# Gaussian Mixtures Proposal Density in Particle Filter for Track-Before-Detect

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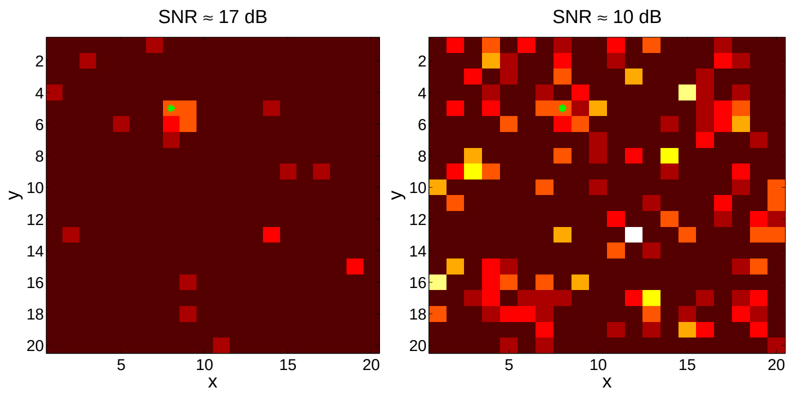
## Introduction to track-before-detect problem

- Classical tracking approaches estimating target state consider measurements, typically position, range, bearing.
- The measurements are extracted by thresholding from the output of sensor signal processing unit.
- However, these approaches are not suitable for tracking targets with low Signal-to-Noise Ratio (SNR), where thresholding has an undesirable effect to disregarding potentially useful data.
- To track low SNR targets, the tracking approach working with raw (unthresholded) data is used. This approach for simultaneous target detection and tracking is known as Track-Before-Detect (TBD) approach.

## Simulated output data with different SNR



## Target state

The state of the target is given by

$$\mathbf{x}_k = [x_k, y_k, \dot{x}_k, \dot{y}_k, I_k]^T,$$

where

- $(x_k,\ y_k)$ and $(\dot{x}_k,\ \dot{y}_k)$ are position and velocity in $x$ and $y$ directions, and
- $I_k$ is return target intensity.

## State equation

The target state evolves according to discrete-time model

$$\mathbf{x}_{k+1} = \mathbf{F}\mathbf{x}_k + \mathbf{e}_k,$$

where

- $\mathbf{F}$ is known transition matrix and
- $p(\mathbf{e}_k) = \mathcal{N}\{\mathbf{e}_k : \mathbf{0}, \mathbf{Q}\}$ with known $\mathbf{Q}$.

Both matrices $\mathbf{F}$ and $\mathbf{Q}$ depend on the sampling period $T$.

## Measurement equation

- The measurement is obtained as a sequence of images consisting of $n_x \times n_y$ cells, i.e. $\mathbf{z}_k = \{z_k^{(i,j)}\}_{i=1,j=1}^{n_x,n_y}$.
- Each cell represents measured intensity and contains a contribution of the target $h^{(i,j)}(\mathbf{x}_k)$ and noise $v_k^{(i,j)}$

$$z_k^{(i,j)} = \begin{cases} h^{(i,j)}(\mathbf{x}_k) + v_k^{(i,j)}, & \text{if target present,} \\ v_k^{(i,j)}, & \text{if target not present.} \end{cases}$$

## Aim of TBD problem

The aim of the track-before-detect problem is to find the filtering probability density function (pdf)

$$p(\mathbf{x}_k, E_k = 1 | \mathbf{z}^k) = ?$$

where $E_k = 1$ represents presence of the target and $\mathbf{z}^k = [\mathbf{z}_0, \ldots, \mathbf{z}_k]$.

## Particle filter for TBD approach

- The basic idea of the Particle Filter (PF) in nonlinear state estimation is to approximate the pdf by an empirical pdf, which is given by $N$ random samples of the state $\{\mathbf{x}_k^{(i)}\}_{i=1}^N$ with associated weights $\{w_k(\mathbf{x}_k^{(i)})\}_{i=1}^N$.
- In the TBD approach, the particles are divided at each time instant $k$ into two groups:
    - "alive" particles (target exists, $E_k = 1$),
    - "dead" particles (target does not exist, $E_k = 0$).
- For "alive" particles the state part $\mathbf{x}_k$ is drawn from several proposal densities.
- For "dead" particles the state part $\mathbf{x}_k$ is not defined.
- The existence variable $E_k$ together with corresponding state part $\mathbf{x}_k$ form the extended state $\tilde{\mathbf{x}}_k$.

## Fundamental steps of PF algorithm for TBD

- Sampling:
  - Several alive particles will die ($E_k^{(i)} = 0$).
  - Remaining alive particles survive with $E_k^{(i)} = 1$ and their state part is drawn from the proposal

$$\mathbf{x}_k^{(i)} \sim \pi(\mathbf{x}_k | \mathbf{x}_{k-1}^{(i)}, E_{k-1}^{(i)} = 1, \mathbf{z}_k).$$

  - Several dead particles remain dead ($E_k^{(i)} = 0$).
  - Remaining dead particles will be born with $E_k^{(i)} = 1$ and their state part is drawn from the proposal

$$\mathbf{x}_k^{(i)} \sim \pi_b(\mathbf{x}_k | E_{k-1}^{(i)} = 0, \mathbf{z}_k).$$

- Weighting: The particles are weighted according to the last measurement $\mathbf{z}_k$.
- Resampling: A new set of samples, where all particles have the same weight, is generated.

## Standard choice of proposal densities

- In the TBD approach there are two proposal densities:
  - $\pi(\mathbf{x}_k|\mathbf{x}_{k-1}^{(i)}, E_{k-1}^{(i)}=1, \mathbf{z}_k) = \pi$ for the surviving particles, and
  - $\pi_b(\mathbf{x}_k^{(i)}|E_{k-1}^{(i)}=0, \mathbf{z}_k) = \pi_b$ for the newborn particles.
- As far as the proposal $\pi$ is concerned, the simplest proposal (transition pdf) $\pi = p(\mathbf{x}_k|\mathbf{x}_{k-1}, E_k = 1, E_{k-1} = 1)$ is usually used (Rutten et al., 2005).
- Concerning the proposal $\pi_b$, there are following possibilities:
  - the proposal spreads the particles uniformly in the state space,
  - the proposal uses available measurements $\mathbf{z}_k$ - particle position is distributed uniformly within $N_c$ highest intensity cells, remaining particle components are distributed uniformly,
  - combination of previous two approaches.
- Thus, newborn particles are standardly drawn from uniform prior distribution $\pi_b = p_b(x_k, y_k, \dot{x}_k, \dot{y}_k, I_k)$.

## Goal of the Paper

The goal of the paper is to present another proposal density $\pi_b = p_b(\mathbf{x}_k|\mathbf{z}_k)$ for the newborn particles, which achieves higher estimation quality with comparable computational demands as the standard proposals.

## Basic idea

- Novel design of the proposal density $\pi_b$ is based on utilisation of more information from the measurement.
- More information is extracted by means of a nonlinear filter, namely Gaussian Mixture (GM) filter.
- The novel proposal $\pi_b = p_b(\mathbf{x}_k|\mathbf{z}_k)$ is denoted as GM proposal.

## Proposal for newborn particles

$$\pi_b = p_b(x_k, y_k) p_b(\dot{x}_k, \dot{y}_k) p_b(I_k),$$

In stage of track initiation

- position, velocity, and intensity are independent,
- velocity components do not influence the measurement, therefore there will be still drawn from prior pdf $p_b(\dot{x}_k, \dot{y}_k)$,
- however, for position and intensity, the measurement can be used to obtain posterior proposal $\pi_b = p_b(x_k, y_k, I_k|\mathbf{z}_k)$.

## Posterior proposal design for newborn particles

- The prior proposal $\pi_b = p_b(x_k, y_k, I_k)$ is considered to be uniform within a whole area covered by measurement, i.e.
  - $[0, n_x]$ and $[0, n_y]$ for x and y directions and
  - $[I_{min}, I_{max}]$ for intensity.
- The prior pdf $p_b(x_k, y_k, I_k)$ can be approximated by a GM with $N$ terms

$$\widehat{p}_b(x_k, y_k, I_k) = \frac{1}{N}\sum_{i=1}^{N} \mathcal{N}\left\{\begin{bmatrix} x_k \\ y_k \\ I_k \end{bmatrix} : \begin{bmatrix} \hat{x}'_{i,k} \\ \hat{y}'_{i,k} \\ \hat{I}'_{i,k} \end{bmatrix}, \mathbf{P}'_k \right\},$$

where $\left[\hat{x}'_{i,k}, \hat{y}'_{i,k}, \hat{I}'_{i,k}\right]^{\mathrm{T}}$ is a position and intensity grid point with covariance matrix $\mathbf{P}'_k$.

## Posterior proposal design for newborn particles (cont'd)

- Each prior grid point is transformed through the Extended Kalman Filter (EKF) and weighted.
- The posterior GM pdf is then given by

$$p_b(x_k, y_k, I_k|\mathbf{z}_k) = \sum_{i=1}^{N} \alpha_{i,k} \mathcal{N}\left\{ \begin{bmatrix} x_k \\ y_k \\ I_k \end{bmatrix} : \begin{bmatrix} \hat{x}_{i,k} \\ \hat{y}_{i,k} \\ \hat{I}_{i,k} \end{bmatrix}, \mathbf{P}_{i,k} \right\},$$

  where filtering means and covariance matrices
  - $\begin{bmatrix} \hat{x}_{i,k} \\ \hat{y}_{i,k} \\ \hat{I}_{i,k} \end{bmatrix} = \begin{bmatrix} \hat{x}'_{i,k} \\ \hat{y}'_{i,k} \\ \hat{I}'_{i,k} \end{bmatrix} + \mathbf{K}_{i,k}(\mathbf{z}_k - \mathbf{h}\left(\begin{bmatrix} \hat{x}'_{i,k} \\ \hat{y}'_{i,k} \\ \hat{I}'_{i,k} \end{bmatrix}\right)),$
  - $\mathbf{P}_{i,k} = (\mathbf{I} - \mathbf{K}_{i,k}\mathbf{H}_{i,k})\mathbf{P}'_k$, and $\mathbf{K}_{i,k}$ is the Kalman gain.
- Position and intensity components will be drawn from posterior proposal $\pi_b = p_b(x_k, y_k, I_k|\mathbf{z}_k)$.

## Computational efficiency

Causes of high computation requirements:

- relatively large number of grid points $N$ to sufficiently cover a whole space of admissible positions and intensity,
- the measurement contains $n_x \times n_y$ elements processed by each EKF (the Jacobian $\mathbf{H}_{i,k}$ has to be evaluated).

## Reduction of computational demands

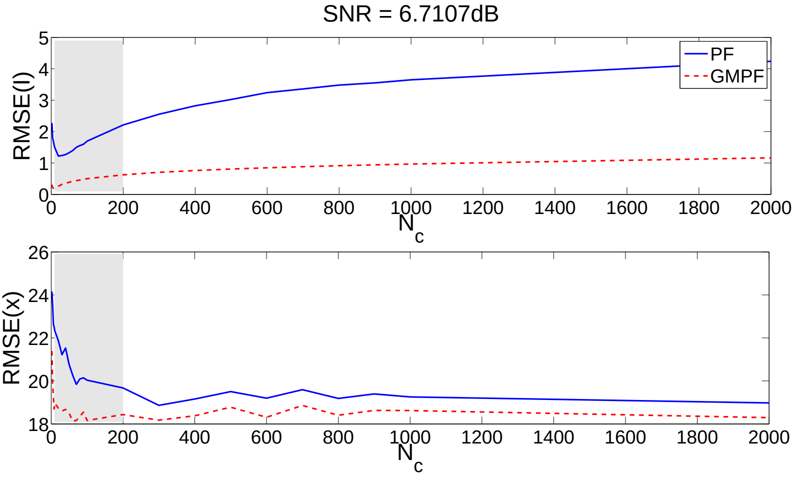
Possibilities to reduce computational requirements:

- reduction of processed measurements - target influences the cells in vicinity and only these measurements are processed,
- reduction of EKF's - grid points are used to cover $N_c$ highest intensity cells only,
- precomputation - Jacobians, Kalman gains, and filtering covariance matrices can be precomputed.

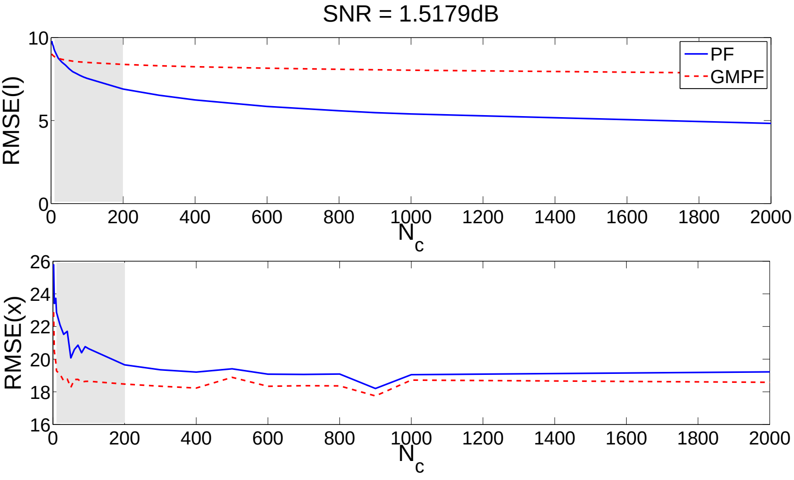
## Outline of numerical experiment

- PF with uniform proposal $\pi_b = p_b(x_k, y_k, I_k)$ and the GMPF with GM proposal $\pi_b = p_b(x_k, y_k, I_k|\mathbf{z}_k)$ were compared.
- Comparison was based on filter performance when the target appeared in the scene.
- Performance was measured in terms of Root Mean Square Error (RMSE) for position and intensity components by means of 1000 Monte Carlo experiments.
- Each frame of data consisted of $n_x = n_y = 64$ cells and the SNR was between 0.69 and 10.23dB.

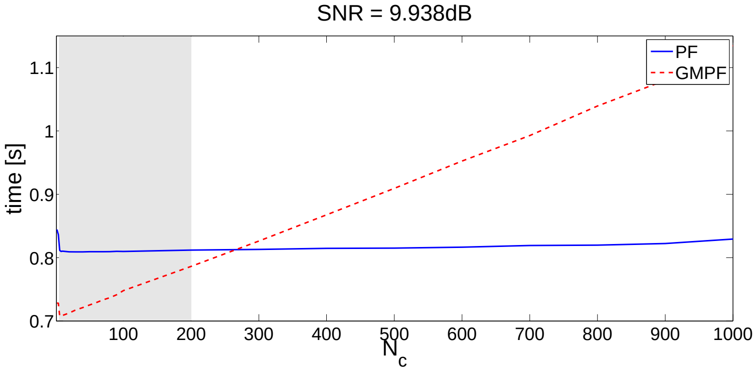
## Root mean square error



# Root mean square error (cont'd)



## Computational time



## Concluding remarks

- The paper dealt with the track-before-detect problem.
- Novel proposal density design for newborn particles was presented.
- The proposal utilises more information from available measurement using bank of the EKF's.
- Resulting target state estimates achieve better quality than estimates based on standard proposal density with comparable computational demands.