Distributed Bayesian Decision-Making: Early Experiments

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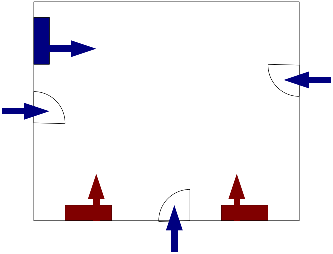
Outline

1. Introduction to Multi-agent Systems
   - Example: room temperature control
   - Theories and issues
2. Distributed Bayesian decision-making
   - Merging of aims
   - Merging of models
3. Experiments
   - Room temperature control

Example: room temperature control

Fictitious room:



Task:

control the room temperature

reliably: failures,

adaptively: changes in the environment

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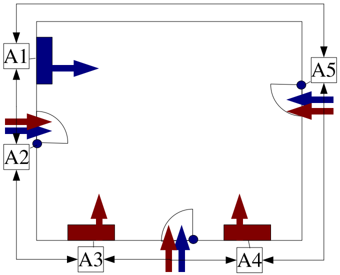

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- scalable: agents can be added
- cheaper: agents in devices
- expensive: in terms of communication
- autonomous: agents follow their *own* aims
- “natural”: living creatures behave this way.

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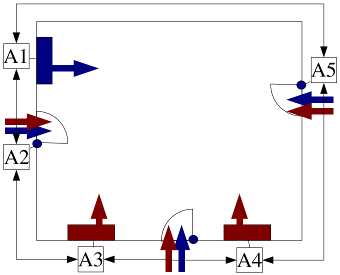

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Keywords: distributed control, *multi-agent systems*, holonic control, autonomous control, etc...

Issues of multi-agent systems

Two **autonomous** agents:

A1 (cooling): aim $10 \pm 1^{\circ}$C

A2 (heating): aim $20 \pm 1^{\circ}$C

What if the current temperature is 18 $^{\circ}$C?

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   Negotiation rules, weights and cost/loss functions.... *intelligence*.

Hard to design these rules, functions, so that these are consistent. The area is dominated by *ad-hoc* and *heuristic* solutions. Verification of design is done via simulation.

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"We need a theory!" – *vice-president of Rockwell Automation, IFAC congress 2005.*

Theories of multi-agent systems

Many theoretical results available based on:

1) Predicate logic,
2) Game theory,
3) Algorithmic information theory.

Provide guarantees of optimality at the cost of:

1. and 2. underrating of uncertainty,

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Underrating the importance of uncertainty in the model may be dangerous, e.g. when we are trying to control variables we do not observe.

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Proper combination of Bayesian decision-making with game theory is not known to us. We propose a heuristic extension of the classical Bayesian theory.

Standard Bayesian decision-makers

Standard approach:

Model: probability density,

$$y_t \sim \mathcal{N}\left(ay_{t-1} + u_t, 1\right).$$

Loss: function of observations,

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Fully probabilistic approach:

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Negotiation: (exchange of knowledge and aims)

- Standard approach: communication of loss functions and their shaping.
- Fully probabilistic approach: communication of ideal densities and their combination. $\Leftarrow$ same calculus, optimization of KL.

Merging of Aims

Aims of participants:

A1: target temperature is ${}^{\lfloor I}f_1(T) = \mathcal{N}(10, 1)$,

A2: target temperature is ${}^{\lfloor I}f_2(T) = \mathcal{N}(20, 1)$,

New target: a common distribution close to both aims.
Linear combination:

$$
\begin{aligned}
\tilde{f}(T) &= \frac{1}{2}\mathcal{N}(10,1) + \frac{1}{2}\mathcal{N}(20,1)\,, \\
&\approx \mathcal{N}(15,26)\,,
\end{aligned}
$$

Geometric combination:

$$
\begin{aligned}
\tilde{f}(T) &= \mathcal{N}(10,1)^{\frac{1}{2}}\,\mathcal{N}(20,1)^{\frac{1}{2}}\,, \\
&= \mathcal{N}(15,1)\,.
\end{aligned}
$$

Merging of Models

Much more demanding, since the agents work with different data, different parameters, etc.

Rule: agents exchange density on variables that are known to both of them.

Optimization results:

$$\tilde{f}\left(\Theta_t | d^{1:t}\right) = f\left(\Theta_t | d^{1:t}\right) \exp\left(\int M(\Psi) \log f\left(d_t | \Theta_t\right)\right) d\Theta_t.$$

Experimental room

Fictions room:

$$y_t = ay_{t-1} + by_{t-2} + u_t - v_t + e_t.$$

Two agents A1 and A2:

A1: assigning values of u_t with model:

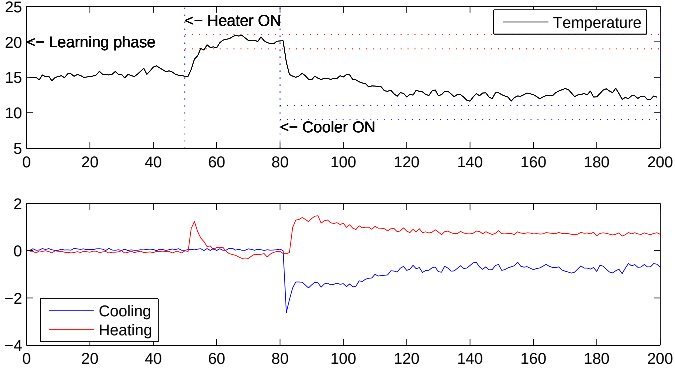
$$y_t = ay_{t-1} + by_{t-2} + u_t \quad + e_t.$$

A2: assigning values of v_t with model:

$$y_t = ay_{t-1} + by_{t-2} \quad - v_t + e_t.$$

Unaware of each others presence by design. Can they cooperate?

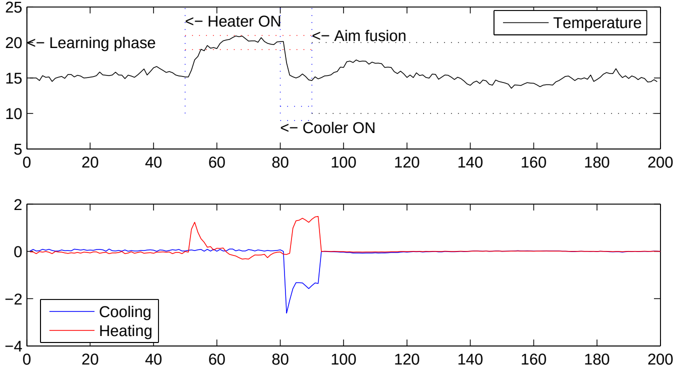
Standard autonomous decision-makers



Initially they push against each other, wasting a lot of energy. Then, they give up a bit. They have *learnt* that their actions has smaller effect then expected, and due to penalization of power they decrease their effort.

Synchronization of aims via linear combination

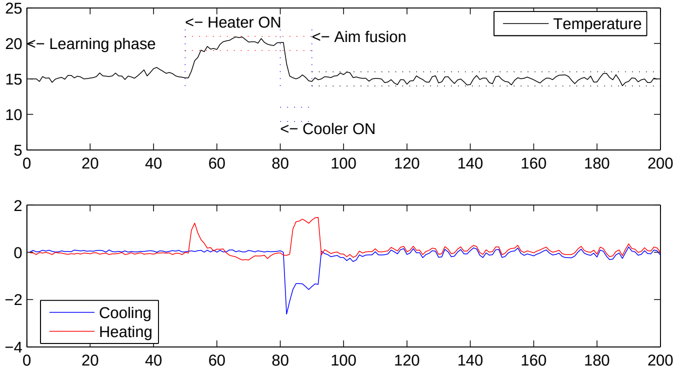
Linear fusion of aims is optimal in terms of preserving information.



Drops of input power due to wider range set for aims.

Synchronization of aims via geometric combination

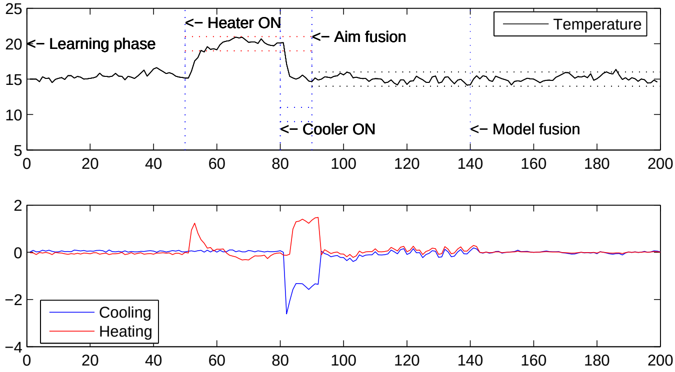
Geometric combination



More narrow aim, agents fully cooperate.

Synchronization of aims via geometric combination

Merging of models



Even lower input power. Models are more unified. Further decrease of beliefs in agents' influence on the environment.

Conclusion

- Distributed Bayesian decision making is an attempt to extend Bayesian theory of decision-making for multiple entities with *limited* abilities.
- Non-standard probabilistic operations are needed for exchange of knowledge and aims.
- Current experiments suggest that the approach is sensible, and indicate directions for more theoretical work.
- Future:
  - more complex systems (more agents, challenging models),
  - negotiation scenarios,
  - heterogenous enironments,
  - theoretical results of optimality.