noises $\mathbf{w}_k$, $\mathbf{v}_k$ are independent, ...
the system state obeys equality or inequality constraints given by

$$\begin{aligned} c_e(\mathbf{x}_k) &= 0, \\ c_n(\mathbf{x}_k) &\leq 0 \end{aligned}$$

the goal is to combine estimates based on the local data and to respect the constraints

Enforcing Constraints

the model approximates the system dynamics

$\Rightarrow$

the estimate need not to obey the constraints

various approaches to restore the constraints exist

- model projection or reduction
- modification of measurement model
  constraint = additive information, dynamics model considered to be accurate $\Rightarrow$ this approach removes inadmissible states
  - pdf truncation (inequality constraints)
  - pseudomeasurements (equality constraints)
- estimate projection
  $\Rightarrow$ this approach refines the approximation

Proposed Density Interpretation

Problem Formulation

true system state follows unknown densities

$$p(\mathbf{x}_{k+1}^{\dagger}|\mathbf{x}_{k}^{\dagger}), p(\mathbf{x}_{0}^{\dagger})$$

but the set $\mathcal{C}_k$ of admissible system states is known for each time k

$$\mathcal{C}_k = \{\forall \mathbf{x}_k^{\dagger} : p(\mathbf{x}_k^{\dagger}) \neq 0\}$$

and some *approximation* of the system dynamics is available,

$$p(\mathbf{x}_{k+1}|\mathbf{x}_k), p(\mathbf{x}_0)$$

The true measurement probability density is given by

$$p(\mathbf{z}_k^{(1)\dagger}, \ldots, \mathbf{z}_k^{(N)\dagger}|\mathbf{x}_k^{\dagger})$$

The model usually assumes that the density exists also for $\mathbf{x}_k \notin \mathcal{C}_k$,

$$p(\mathbf{z}_k^{(1)}, \ldots, \mathbf{z}_k^{(N)}|\mathbf{x}_k)$$

Proposed Density Interpretation

Enforcing Constraints

- model projection or reduction = transition density transformation

$$\mathrm{C}_S : p(\mathbf{x}_{k+1}|\mathbf{x}_k) \rightarrow p_{\mathcal{C}}(\mathbf{x}_{k+1}|\mathbf{x}_k), \quad p_{\mathcal{C}}(\mathbf{x}_{k+1}|\mathbf{x}_k) \neq 0 \Rightarrow \mathbf{x}_{k+1} \in \mathcal{C}_{k+1}$$

- modification of measurement model – fictive measurement $\mathbf{c}_k$

$$p(\mathbf{x}_k|\mathcal{Z}_k, \mathbf{c}_k) \propto \mathrm{I}_{\mathcal{C}_k}(\mathbf{x}_k) p(\mathbf{x}_k|\mathcal{Z}_k),$$

where $\mathrm{I}_{\mathcal{C}_k}$ is the indicator function, the relation results from

  - fusion with noninformative constraining density

$$p(\mathbf{x}_k|\mathbf{c}_k) \propto \mathrm{I}_{\mathcal{C}_k}(\mathbf{x}_k)$$

  - zero noise measurements $\mathbf{c}_k = 0$

$$p(\mathbf{c}_k|\mathbf{x}_k) = \delta(\mathbf{c}_k - c_k(\mathbf{x}_k))$$

- estimate projection = estimated density transformation

$$\mathrm{C}_k : p(\mathbf{x}_k|\mathcal{Z}_k) \rightarrow p(\mathbf{x}_k|\mathcal{Z}_k, \mathbf{c}_k), \quad p(\mathbf{x}_k|\mathcal{Z}_k, \mathbf{c}_k) \neq 0 \Rightarrow \mathbf{x}_k \in \mathcal{C}_k$$

Fusion Problem

fusion of estimates

local measurements	$\longrightarrow$	local estimates
local estimates	$\longrightarrow$	fused estimate

uncostrained fusion

- fusion of decorrelated information
- computing the dependences of estimates
- respecting the unknown dependences

constrained estimation – the meaning of dependences unclear
$\Rightarrow$ respecting the dependences

Covariance Intersection

- having consistent local estimates $\{\hat{\mathbf{x}}_1, \mathbf{P}_1\}$, $\{\hat{\mathbf{x}}_2, \mathbf{P}_2\}$, the fused estimate $\{\hat{\mathbf{x}}, \mathbf{P}\}$ is consistent, i.e. $\mathbf{P} - \mathrm{E}[(\mathbf{x} - \hat{\mathbf{x}})(\mathbf{x} - \hat{\mathbf{x}})^{\mathrm{T}}] \geq 0$, regardless the dependence of the local estimates

$$\begin{aligned} \mathbf{P}^{-1}\hat{\mathbf{x}} &= \omega \mathbf{P}_1^{-1}\hat{\mathbf{x}}_1 + (1-\omega)\mathbf{P}_2^{-1}\hat{\mathbf{x}}_2, \\ \mathbf{P}^{-1} &= \omega \mathbf{P}_1^{-1} + (1-\omega)\mathbf{P}_2^{-1}, \end{aligned}$$

  for $\omega \in [0, 1]$
- works well for unconstrained estimates (regular covariances)
- equality constrained estimates have singular covariances if a linearised projection have been used

Fusion of Singular Densities

estimates constrained in the same direction

fusion of approximated constrained estimates

$$\left(\begin{bmatrix} 1 & \\ 0 & 0.01 \end{bmatrix}^{-1} + \begin{bmatrix} 2 & 0 \\ 0 & 0.01 \end{bmatrix}^{-1} \right)^{-1} \doteq \begin{bmatrix} 0.666 & 0 \\ 0 & 0.005 \end{bmatrix}$$

the pseudoinverse solution is OK

$$\left(\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}^{-1_{MP}} + \begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix}^{-1_{MP}} \right)^{-1_{MP}} = \begin{bmatrix} \frac{2}{3} & 0 \\ 0 & 0 \end{bmatrix}$$

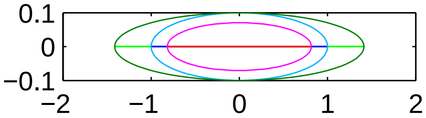


Figure: Covariance ellipses, $\mathbf{x}^{\mathrm{T}}\mathbf{P}\mathbf{x} = 1$ (lines for singular covariances)

Fusion of Singular Densities

estimates constrained in different directions

the approximation gives approximative results

$$\left(\begin{bmatrix}1 & 0\\ 0 & 0.01\end{bmatrix}^{-1} + \begin{bmatrix}0.01 & 0\\ 0 & 1\end{bmatrix}^{-1}\right)^{-1} \doteq \begin{bmatrix}0.01 & 0\\ 0 & 0.01\end{bmatrix} \to \begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$$

while the use of pseudoinverses spoils the fusion

$$\left(\begin{bmatrix}1 & 0\\ 0 & 0\end{bmatrix}^{-1_{MP}} + \begin{bmatrix}0 & 0\\ 0 & 1\end{bmatrix}^{-1_{MP}}\right)^{-1_{MP}} = \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} \gg \begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$$

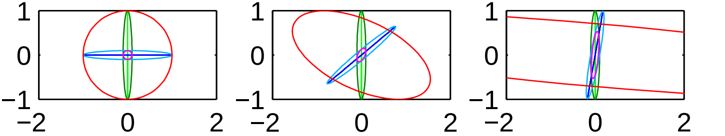
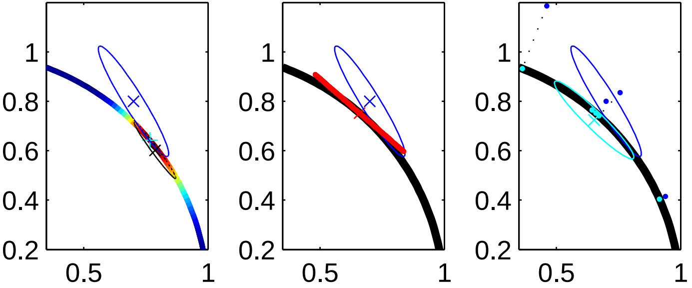


Figure: Closer constrained direction ⇒ worse pseudoinverse solution

Projection by Unscented Transformation

nonlinear equality constraint ⇒ nonlinear projection needed



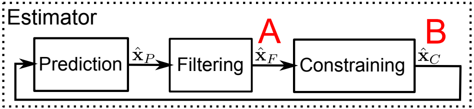
(a) pdf truncation (b) linear projection (c) projection by UT

a) pdf truncation (pseudomeasurements) gives inadequate results
b) linear projection leads to singular covariances
c) unscented transformation respects the constraint nonlinearity

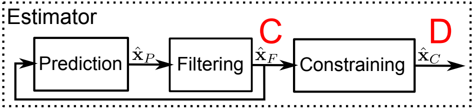
Estimators Architecture

Local Estimators

constraint in the loop

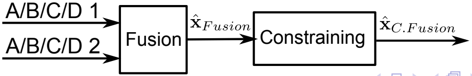


constraint outside the loop



Fusion at the centre

A, C	fusion of unconstrained estimates
B, D	fusion of constrained estimates



Ground Tracking Example

- nearly constant velocity model – state $\mathbf{x} = [x, v_x, y, v_y]^{\mathrm{T}}$
- position measured by two sensors (different error covariances)
- circular road, tangent velocity

$$\begin{aligned} x^2 + y^2 - R^2 &= 0 && \text{position constraint} \\ xv_x + yv_y &= 0 && \text{velocity constraint} \end{aligned}$$

- estimate projection aprroach

$$\pi(\mathbf{x}) = \left[\frac{Rx}{\sqrt{x^2+y^2}} \quad \frac{v_x y^2 - v_y xy}{x^2+y^2} \quad \frac{Ry}{\sqrt{x^2+y^2}} \quad \frac{-v_x xy + v_y x^2}{x^2+y^2} \right]^{\mathrm{T}}$$

- the proposed approaches compared

Example Results

Table: Mean position and velocity errors at local estimator

	C in the loop	C outside the loop
$\bar{\varepsilon}(\hat{\mathbf{x}}_P)_{position/velocity}$	4.34/2.41	*12.78/6.90*
$\bar{\varepsilon}(\hat{\mathbf{x}}_F)_{position/velocity}$	A 3.52/2.45	C *7.65/5.00*
$\bar{\varepsilon}(\hat{\mathbf{x}}_C)_{position/velocity}$	B 3.29/1.34	D 3.76/1.51

Table: Mean position/ velocity errors after the fusion of local estimates

approach	A	B	C	D
$\bar{\varepsilon}(\hat{\mathbf{x}}_{Fusion})_{pos./vel.}$	2.75/2.27	2.40/1.01	*6.88/4.86*	2.91/1.20
$\bar{\varepsilon}(\hat{\mathbf{x}}_{C.Fusion})_{pos./vel.}$	2.49/1.04	2.39/1.01	3.53/1.43	2.91/1.20

+ opened questions (divergence between model and system, incomplete constraint utilisation, projection design)

Summary

- constrained estimation formulated in density framework
- estimate projection preferred to pseudomeasurements
- problems with singular densities referred
- derivative-free solution proposed
- various estimators designed
- their performance compared

Thank you for the attention.