APPLICATION OF THE KALMAN FILTER IN TRAFFIC

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Abstract: In this paper, we evaluate the use of extended Kalman filtr (EKF), its inovation known as DD2, and Unscented Kalman Filtr (UKF) for real-time estimation of state-space model for traffic control. The model is non-linear since it contains non-linear relation between queue length and occupation, and unknown parameters. Hence, the standard Kalman filter is not suitable and we have to use EKF and its inovations for estimation of the traffic model.

Keywords: traffic control, non-linear estimation, EKF

1. INTRODUCTION

In majority of cities, the number of cars in streets is on the increase but the current structure of the urban road network is not sufficient. In historical heart of the city, construction of new roads is impossible. We approach this problem using better traffic control.

Our approach is to increase capacity of crossroads, since they are critical points in the traffic network. For the increase of capacity of crossroads, an algorithm was developed using the criteria of total queue length reduction, i. e. sum of queues in each arm of all crossroads. It is described in Kratochvílová and Nagy (2004a); Kratochvílová and Nagy (2004b).

At present, Kalman filter can be used to estimate model parameters because the current model is simplified by several limiting condition. At least two of these conditions—(i) the linear relation between queue length and occupation, and (ii) unknown input/output—can be relaxed. Then, some form of the extended Kalman filtr must be used to estimate the model.

2. STATE SPACE MODEL OF TRAFFIC MICROREGION

State of the model is composed of: (i) queue length, (ii) intensity of traffic flow, and (iii) occupancy. Observability of the model is achieved by introduction of relations between queue length, intensity and occupancy.

The state vector for this model is

$$x_{t+1} = [\xi_{1;t+1}, \xi_{1;t}, \xi_{2;t+1}, \xi_{2;t}, \tilde{I}_{1;t}, \tilde{I}_{2;t}, \tilde{O}_{1;t}, \tilde{O}_{2;t}, \tilde{I}_{3;t}]' \quad (1)$$

where:

ξ is a queue length,
$\tilde{I}$ is a deviation from typical day intensity,
$\tilde{O}$ is a deviation from typical day occupancy.

The deviation from typical day intensity or occupancy is defined as:

$$\tilde{I}_{k;t+1} = I_{k;t+1} - \bar{I}_{k;t+1}, \quad (2)$$
$$\tilde{O}_{k;t+1} = O_{k;t+1} - \bar{O}_{k;t+1}. \quad (3)$$

Where:
$I(O)$ is the current intensity (occupancy),
$\bar{I}(\bar{O})$ is a typical day intensity (occupancy),
k is an index of arms in crossroads.

Using (1), the state model can be written as

$$x_{t+1} = A_t x_t + B_t \begin{bmatrix} z_{1;t} \\ z_{2;t} \end{bmatrix} + F_t + e_t \quad (4)$$

where
the matrices A, B and F are displayed in figure 1,
z_t is green time,
e_t is the noise.

δ- function is used to determine type of state on an arm of a crossroad. We consider the following options:

(1) $\delta = 0$, if the capacity of the crossroad is bigger than number of cars entering the crossroad,
(2) $\delta = 1$, in other cases.

Simply said, if all cars on one arm go through the crossroads in the period of green, then $\delta = 0$. If some car remain before the crossroad when the red comes, then $\delta = 1$. The value of each δ-function is set deterministically using estimates of the queue length.

Intensity and occupancy are measured in real time by road detectors. The vector of measurements is

$$Y_{5;t} = \begin{bmatrix} I_{1;t+1} \\ I_{2;t+2} \\ O_{1;t+1} \\ O_{2;t+2} \\ y_{5;t} \end{bmatrix}. \quad (5)$$

Where:

$y_{5;t}$ is output from the crossroad.

Using (5), the output equation is given by

$$Y_{5;t} = C x_{t+1} + G_t + e_t, \quad (6)$$

where:

$$C = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ -\alpha_{14} & \alpha_{14} & -\alpha_{24} & \alpha_{24} & \alpha_{14} & \alpha_{24} & 0 & 0 & \alpha_{35} \end{bmatrix},$$

$$G_t = \begin{bmatrix} \bar{I}_{1;t+1} \\ \bar{I}_{2;t+1} \\ \bar{O}_{1;t+1} \\ \bar{O}_{2;t+1} \\ \alpha_{14}\bar{I}_{1;t+1} + \alpha_{24}\bar{I}_{2;t+1} + \alpha_{35}\bar{I}_{3;t+} \end{bmatrix},$$

α is the ratio of cars which go from arm i to arm j in the total number of cars on the input
e_t is the output noise.

3. THE EXTENDED KALMAN FILTER

The Kalman filter is a well known tool for estimation of parameters of linear stochastic system. However, it can not be used when the relations between parameters are nonlinear or we do not know all input/output for at some time. For this purpose, the Kalman filter was extended and is known as the Extended Kalman Filter (EKF) Julier and Uhlmann (1997); Maybeck (1993); Welch and Bishop (2004).

Using EKF, we can estimate parameters of nonlinear model using linearization of partial derivatives of the model function.

Here, we define the basic stochastic difference equation as follows,

$$x_{t+1} = f(x_t, u_t, v_t), \quad (7)$$

with a measurement equation,

$$y_t = h(x_t, u_t, e_t). \quad (8)$$

Here:
v_t is the process noise,
e_t is the measurement noise.

The nonlinear function f in equation (7) relates the state x in time t and $t+1$. The nonlinear function h in equation (8) relates the state x to the measurement y in time t.

Translation of our basic equations for the traffic control to the non-linear notation is not so transparent because the equations must be linearized using partial derivatives. It can be done as follows:

$$x_{t+1} \approx \tilde{x}_{t+1} + A(x_t - \hat{x}_t) + V v_t, (9)$$

$$y_t \approx \tilde{y}_t + H(x_t - \tilde{x}_t) + E e_t. \quad (10)$$

Here:

$$A_t = \begin{bmatrix} \delta_{1;t} & \delta_{1;t}\kappa^I_{1;t} & 0 & 0 & \delta_{1;t}\beta^I_{1;t} & 0 & \delta_{1;t}\omega^I_{1;t} & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \delta_{2;t} & \delta_{2;t}\kappa^I_{2;t} & 0 & \delta_{2;t}\beta^I_{2;t} & 0 & \delta_{2;t}\omega^I_{2;t} & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \kappa^I_{1;t} & 0 & 0 & \beta^I_{1;t} & 0 & \omega^I_{1;t} & 0 & 0 \\ 0 & 0 & 0 & \kappa^I_{2;t} & 0 & \beta^t_{2;t} & 0 & \omega^I_{2;t} & 0 \\ 0 & \kappa^O_{1;t} & 0 & 0 & \omega^O_{1;t} & 0 & \beta^O_{1;t} & 0 & 0 \\ 0 & 0 & 0 & \kappa^O_{2;t} & 0 & \omega^O_{2;t} & 0 & \beta^O_{2;t} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix},$$

$$B_t = \begin{bmatrix} -\delta_{1;t}S_1 & 0 \\ 0 & 0 \\ 0 & -\delta_{2;t}S_2 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \; F_t = \begin{bmatrix} \delta_{1;t}\bar{I}_{1;t} \\ 0 \\ \delta_{2;t}\bar{I}_{2;t} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix},$$

Figure 1. Matrix of state space model

x_t and y_t are the actual state and measurement vectors, respectively,
$\tilde{x}_t$ and $\tilde{y}_t$ are the approximate state and measurement vectors,
$\hat{x}_t$ is the a posteriori estimate of state in time t,
v_t and e_t are process and measurement noises in time t, respectively,
A is the Jacobian matrix of partial derivates of f with respect to x,
$A_{[i,j]} = \frac{\partial f_{[i]}}{\partial x_{[j]}}(\hat{x}_t, u_t, 0)$,
V is the Jacobian matrix of partial derivatives of f with respect to w,
$V_{[i,j]} = \frac{\partial f_{[i]}}{\partial v_{[j]}}(\hat{x}_t, u_t, 0)$,
H is the Jacobian matrix of partial derivatives of h with respect to x,
$H_{[i,j]} = \frac{\partial h_{[i]}}{\partial x_{[j]}}(\tilde{x}_t, 0)$,
E is the Jacobian matrix of partial derivatives of h with respect to v,
$E_{[i,j]} = \frac{\partial h_{[i]}}{\partial e_{[j]}}(\tilde{x}_t, 0)$.

Using the above equations, time update and measurement update equations can be formulated. It is necessary to recalculate the prediction error, process error, etc., as described in Welch and Bishop (2004).

Time update equations are given by:

$$\hat{x}^{\triangleleft}_{t+1} = f(\hat{x}_t, u_t, 0), \tag{11}$$
$$P^{\triangleleft}_{t+1} = A_{t+1}P_tA^T_{t+1} + V_{t+1}Q_tV^T_{t+1}. \tag{12}$$

Here:
$\triangleleft$ is a priori knowledge,
P is estimate error covariance,
P is process noise covariance,

Measurement update equations are:

$$K_t = P^{\triangleleft}_t H^T_t (H_t P^{\triangleleft}_t H^T_t + E_t R_t V^T_t)^{-1}, \tag{13}$$
$$\hat{x}_t = \hat{x}^{\triangleleft}_t + K_t(y_t - h(\hat{x}^{\triangleleft}_t, 0)), \tag{14}$$
$$P_t = (I - K_tH_t)P^{\triangleleft}_t. \tag{15}$$

Here:
K_t is the Kalman gain,
R is the covariance of the output noise.

The standard state space model has not the input in the measurement equation, but in the traffic state space model has one. It is not problem for the EKF. This algorithm is quick and exact and is described very often.

The main problem of this filter is cause by the linearization operation which may transform stable nonlinear relation into unstable linear relations. This is not acceptable for the task of traffic control. We require the system to be stable at all times.

Therefore, this filter was found to be unsuitable and we proceed with testing UKF and DD2 filters.

4. THE UKF AND THE DD2 FILTER

4.1 *The Unscented Kalman Filter*

In the EKF, the state is approximated via a Gaussian distribution using linearization. The Unscented Kalman Filter (UKF) solves this problem using deterministic sampling approach. The state is also approximated by a Gaussian distribution but a it requires to evaluate minimal set of points which are selected according to given rules. Computational complexity of the UKF is comparable to that of EKF.

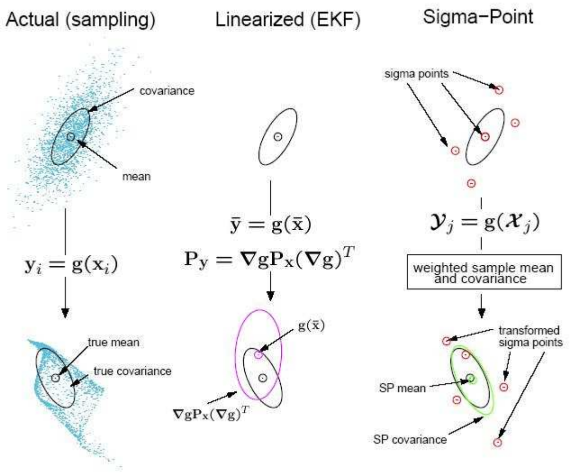

Figure 2. Comparison of nonlinear transformations.

The UKF is based on unscented transformations (UT). This is a method for calculating statistics of a random variable under nonlinear transformation. We assume that $\boldsymbol{x}$ has mean value $\overline{\boldsymbol{x}}$ and covariance $\boldsymbol{P_x}$. The statistics are defined by a matrix $\boldsymbol{\chi}$ of $2L+1$ sigma vectors χ_i as follows:

$$\begin{aligned} \chi_0 &= \overline{\boldsymbol{x}}, & W_0 &= \kappa/(n+\kappa), \\ \chi_i &= \overline{\boldsymbol{x}} + \boldsymbol{\chi}_i, & W_i &= 1/(2(n+\kappa)), \\ & & & \quad i = 1, 2, \cdots, n \qquad (16) \\ \chi_i &= \overline{\boldsymbol{x}} - \boldsymbol{\chi}_i, & W_i &= 1/(2(n+\kappa)), \\ & & & \quad i = n+1, \cdots, 2n+1 \end{aligned}$$

Comparison of UT with another trasformations is displayed in Figure 2.

The time update and measurement update equations and each derivation can be found in Wan and van der Merwe; van der Merwe (2004).

The UKF contains a tuning parameter κ. It can be shown, that optimal value of this paramater for a Gaussian distribution is $n + \kappa = 3$, where n is dimension of the state, Duník (2005). However, for $n > 3$ the coefficient $\kappa$is negative. This situation can cause instability of the algorithm. Extra innovations of the UKF were developed to avoid this problem. However, it is also possible to set $\kappa = 0$. Then, we can use the standard UKF without further inovations.

4.2 *The DD2 Filter*

The DD2 filter for nonlinear estimation is based on Taylor approximations. This filter is based on the idea of the DD1 filter, which is described in Schei (1997). The difference between the DD1 and DD2 filters is in the order of approximation. The DD1 filter is based on first-order approximation and the DD2 is based on second-order approximation. The filter works with general nonlinear model of a dynamic system,

$$x_{t+1} = f(x_t, u_t, v_t),$$
$$y_t = h(x_t, e_t),$$

where: v_t is process noise,
e_t is measurement noise.

In principle, the DD2 filter corresponds to the EKF except that the Jacobians (9), (10) are replaced by divided differences. The state update is therefore the same as in the EKF. The difference can also be found in the update of various covariance matrices.

In the DD2 filter, all matrices are decomposed into Cholesky factors . Then, evaluation of filtration and prediction covariance matrices has the advantage in much smaller computational demands. All operations of the filter are defined directly on the Cholesky factorsNorgaard et al. (1998); Duník (2005).

The only problem with this algorithm is in definition of a nonlinear model of a dynamic system. Our model has an input in the measurement equation which is not present in the model of the DD2 filter. This can be solved by transformation of the traffic model.

5. CONCLUSION

In this paper, we presented tree filters for nonlinear state estimation of the state-space model for traffic control. The EKF algorithm is simple but it can become unstable. This is not acceptable and two more sophisticated filters were tested. First, the UKF is based on sampling, and its computational demands is comparable to that of EKF. Second, the DD2 filter which replaces the derivation by divided differences.

It appears that neither UKF or DD2 can be directly applied to our problem. Further modifications of these filters are needed.

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